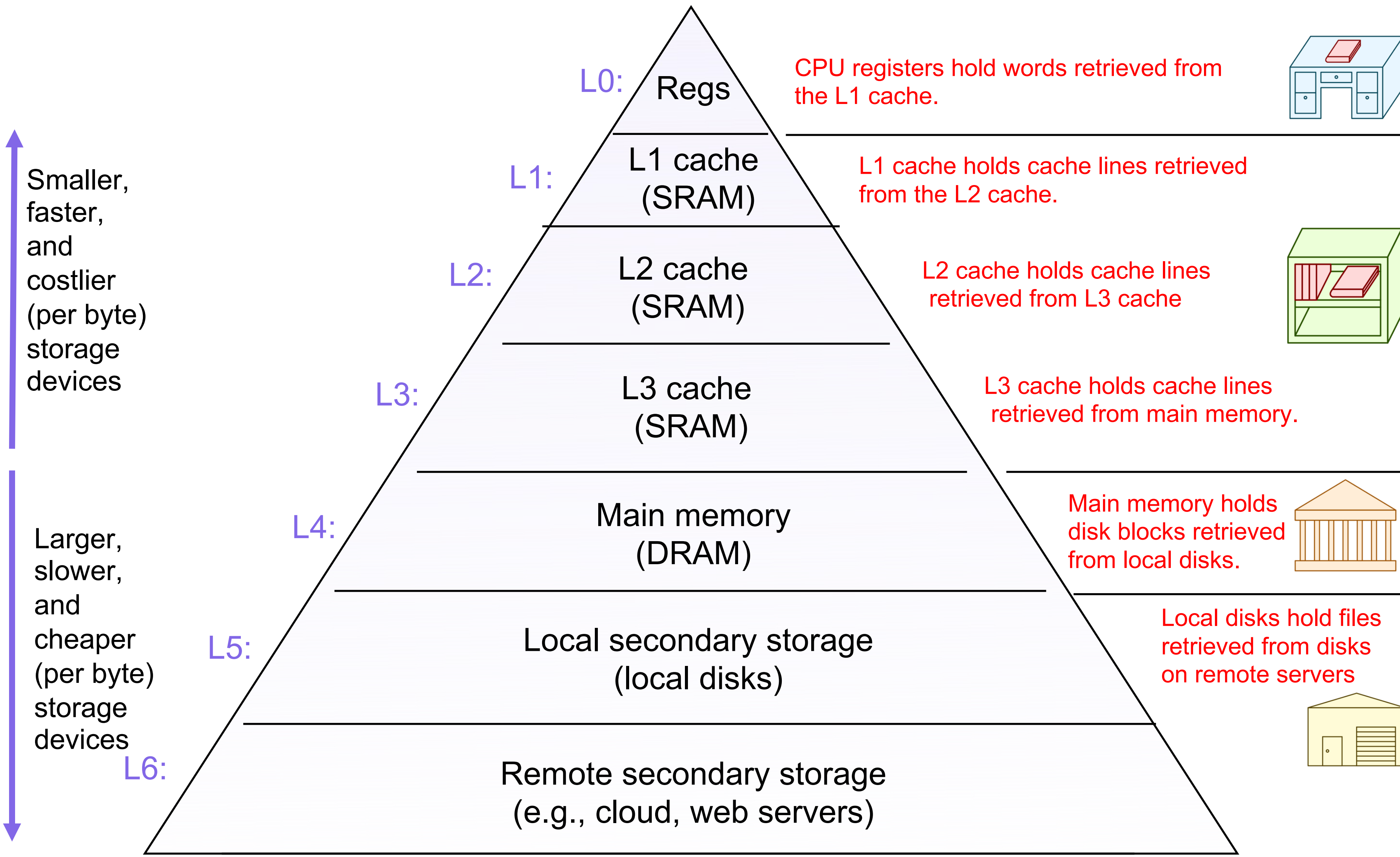
Lecture 14: Optimization with Caches

CS 105 Spring 2024

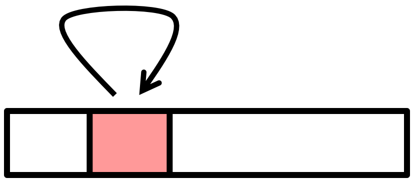
Review: Memory Hierarchy



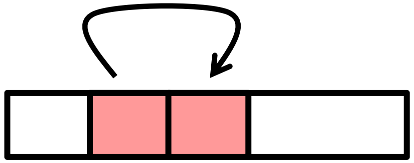
Review: Principle of Locality

Programs tend to use data and instructions with addresses near or equal to those they have used recently

- **Temporal locality:**
  - Recently referenced items are likely to be referenced again in the near future



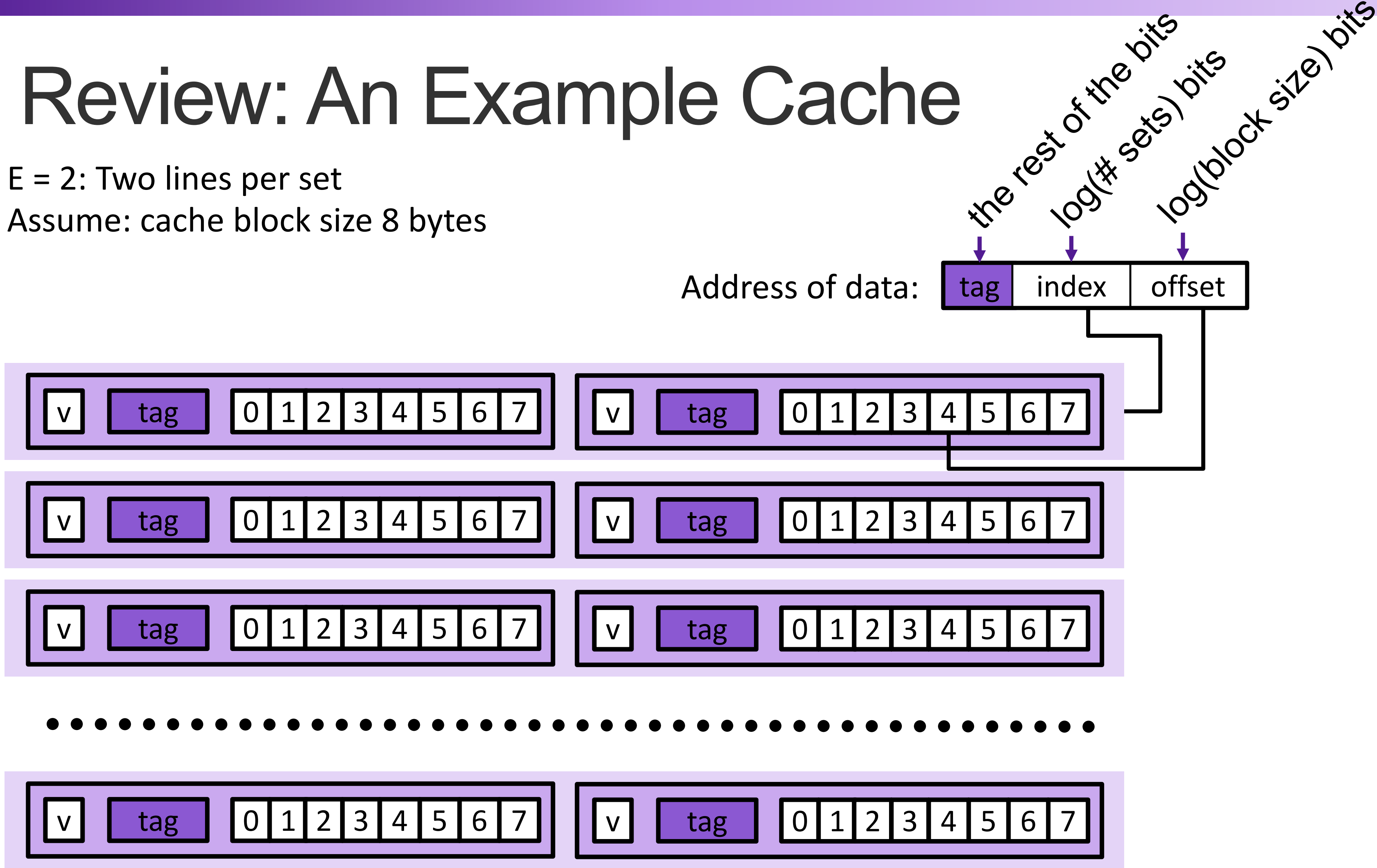
- **Spatial locality:**
  - Items with nearby addresses tend to be referenced close together in time



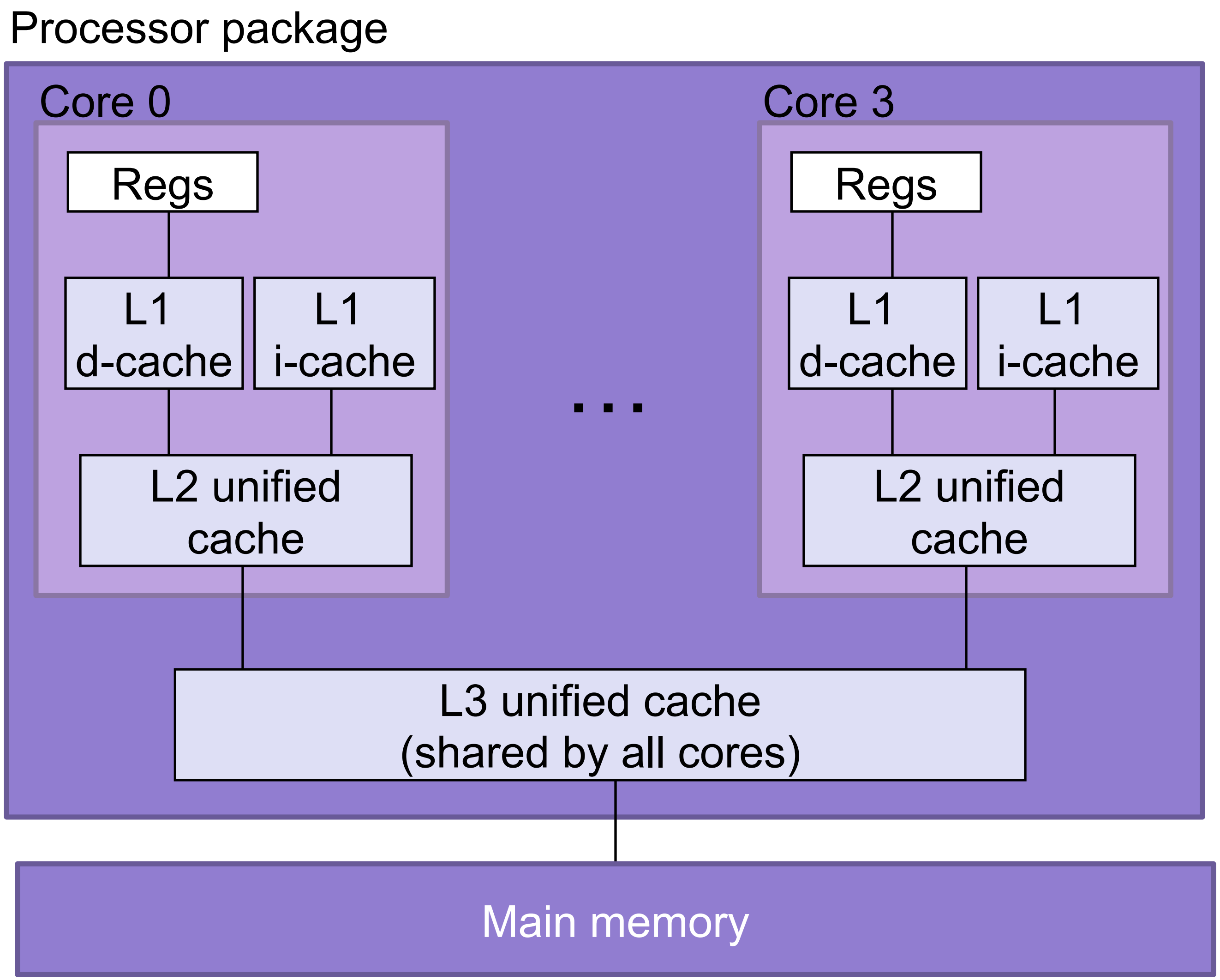
Review: An Example Cache

E = 2: Two lines per set

Assume: cache block size 8 bytes



Typical Intel Core i7 Hierarchy



L1 i-cache and d-cache:
32 KB, 8-way,
Access: 4 cycles

L2 unified cache:
256 KB, 8-way,
Access: 10 cycles

L3 unified cache:
8 MB, 16-way,
Access: 40-75 cycles

Block size: 64 bytes for all caches.

Cache Performance Metrics

- Miss Rate
  - Fraction of memory references not found in cache (misses / accesses)
  - Typically 3-10% for L1
  - can be quite small (e.g., < 1%) for L2, depending on size, etc.
- Hit Time
  - Time to deliver a line in the cache to the processor
    - includes time to determine whether the line is in the cache
  - Typically 4 clock cycles for L1, 10 clock cycles for L2
- Miss Penalty
  - Additional time required because of a miss
    - typically 50-200 cycles for main memory (Trend: increasing!)

Memory Performance with Caching

- **Read throughput (aka read bandwidth):** Number of bytes read from memory per second (MB/s)

- **Memory mountain:** Measured read throughput as a function of spatial and temporal locality.
  - Compact way to characterize memory system performance.

Memory Mountain Test Function

Call `test()` with many combinations of `elems` and `stride`.

For each elems and stride:

1. Call test() once to warm up the caches.

2. Call test() again and measure the read throughput(MB/s)

```
long data[MAXELEMS];  /* Global array to traverse */

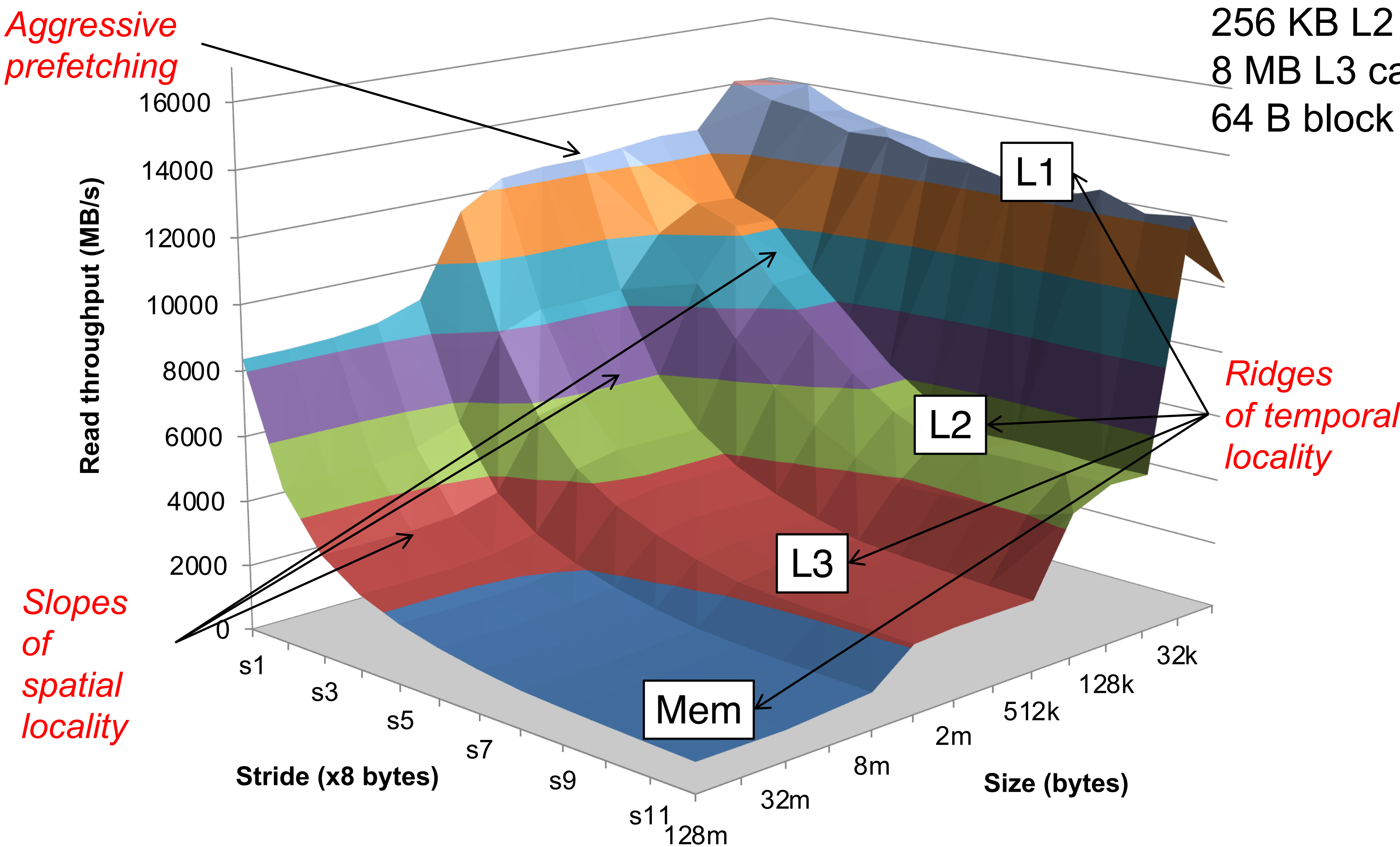
/* test - Iterate over first "elems" elements of
 *        array “data” with stride of "stride", using
 *        using 4x4 loop unrolling.
 */
int test(int elems, int stride) {
    long i, sx2=stride*2, sx3=stride*3, sx4=stride*4;
    long acc0 = 0, acc1 = 0, acc2 = 0, acc3 = 0;
    long length = elems, limit = length - sx4;

    /* Combine 4 elements at a time */
    for (i = 0; i < limit; i += sx4) {
        acc0 = acc0 + data[i];
        acc1 = acc1 + data[i+stride];
        acc2 = acc2 + data[i+sx2];
        acc3 = acc3 + data[i+sx3];
    }

    /* Finish any remaining elements */
    for (; i < length; i++) {
        acc0 = acc0 + data[i];
    }
    return ((acc0 + acc1) + (acc2 + acc3));
}
```

The Memory Mountain

Core i7 Haswell
2.1 GHz
32 KB L1 d-cache
256 KB L2 cache
8 MB L3 cache
64 B block size



Exercise 1: Locality

- Which of the following functions is better in terms of locality with respect to array src?

```
void copyij(int src[2048][2048],
            int dst[2048][2048])
{
  int i,j;
  for (i = 0; i < 2048; i++)
    for (j = 0; j < 2048; j++)
      dst[i][j] = src[i][j];
}
```

```
void copyji(int src[2048][2048],
            int dst[2048][2048])
{
  int i,j;
  for (j = 0; j < 2048; j++)
    for (i = 0; i < 2048; i++)
      dst[i][j] = src[i][j];
}
```

Exercise 1: Locality

- Which of the following functions is better in terms of locality with respect to array src?

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void copyij(int src[2048][2048],
            int dst[2048][2048])
{
  int i,j;
  for (i = 0; i < 2048; i++)
    for (j = 0; j < 2048; j++)
      dst[i][j] = src[i][j];
}
```

```
void copyji(int src[2048][2048],
            int dst[2048][2048])
{
  int i,j;
  for (j = 0; j < 2048; j++)
    for (i = 0; i < 2048; i++)
      dst[i][j] = src[i][j];
}
```

4.3ms

81.8ms

2.0 GHz Intel Core i7 Haswell

Writing Cache-Friendly Code

- Make the common case go fast
  - Focus on the inner loops of the core functions

- Minimize the misses in the inner loops
  - Repeated references to variables are good (**temporal locality**)
  - Stride-1 reference patterns are good (**spatial locality**)

Exercise: Miss Rate Analysis

```
int sum_array(int* array, int n){
  int sum = 0;
  for(int i = 0; i < n; i++){
    sum += array[i];
  }
  return sum;
}
```

assume `n`, `sum` and `i` are stored in registers and only `array` is stored in memory, assume `n=16`

assume `array = 0x600090`

assume 256 byte direct-mapped cache w/ 16-byte cache lines

Exercise: what is the sequence of memory accesses made by this program?
Exercise: what is the hit rate of this program?

Review: Matrix Multiplication

- $a = \begin{bmatrix} 2.0 & 1.5 \\ 1.0 & 0.5 \end{bmatrix}$
- $b = \begin{bmatrix} 1.5 & 2.0 \\ 2.0 & 1.5 \end{bmatrix}$

- What is a x b?

```
/* ijk */
for(int i=0; i<n; i++)  {
  for(int j=0; j<n; j++) {
    sum = 0.0;
    for(int k=0; k<n; k++){
      sum += a[i][k] * b[k][j];
    }
    c[i][j] = sum;
  }
}
```

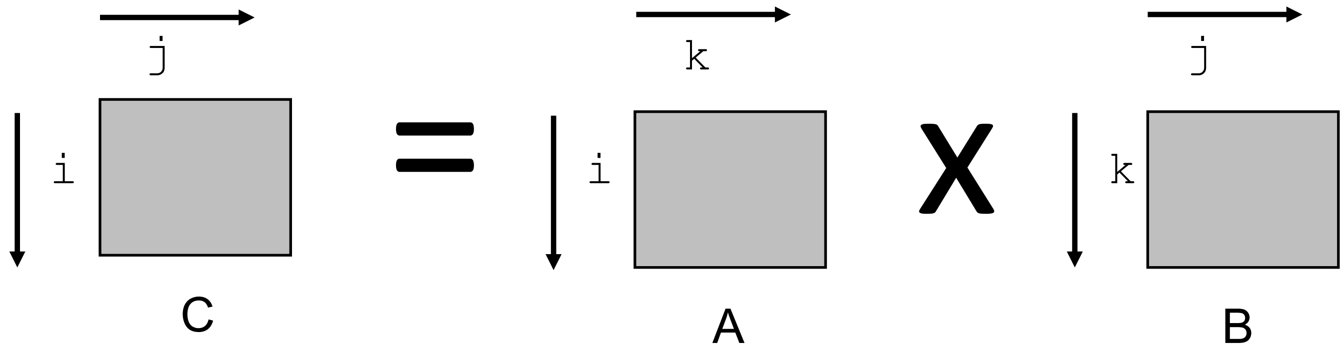
Example: Matrix Multiplication

- Multiply N x N matrices
- Matrix elements are doubles (8 bytes)
- $O(N^3)$ total operations
- N reads per source element
- N values summed per destination

```
/* ijk */
for(int i=0; i<n; i++)  {
  for(int j=0; j<n; j++) {
    sum = 0.0;
    for(int k=0; k<n; k++){
      sum += a[i][k] * b[k][j];
    }
    c[i][j] = sum;
  }
}
```

Miss Rate Analysis for Matrix Multiply

- Assume:
  - datablock size = 32 bytes (big enough for four doubles)
  - Matrix dimension (N) is very large
    - Approximate 1/N as 0.0
  - Cache is not even big enough to hold multiple rows
- Analysis Method:
  - Look at access pattern of inner loop



Review: Layout of C Arrays in Memory

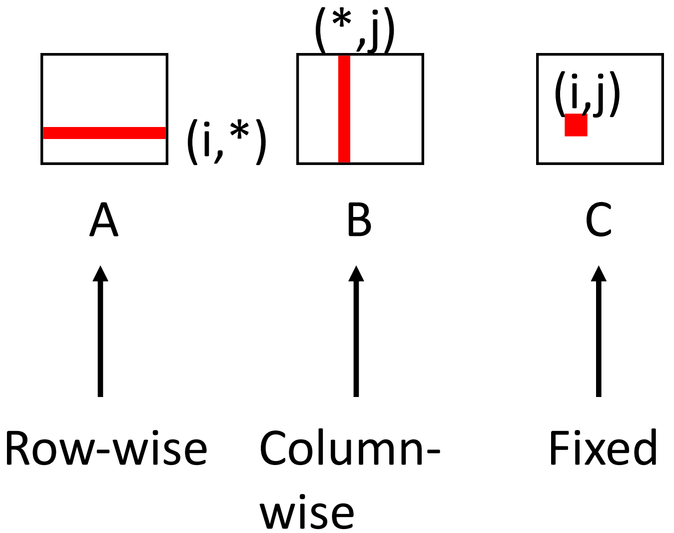
- C arrays allocated in row-major order
  - each row in contiguous memory locations
- Stepping through columns in one row:
  - accesses successive elements
  - if data block size (B) > sizeof(a_{ij}) bytes, exploit spatial locality
    - miss rate = sizeof(a_{ij}) / B
- Stepping through rows in one column:
  - accesses distant elements
  - no spatial locality!
    - miss rate = 1 (i.e. 100%)

Matrix Multiplication (ijk)

(jik is similar)

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

Inner loop:



Average Misses per inner loop iteration:

A	B	C	Total
.25	1.0	0.0	1.25

2 reads, 0 writes per inner loop iteration

Exercise: Alternative Matrix Multiplication Algs

- $a = \begin{bmatrix} 2.0 & 1.5 \\ 1.0 & 0.5 \end{bmatrix}$
- $b = \begin{bmatrix} 1.5 & 2.0 \\ 2.0 & 1.5 \end{bmatrix}$

- Use this algorithm to compute a x b

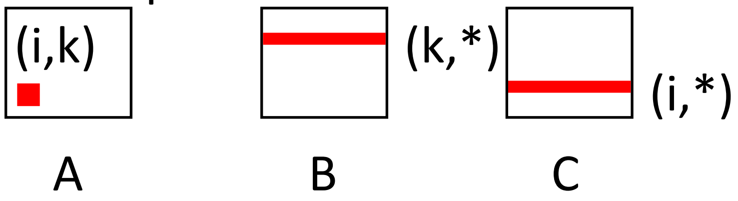
```
/* kij */
for (k=0; k<n; k++) {
 for (i=0; i<n; i++) {
  r = a[i][k];
  for (j=0; j<n; j++)
   c[i][j] += r * b[k][j];
 }
}
```

```
/* jki */
for (j=0; j<n; j++) {
 for (k=0; k<n; k++) {
   r = b[k][j];
   for (i=0; i<n; i++)
    c[i][j] += a[i][k] * r;
 }
}
```

Exercise: Matrix Multiplication

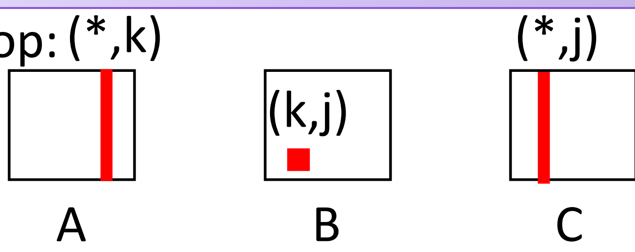
```
/* kij */
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

Inner loop:




```
/* jki */
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```

Inner loop:



Summary of Matrix Multiplication

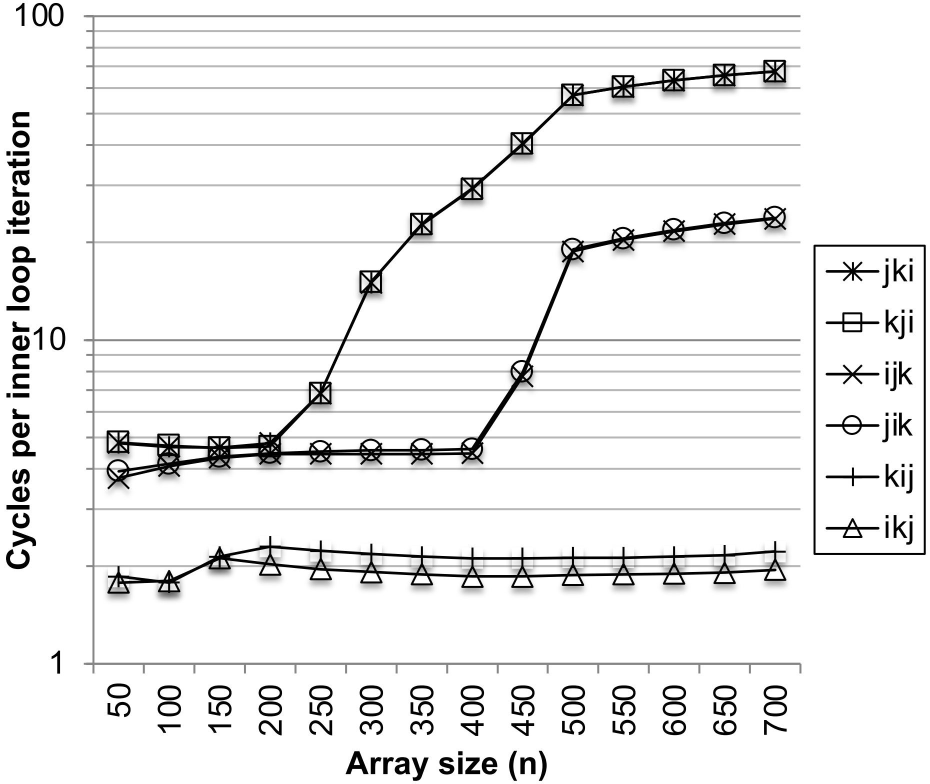
```
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
   sum = 0.0;
   for (k=0; k<n; k++)
     sum += a[i][k] * b[k][j];
   c[i][j] = sum;
 }
}
```

```
for (k=0; k<n; k++) {
 for (i=0; i<n; i++) {
  r = a[i][k];
  for (j=0; j<n; j++)
   c[i][j] += r * b[k][j];
 }
}
```

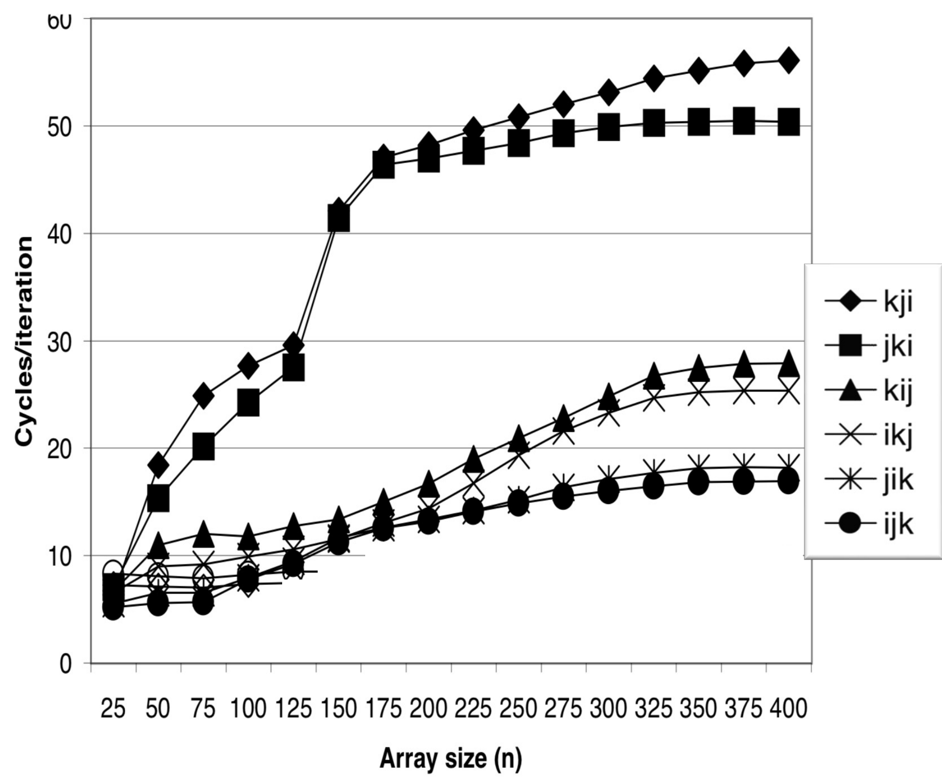
```
for (j=0; j<n; j++) {
 for (k=0; k<n; k++) {
   r = b[k][j];
   for (i=0; i<n; i++)
    c[i][j] += a[i][k] * r;
 }
}
```

Matrix Multiply Performance

Core i7



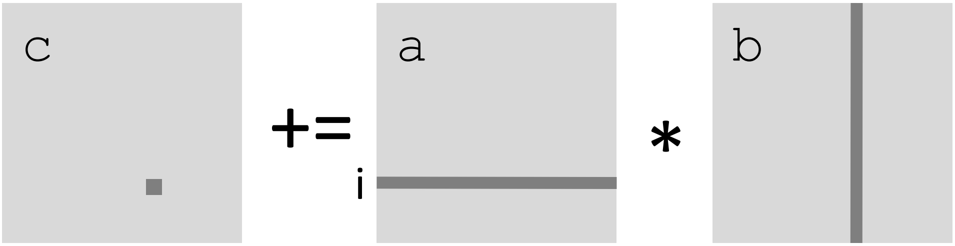
Pentium III Xeon



Can we do better?

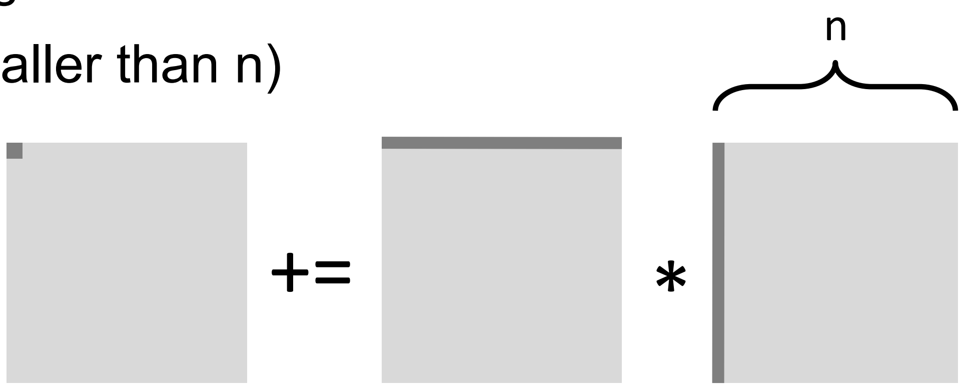
```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b  */
void mmm(double* a, double* b, double* c, int n) {
    int i, j, k;
    for (i = 0; i < n; i++)
        for (j = 0; j < n; j++)
            for (k = 0; k < n; k++)
                c[i*n + j] += a[i*n + k] * b[k*n + j];
}
```



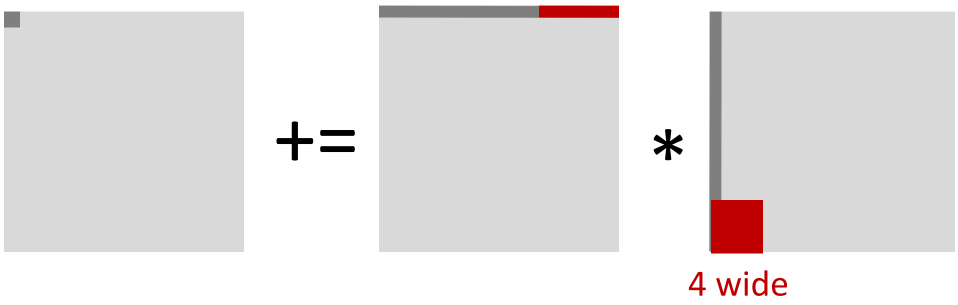
Cache Miss Analysis

- Assume:
  - Matrix elements are doubles
  - Cache data block = 4 doubles
  - Cache size C << n (much smaller than n)



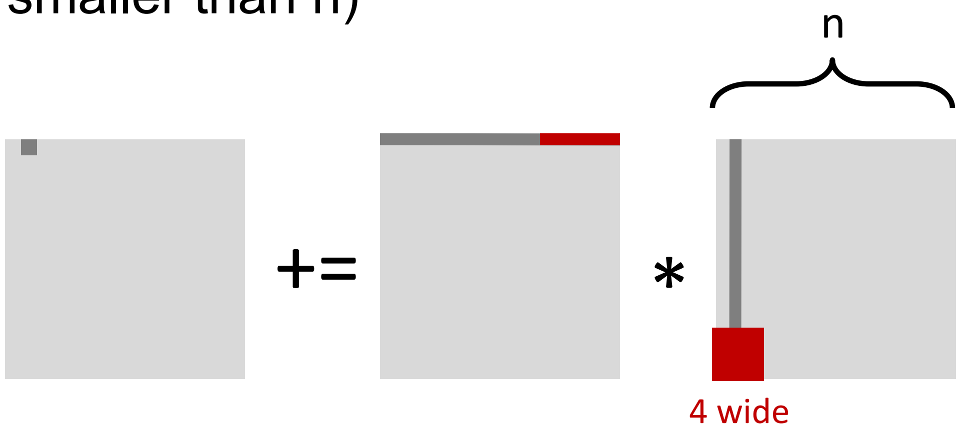
- First iteration:
  - n/4 + n = 5n/4 misses

  - Afterwards in cache: (schematic)



Cache Miss Analysis

- Assume:
  - Matrix elements are doubles
  - Cache data block = 4 doubles
  - Cache size C << n (much smaller than n)
- Second iteration:
  - n/4 + n = 5n/4 misses

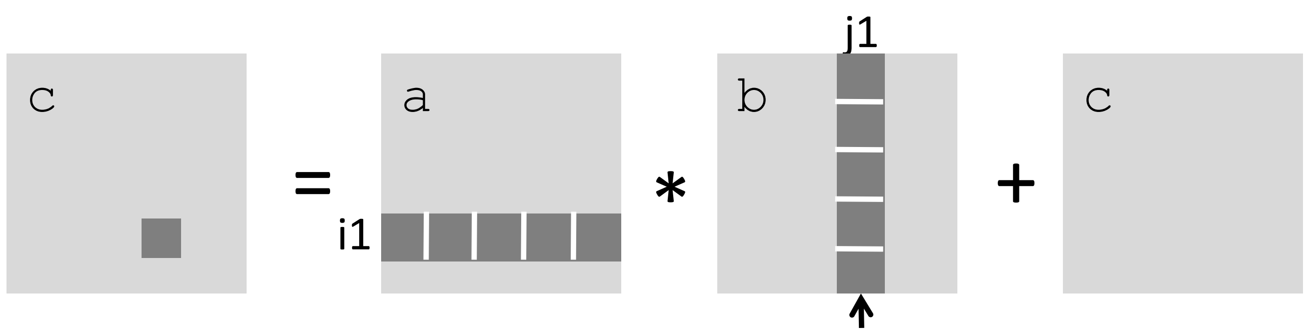


- Total misses:
  - $5n/4 * n^2 = (5/4) * n^3$

Blocked Matrix Multiplication

```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b  */
void mmm(double* a, double* b, double* c, int n) {
    int i, j, k;
    for (i = 0; i < n; i+=B)
        for (j = 0; j < n; j+=B)
             for (k = 0; k < n; k+=B)
                 /* B x B mini matrix multiplications */
                   for (i1 = i; i1 < i+B; i++)
                       for (j1 = j; j1 < j+B; j++)
                           for (k1 = k; k1 < k+B; k++)
                               c[i1*n+j1] += a[i1*n + k1]*b[k1*n + j1];
}
```



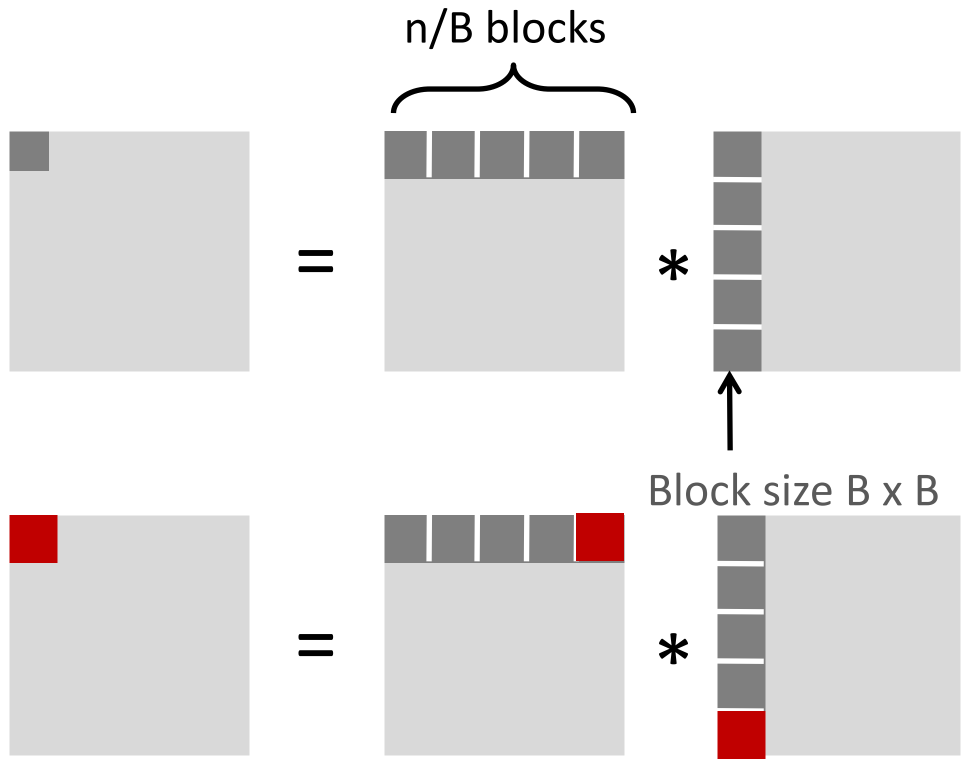
Block size B x B

Cache Miss Analysis

- Assume:
  - Cache data block = 4 doubles
  - Cache size C << n (much smaller than n)
  - Three blocks  fit into cache: $3B^2 < C$

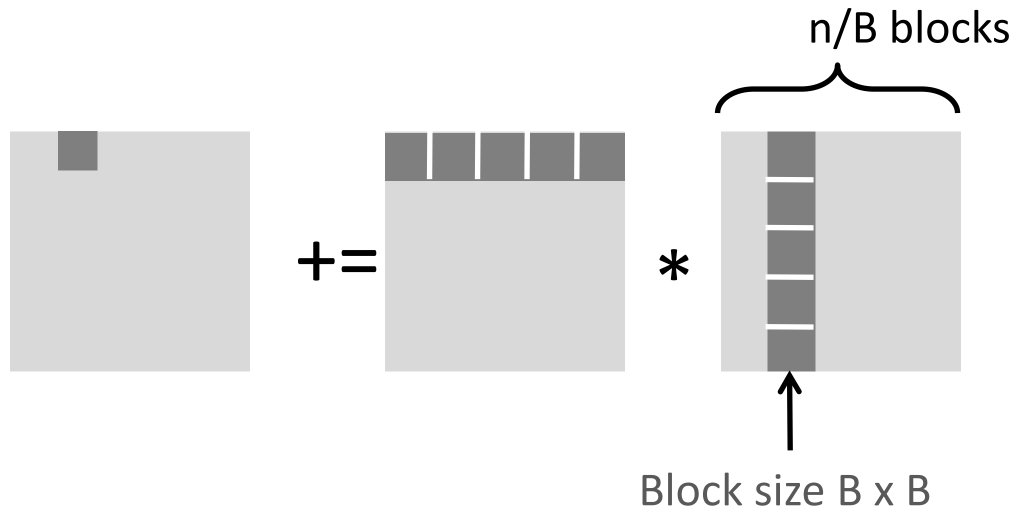
- First (block) iteration:
  - B^2 elements in each block, so $B^2/4$ misses for each block
  - n/B blocks in each row/col, so $2 * n/B * B^2/4 = nB/2$ misses in first iteration (omitting matrix c)

  - Afterwards in cache (schematic)



Cache Miss Analysis

- Assume:
  - Cache block = 4 doubles
  - Cache size C << n (much smaller than n)
  - Three blocks fit into cache: $3B^2 < C$
- Second (block) iteration:
  - Same as first iteration
  - $2 * n/B * B^2/4 = nB/2$



- Total misses:
  - $nB/2 * (n/B)^2 = n^3/(2B)$

Blocking Summary

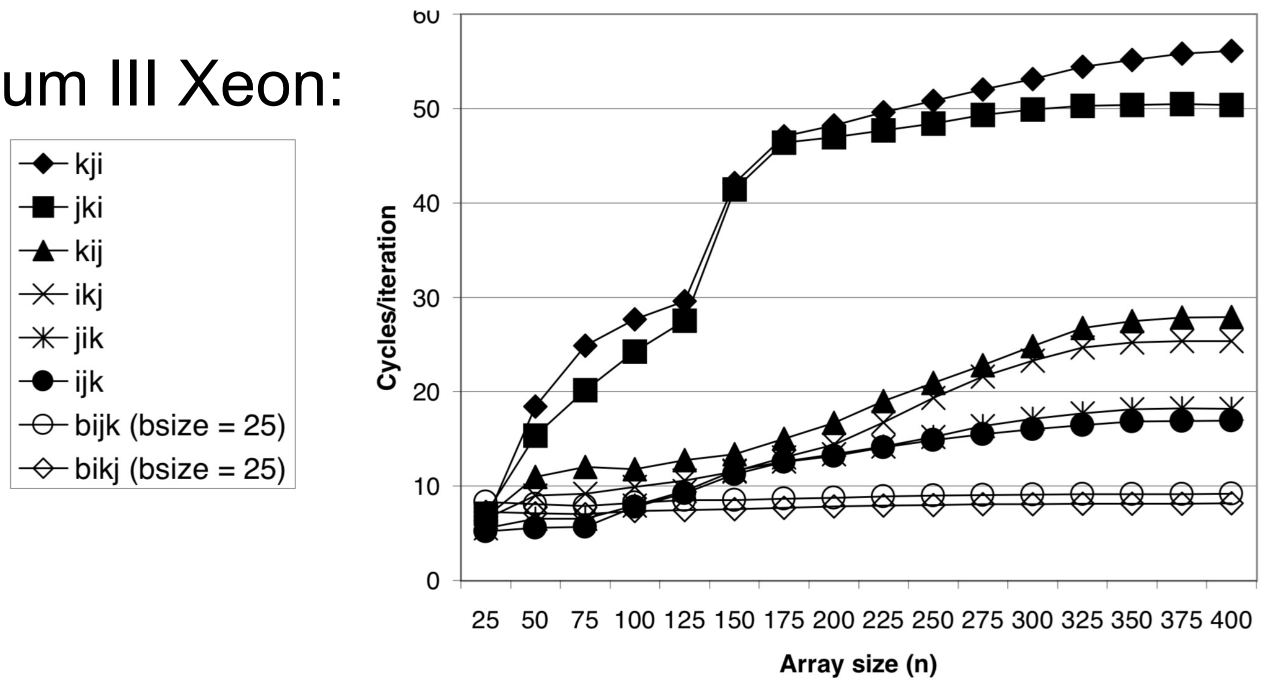
- No blocking: $(5/4) * n^3$
- Blocking: $n^3 / (2B)$

- Suggest largest possible block size B, but limit $3B^2 < C$!

- Reason for dramatic difference:
  - Matrix multiplication has inherent temporal locality:
    - Input data: $3n^2$, computation $2n^3$
    - Every array elements used O(n) times!
  - But program has to be written properly

A reality check

- This analysis only holds on some machines!
- Intel Core i7 does aggressive pre-fetching for one-stride programs, so blocking doesn't actually improve performance
- But on a Pentium III Xeon:



And that's the end of Part 1