

## CS054: Relations

The goal of this worksheet is to give you practice with relations, functions, and their properties. It's not for a grade—no need to turn it in! I'll post solutions, but you'll get the most out of it if you don't peek.

1. What's a descriptive name for the following relation  $T \subseteq \text{bool} \times \text{bool}$ ?

$$T = \{(\top, \top), (\top, \perp), (\perp, \perp)\}$$

**Answer:**

2. Construct a relation that is reflexive but not symmetric. It can be on any pair of sets you like.

**Answer:**

3. Construct a relation that is reflexive but not transitive. It can be on any pair of sets you like.

**Answer:**

4. Prove that the symmetric closure of a relation  $R \subseteq A \times A$  is symmetric.

**Proof:**

5. Write a relation  $R \subseteq \mathbb{N} \times \mathbb{N}$  that is total but not deterministic.

**Answer:**

6. Write a relation  $R \subseteq \mathbb{N} \times \mathbb{N}$  that is deterministic but not total.

**Answer:**

7. Prove that  $\text{map}(f, \text{map}(g, l)) = \text{map}(f \circ g, l)$ .

**Proof:**

8. Prove that if  $f : A \rightarrow B$  is a bijection, then  $f^{-1} : B \rightarrow A$  and is also a bijection. (Some theorems from the book will help, but you'll learn the most if you do it all by hand.)

**Proof:**