CS131: Higher-order functions

Due at the beginning of class on Monday, February 6th

Name: _____

CAS ID (e.g., abc01234@pomona.edu): _____

I encourage you to collaborate. Please record your collaborations below.

I encourage you to work these problems by hand, on the board. You'll learn the most, not to mention getting the best practice for exams.

Most solutions using higher-order functions be written in a single-line. Some solutions may take as many as four or five lines, but any more and you're off the scent.

Feel free to use Prelude definitions that *help*... but don't make the question trivial.

Each question is worth one point.

Collaborators: _____

1 Maps

1.1 Say hello!

The function `sayHello` takes a list of names and produces a list of greetings, as in:

```
> sayHello []
[]
> sayHello ["Yuji","Emiliano","Helen"]
["Hello, Yuji!","Hello, Emiliano!","Hello, Helen!"]
```

Write the type of `sayHello`.

Write two implementations of `sayHello`: one using natural recursion and one using `map`.

1.2 Make it a double

The function `doubleAll` takes a list of arithmetic expressions (as in HW02) and produces another list of arithmetic expressions that will evaluate to double the value. For example:

```
> doubleAll [Num 3,Neg (Num 5)]
[Times (Num 2) (Num 3),Times (Num 2) (Neg (Num 5))]
```

Write the type of `doubleAll`.

Write two implementations of `doubleAll`: one using natural recursion and one using `map`.

2 Filters

2.1 It's all relative

The function `relPrime` takes a number and a list of numbers, returning those numbers in the list that are relatively prime, e.g.:

```
> relPrime 5 [1..10]
[1,2,3,4,6,7,8,9]
> relPrime 2 [1..10]
[1,3,5,7,9]
```

There are many different types we can assign to `relPrime`; write a type that will allow at least the above examples to work.

Write two implementations of `relPrime`: one using natural recursion and one using `filter`.

2.2 A rock and a hard place

The function `between` takes a lower and upper bound and a list, returning those elements of the list that are between (inclusive) the two bounds.

```
> between 5 7 [1..10]
[5,6,7]
> between 'g' 'l' ['a'..'z']
"ghijkl"
```

Write the type of `between`.

Write two implementations of `between`: one using natural recursion and one using `filter`.

3 Folds

Many functions in the Prelude reference `Foldable t`; for the purposes of this homework, please just use lists.

3.1 Long story short

The `length` function computes the length of a list, returning an `Int`. Write three versions of `length`: one using natural recursion, one using `foldr`, and one using `foldl`.

3.2 It's so nice we'll say it twice

The function `stutter` takes a list and returns a list with each item appearing twice:

```
> stutter []
[]
> stutter [1..4]
[1,1,2,2,3,3,4,4]
```

Write two versions of `stutter`: one using natural recursion, one using `foldr`.

3.3 You've got this down backwards and forwards

Write `reverse` four ways: using natural recursion, using accumulating recursion, using `foldr`, and using `foldl`.

3.4 Two great tastes that taste great together

Write two implementations of `concatMap :: (a -> [b]) -> [a] -> [b]`: one using natural recursion and one using `foldr`. Make sure you pass over the list only once.

3.5 Two great tastes that taste weird together

Just as `concatMap f` behaves like `concat . map f` (but passes over the list only once), the function `reverseMap f` behaves like `reverse . map f` (but passes over the list only once). Write two implementations of `reverseMap :: (a -> b) -> [a] -> [b]`: one using natural recursion and one using `foldl`.

3.6 Better keep 'em separated

In `Data.List`, the function `intercalate :: [a] -> [[a]] -> [a]` is useful for text processing, as in:

```
> intercalate ", " ["A one","a two","a one two three"]
"A one, a two, a one two three"
```

Write two implementations of `intercalate`: one using natural recursion and one using `foldr`.

4 Composing higher-order functions

In these questions, you can't define your function *directly in terms* of a higher-order function. You might have to use more than one higher-order function to get the answer, or you might have to pre- or post-process your data.

4.1 Saying it twice, again

Write a version of `stutter` (Problem 3.2) that uses `map` and `concat`.

4.2 Conjunction junction

Write a version of the Prelude function `all :: (a->Bool) -> [a] -> Bool` using `map`, `filter`, and `length`.

4.3 Ducks in a row

Write a function `isSorted` that determines whether a list is sorted ascending:

```
> isSorted [1..10]
True
> isSorted ['a'..'z']
True
> isSorted "algebra"
False
> isSorted "sty"
True
```

Use `foldl` (and whatever else is handy). Your solution should be $O(n)$ and pass over the list only once.

4.4 Mean means average

Write a function `mean` that computes the arithmetic mean of a list of numbers. There are many ways to write this function... so *say which type yours has*. Your function should be $O(n)$ and pass over the list only once.

Write `mean` two ways: using natural recursion and using either `foldr` or `foldl`.

4.5 A change of key

The function `transpose` "pivots" a list of lists, as in:

```
> transpose [[1,2,3],[4,5,6]]
[[1,4],[2,5],[3,6]]
> transpose [[1..5],[10],[],[20..30]]
[[1,10,20],[2,21],[3,22],[4,23],[5,24],[25],[26],[27],[28],[29],[30]]
```

Write `transpose`. Make it as concise and clear as possible, using higher-order functions (and other Prelude functions) as necessary. Remember: the most powerful tool isn't always the right one for the job.

4.6 Taking attendance

Write three versions of `elem :: Eq a => a -> [a] -> Bool`: one using natural recursion, one using a fold of your choice, and one using a combination of `map`, `filter`.

4.7 Delete your account

Write two versions of `delete :: Eq a => a -> [a] -> [a]`: one using natural recursion, one using `foldr`.

4.8 A man, a plan, a canal: Haskell

Write a function `palindrome :: Eq a => [a] -> Bool` that determines whether a given list is a palindrome, i.e., the same backwards and forwards.

Bonus: write a palindrome in English. If (a) it's cool and (b) no one else writes the same one, I'll give you a bonus quarter of a point.

4.9 The heart of the matter

The function `nub` de-duplicates a list, as in:

```
> nub [1..10]
[1,2,3,4,5,6,7,8,9,10]
> nub ([1..10] ++ [20,22..30] ++ [1..10])
[1,2,3,4,5,6,7,8,9,10,20,22,24,26,28,30]
> nub ([20,22..30] ++ [1..10] ++ [41,43..49] ++ [1..10])
[20,22,24,26,28,30,1,2,3,4,5,6,7,8,9,10,41,43,45,47,49]
```

Write three versions of `nub`: one using natural recursion, one using `foldr`, and one using `foldl`.