

CS054: Sets

The goal of this worksheet is to give you practice with proofs about sets, using both the element-wise and algebraic approaches. It's not for a grade—no need to turn it in! I'll post solutions, but you'll get the most out of it if you don't peek.

1. Prove that $A \cap A = A$ both ways.

Element-wise proof:

Algebraic proof:

2. Prove that $\overline{\emptyset} = U$ both ways.

When doing proofs about complement, U represents the universe of discourse; you can assume $\forall x, x \in U$. (Try to avoid making such an assumption in proofs that don't involve set complement, though!) You can also assume that $U \neq \emptyset$, and, as a consequence, you can introduce arbitrary sets A into your proof as needed.

Element-wise proof:

Algebraic proof:

3. Prove that $\overline{U} = \emptyset$ both ways.

Element-wise proof:

Algebraic proof:

4. Prove that $\overline{\overline{A}} = A$ both ways. Your element-wise proof will need the law of the excluded middle, i.e., you may assume that $x \in A \vee x \notin A$ for all x and A —that is, you can use the law of the excluded middle.

Element-wise proof:

Algebraic proof: