

# Lecture 21: Differential Privacy

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CS 181S

Spring 2024

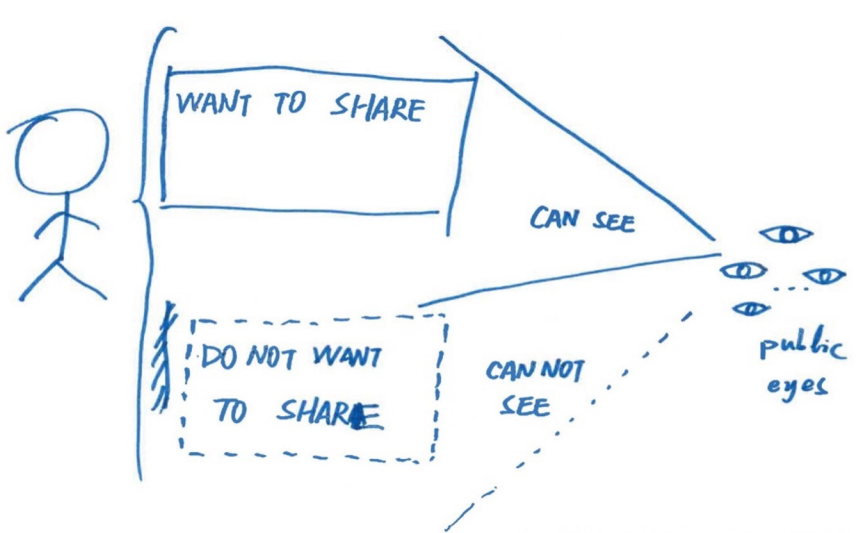
# What is Privacy?



# Privacy

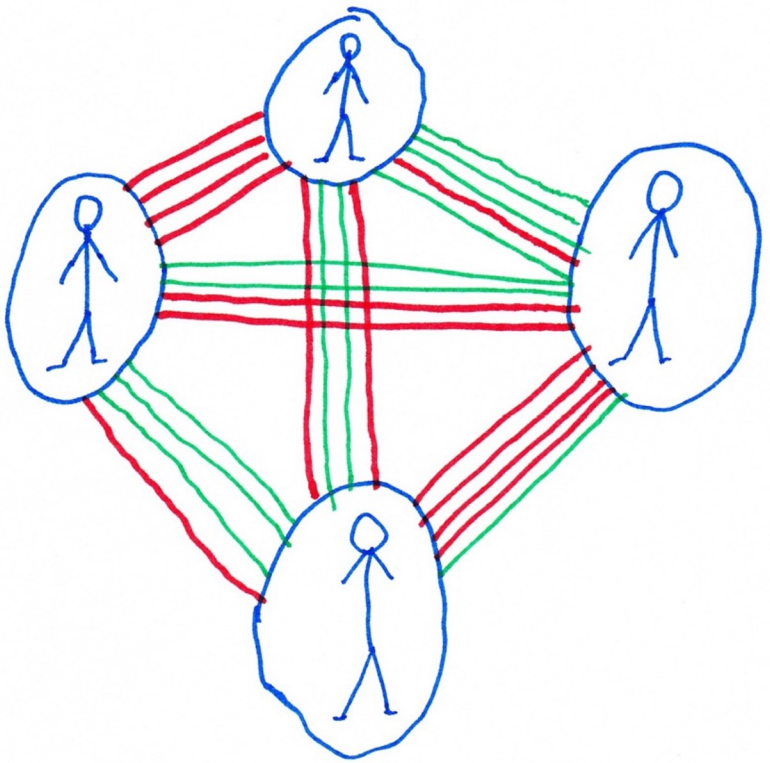
*Privacy* concerns information about individuals (people, organizations, etc.)

- Often construed as legal right
- *Privacy* is not a synonym for confidentiality or for secrecy



People have right to keep what they do not want to share invisible.

– AC, age 24



Privacy is a network: I share what I want with whom I want and trust and what matches with those in the network....

Green = share.  
Red = don't.

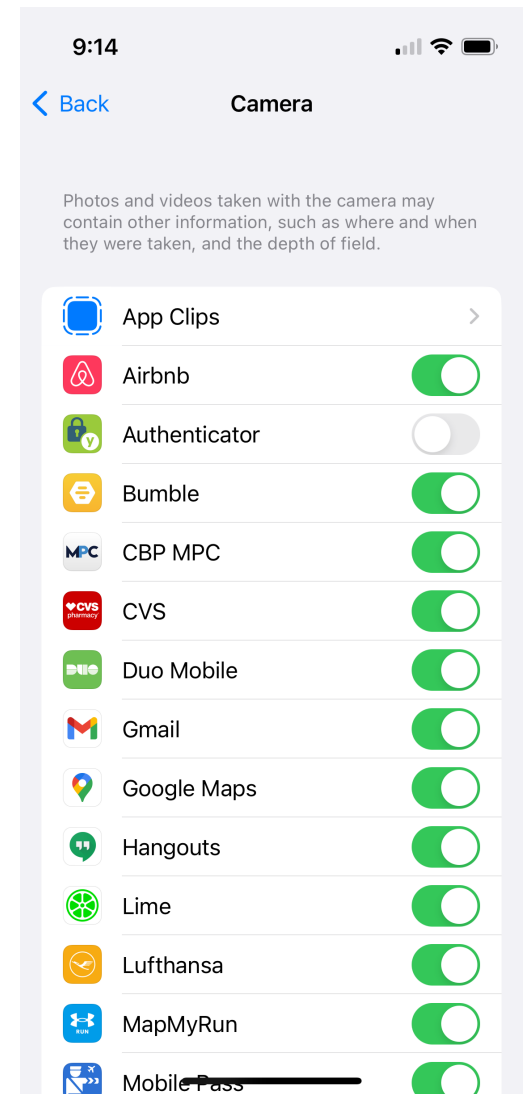
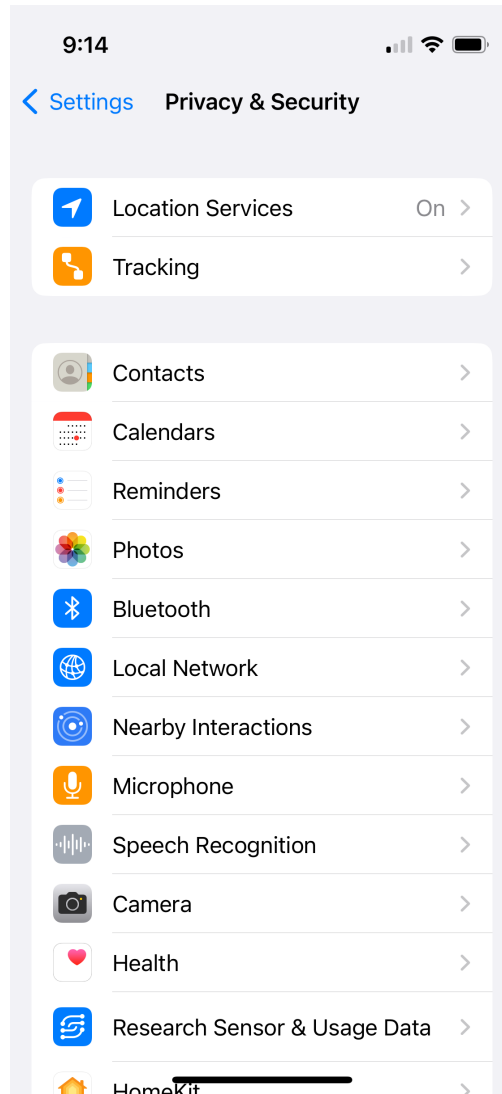
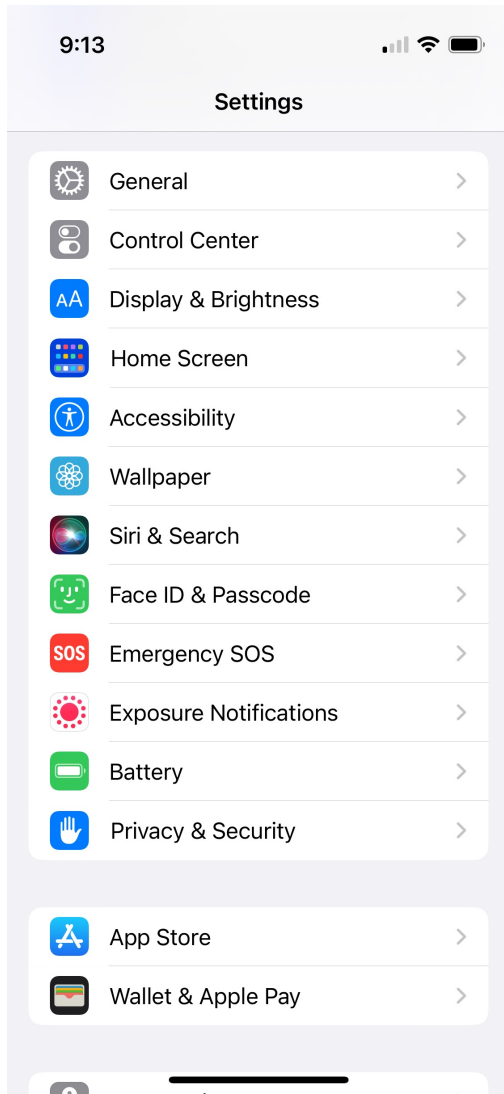
# Privacy and Freedom (1967)

Privacy is the claim of individuals, groups or institutions to determine for themselves when, how, and to what extent information about them is communicated to others.

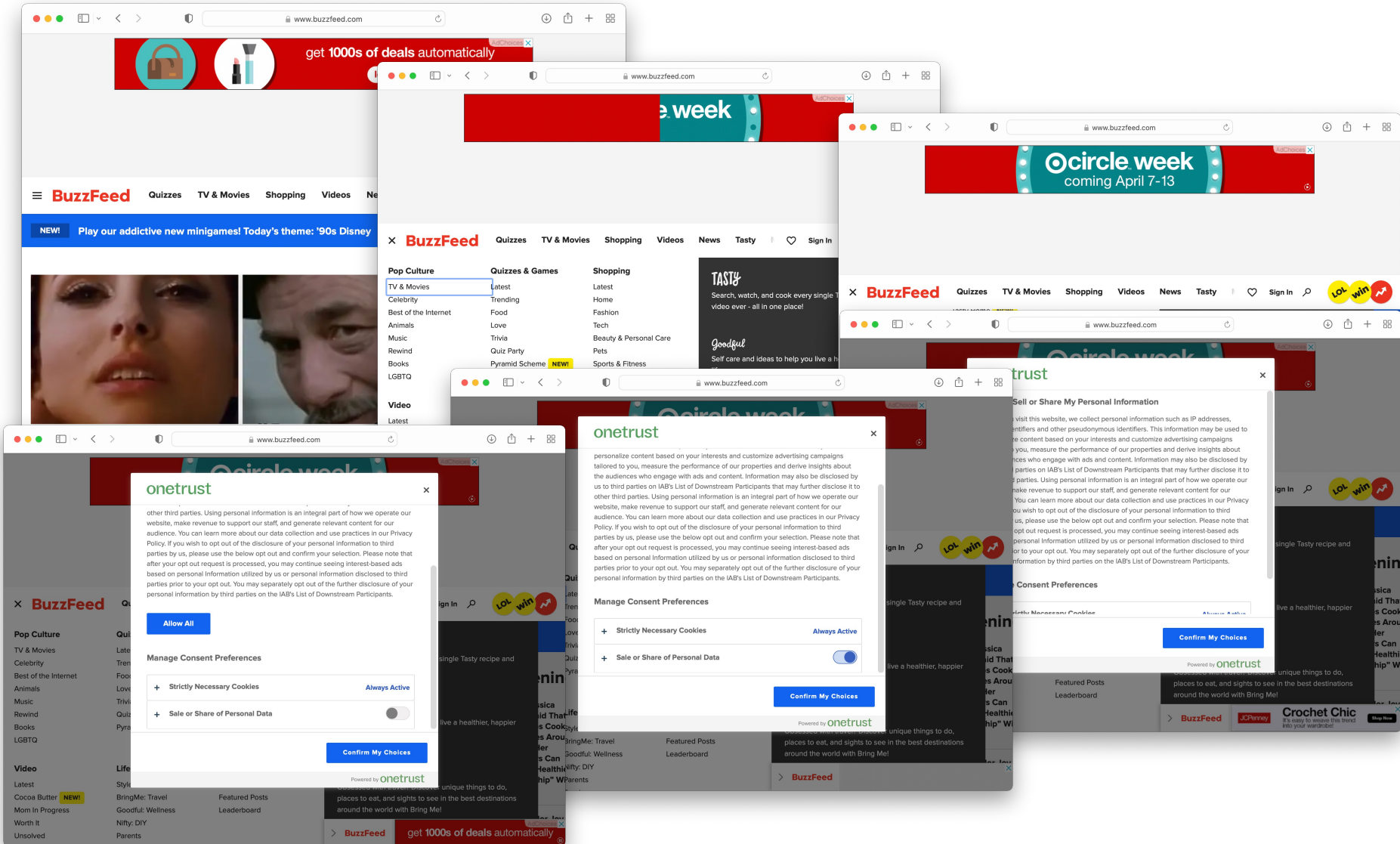
– Alan Westin



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- Make sure the website looks consistent
- Allow you to share pages with social networks
- Allow you to post comments
- Serve ads relevant to your interests

This page transmits information using HTTPS protocol. Some vendors cannot support HTTPS opt-out requests. TrustArc will submit your preferences through HTTP in a pop-up window.

CANCEL

SUBMIT PREFERENCES

ADVANCED SETTINGS

# Manipulation of privacy behavior

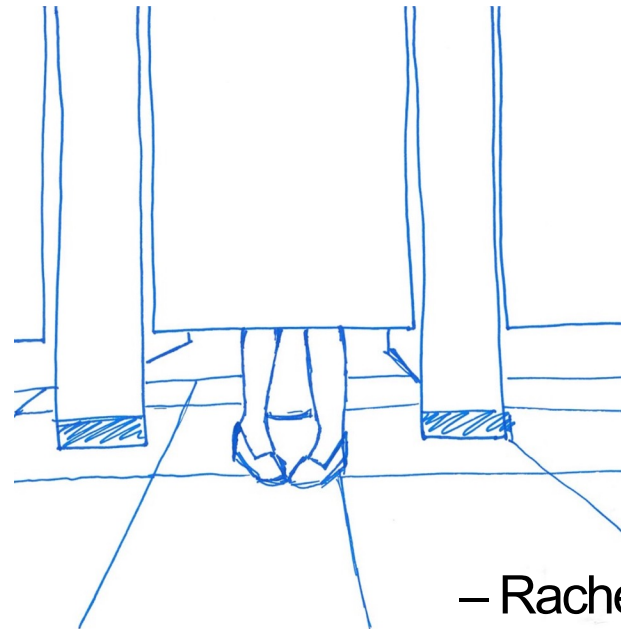
A few common examples of manipulative design in privacy-related interfaces

- Defaults are not privacy protective
- Buttons have confusing labels
- Framing - including wording that shames users to influence their decisions or makes them feel like they will be missing out
- Highlighting – visually emphasizing opt-in
- Cumbersome privacy choices - more difficult to choose privacy options



Your room is private.

– Alexia, age 11



– Rachel, age 20

# Contextual Integrity



- defines privacy relative to appropriate context
- considers information type, time, location, purpose, principals involved (subject, sender, receiver)
- dependent on social norms
- norms can change over time

# General Guidelines

The FTC's Fair Information Practice Principals (FIPPs) are the most broadly recognized guidelines for handling private data in information systems

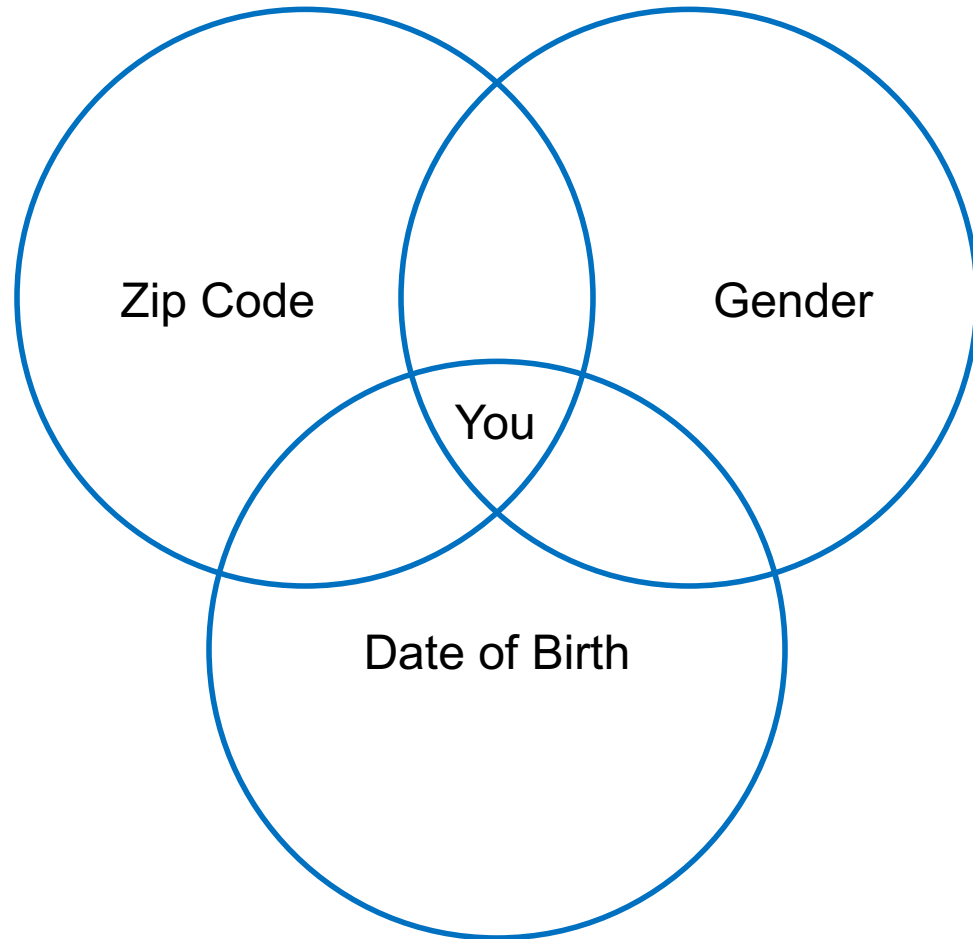
- Seek consent
- Minimize data use
- Limit storage
- Avoid linking

# General Data Protection Regulation

**Goal:** Codify fundamental right to protection of personal data.

- Introduced individual rights
  1. The right to transparency
  2. The right to access
  3. The right to correct
  4. The right to delete
  5. The right to data portability
  6. The right to withdraw consent
  7. The right to object
- Additional obligations
  - Legal basis for processing
  - Purpose limitation
  - Data Minimization
  - Storage limitation
  - Security requirements
  - Privacy by design
- Adopted: April 14, 2016
- Effective: May 25, 2018

# Deanononymization



# Deanononymization



**NETFLIX**



# k-Anonymity

Name	Pronouns	Year	Grade
Alice	she/her	2025	95
Bob	he/him	2025	80
Charlie	they/them	2025	95
David	he/him	2025	60
Edward	he/him	2026	80
Flora	she/her	2026	99
Georgia	she/her	2026	60

- **Quasi-identifiers (QIs)** are sets of attributes that can be exploited for linking
- A database is **k-anonymous** if each QI maps to at least k different individuals
- Techniques: suppression and generalization

# Exercise 2: k-anonymity

- Modify this dataset to make it 2-anonymous with respect to Race/DOB/Sex

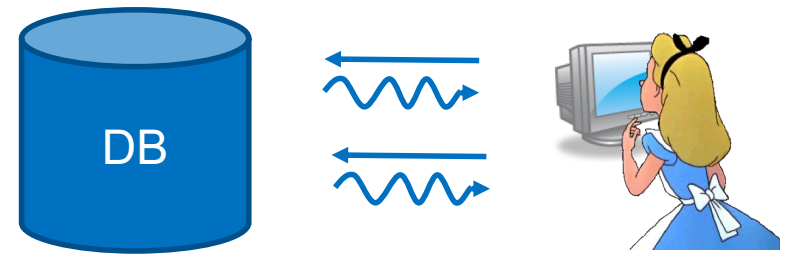
Race	DOB	Sex	Marital Status	Health Issues
asian	9/27/00	female	divorced	hypertension
asian	9/30/00	female	divorced	obesity
asian	4/18/00	male	married	chest pain
asian	4/15/00	male	married	obesity
black	3/13/99	male	married	hypertension
black	3/18/99	male	married	shortness of breath
black	9/13/00	female	married	shortness of breath
black	9/07/00	female	married	obesity
white	5/14/01	male	single	chest pain
white	4/08/01	male	single	obesity
white	9/15/01	female	married	shortness of breath

# Database Privacy

## Offline Privacy

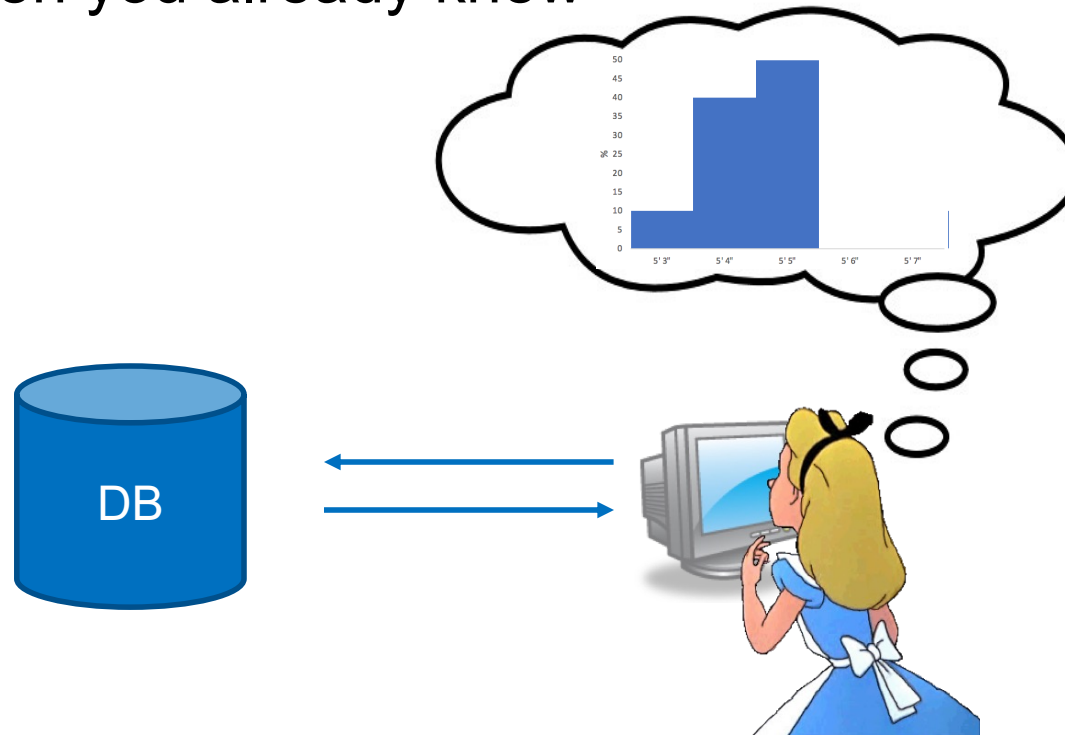


## Online Privacy

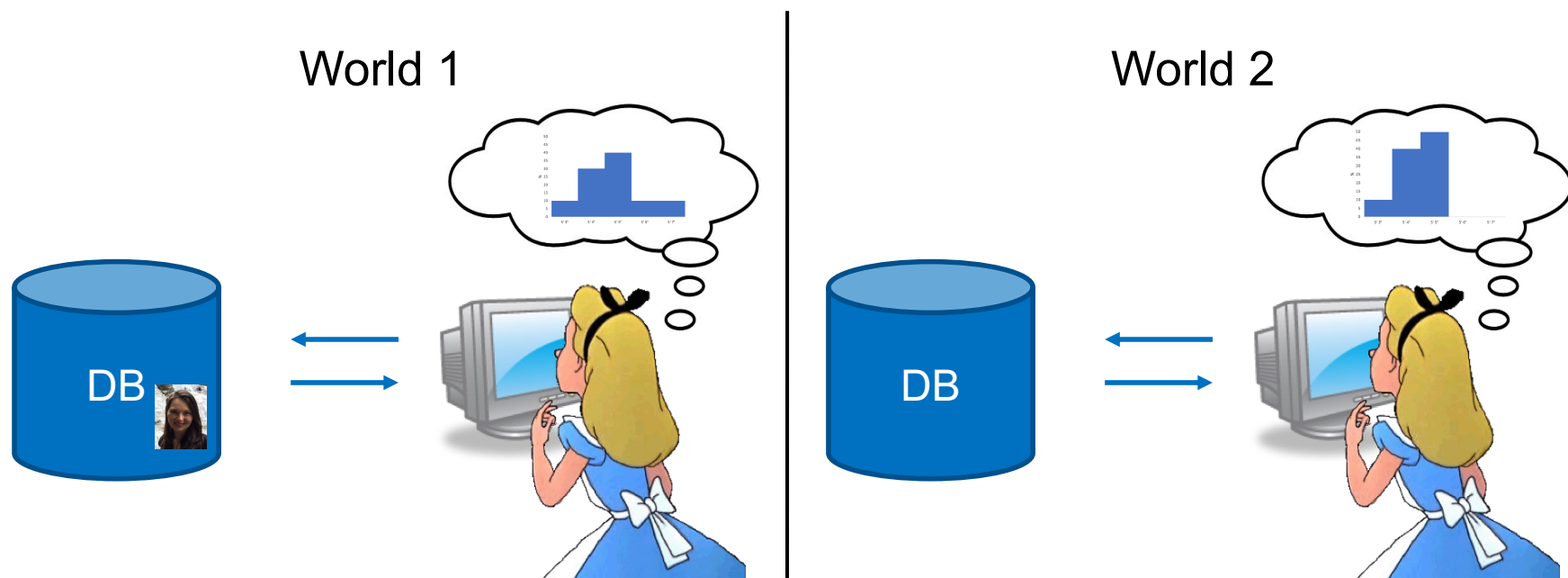


# Defining Privacy: Try #1

- You don't know anything more after interacting with the database than you already knew



# Differential Privacy



A query  $Q$  is  $\epsilon$ -differentially private if  $\forall D, r \in D,$   
 $\Pr[Q(D) = x] \leq e^\epsilon \cdot \Pr[Q(D - r) = x]$



# Sensitivity

- The sensitivity  $\Delta$  of a query  $Q$  is the maximum the answer to  $Q$  can possibly change between two databases that differ only by one person
- $Q = \text{number of people taller than 6 ft}$        $\Delta = 1$
- $Q = \text{maximum height of a person}$        $\Delta = 48$

# Exercise 3: Sensitivity

- Assume you have a database containing the heights of 100 users specified in inches. You may assume that all heights are between 48 in and 96in.
- What is the sensitivity of the following queries?
  1. The number of people who are 5' 4"
  2. The median height in the dataset
  3. The mean height in the dataset

# Exercise 3: Sensitivity

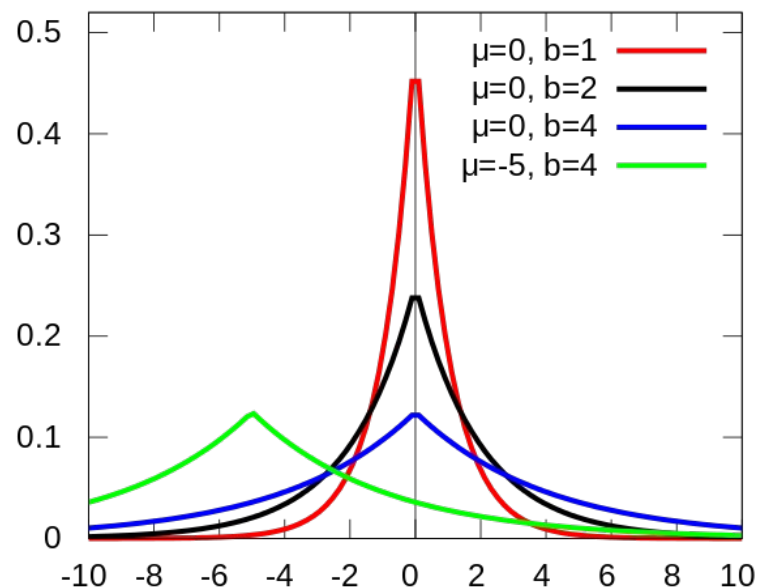
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# Laplacian Distribution

- $Lap(b)$  is the probability distribution with the property that

$$\Pr[ Lap(b) = x ] = \frac{1}{2b} \cdot e^{-\frac{|x|}{b}}$$

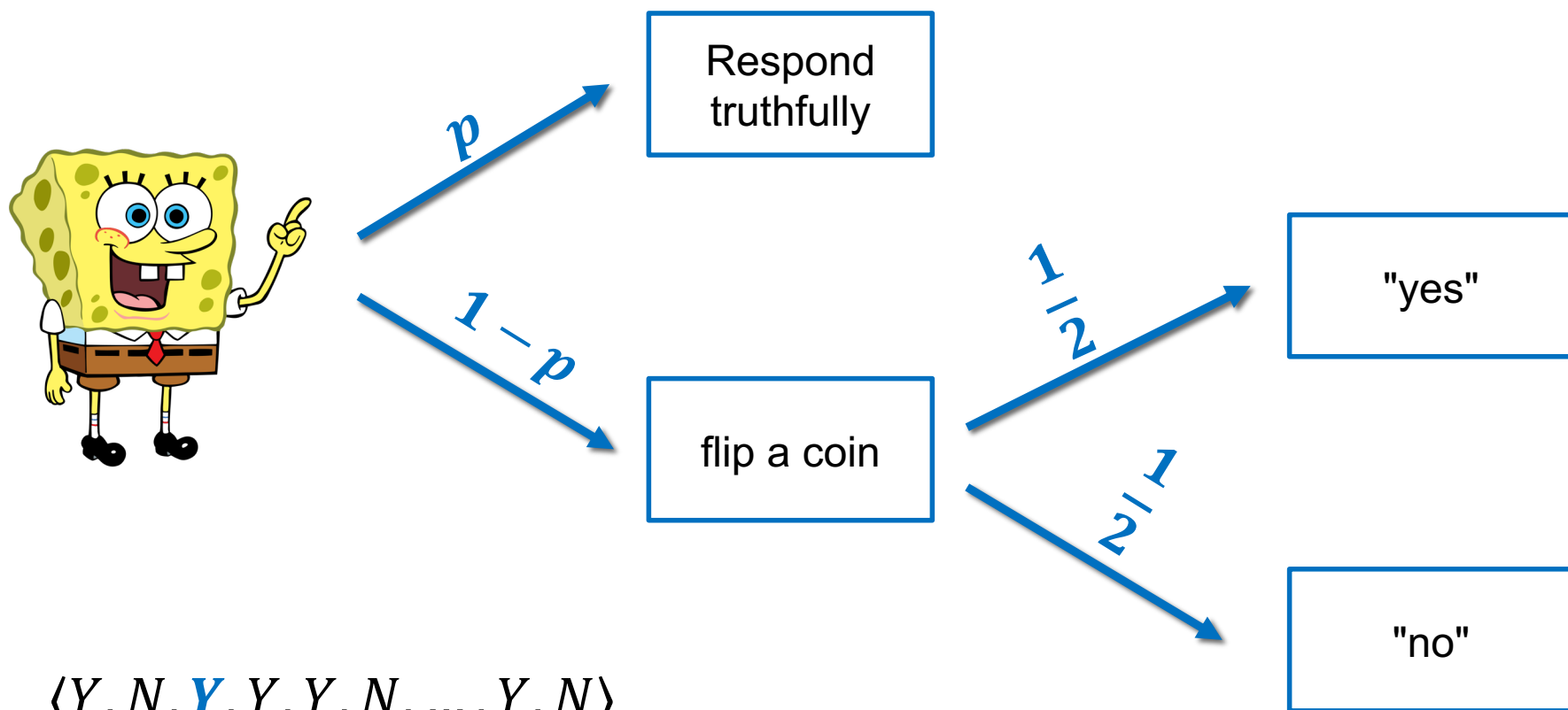


# Laplacian Mechanism

- Given a query  $Q$  on a database  $D$  that has sensitivity  $\Delta$ , respond with  $Q(D)+Y$  where  $Y$  is drawn from the distribution  $Lap\left(\frac{\Delta}{\epsilon}\right)$
- Theorem: this mechanism satisfies  $\epsilon$ -differential privacy

$$\begin{aligned}\frac{\Pr[Q(D) + Y = x]}{\Pr[Q(D-r) + Y = x]} &= \frac{\Pr[Y = x - Q(D)]}{\Pr[Y = x - Q(D-r)]} = \frac{\frac{1}{2(\Delta/\epsilon)} \cdot e^{-\frac{|x-Q(D)|}{\Delta/\epsilon}}}{\frac{1}{2(\Delta/\epsilon)} \cdot e^{-\frac{|x-Q(D-r)|}{\Delta/\epsilon}}} = \frac{e^{-\frac{|x-Q(D)|}{\Delta/\epsilon}}}{e^{-\frac{|x-Q(D-r)|}{\Delta/\epsilon}}} \\ &= e^{\frac{|x-Q(D-r)|}{\Delta/\epsilon} - \frac{|x-Q(D)|}{\Delta/\epsilon}} = e^{\left(\frac{\epsilon}{\Delta}\right) \cdot (|x-Q(D-r)| - |x-Q(D)|)} \\ &\leq e^{\left(\frac{\epsilon}{\Delta}\right) \cdot (|x-Q(D-r) - x + Q(D)|)} = e^{\left(\frac{\epsilon}{\Delta}\right) \cdot (|Q(D) - Q(D-r)|)} \\ &\leq e^{\left(\frac{\epsilon}{\Delta}\right) \cdot \Delta} = e^{\epsilon}\end{aligned}$$

# Randomized Response



Theorem: this mechanism satisfies  $\epsilon$ -differential privacy

# Randomized Response

- Theorem: this mechanism satisfies  $\epsilon$ -differential privacy

$$\frac{\Pr[\langle Y, N, \mathbf{Y}, Y, Y, N, \dots, Y, N \rangle \mid f(\text{Bob}) = Y]}{\Pr[\langle Y, N, \mathbf{Y}, Y, Y, N, \dots, Y, N \rangle \mid f(\text{Bob}) = N]}$$

$$= \frac{\Pr[Y \mid f(P_0)] \cdot \Pr[N \mid f(P_1)] \cdot \Pr[Y \mid f(\text{Bob}) = Y] \cdot \dots \cdot \Pr[N \mid f(P_{n-1})]}{\Pr[Y \mid f(P_0)] \cdot \Pr[N \mid f(P_1)] \cdot \Pr[Y \mid f(\text{Bob}) = N] \cdot \dots \cdot \Pr[N \mid f(P_{n-1})]}$$

$$= \frac{\Pr[Y \mid f(\text{Bob}) = Y]}{\Pr[Y \mid f(\text{Bob}) = N]}$$

$$= \frac{p \cdot 1 + (1 - p) \cdot \frac{1}{2}}{p \cdot 0 + (1 - p) \cdot \frac{1}{2}} = \frac{(1 + p) \cdot \frac{1}{2}}{(1 - p) \cdot \frac{1}{2}} = \frac{(1 + p)}{(1 - p)}$$

$$= e^{\ln\left(\frac{1+p}{1-p}\right)}$$

# DP in action...

