The goal of this worksheet is to give you practice with proofs about sets, using both the element-wise and algebraic approaches. It’s not for a grade—no need to turn it in! I’ll post solutions, but you’ll get the most out of it if you don’t peek.

1. Prove that $A \cap A = A$ both ways.
   
   **Element-wise proof:**

   **Algebraic proof:**
2. Prove that $\emptyset = U$ both ways.

When doing proofs about complement, $U$ represents the universe of discourse; you can assume $\forall x, x \in U$. (Try to avoid making such an assumption in proofs that don’t involve set complement, though!) You can also assume that $U \neq \emptyset$, and, as a consequence, you can introduce arbitrary sets $A$ into your proof as needed.

Element-wise proof:

Algebraic proof:

3. Prove that $\overline{U} = \emptyset$ both ways.

Element-wise proof:

Algebraic proof:
4. Prove that $\overline{A} = A$ both ways. Your element-wise proof will need the law of the excluded middle, i.e., you may assume that $x \in A \lor x \not\in A$ for all $x$ and $A$—that is, you can use the law of the excluded middle.

Element-wise proof:

Algebraic proof: