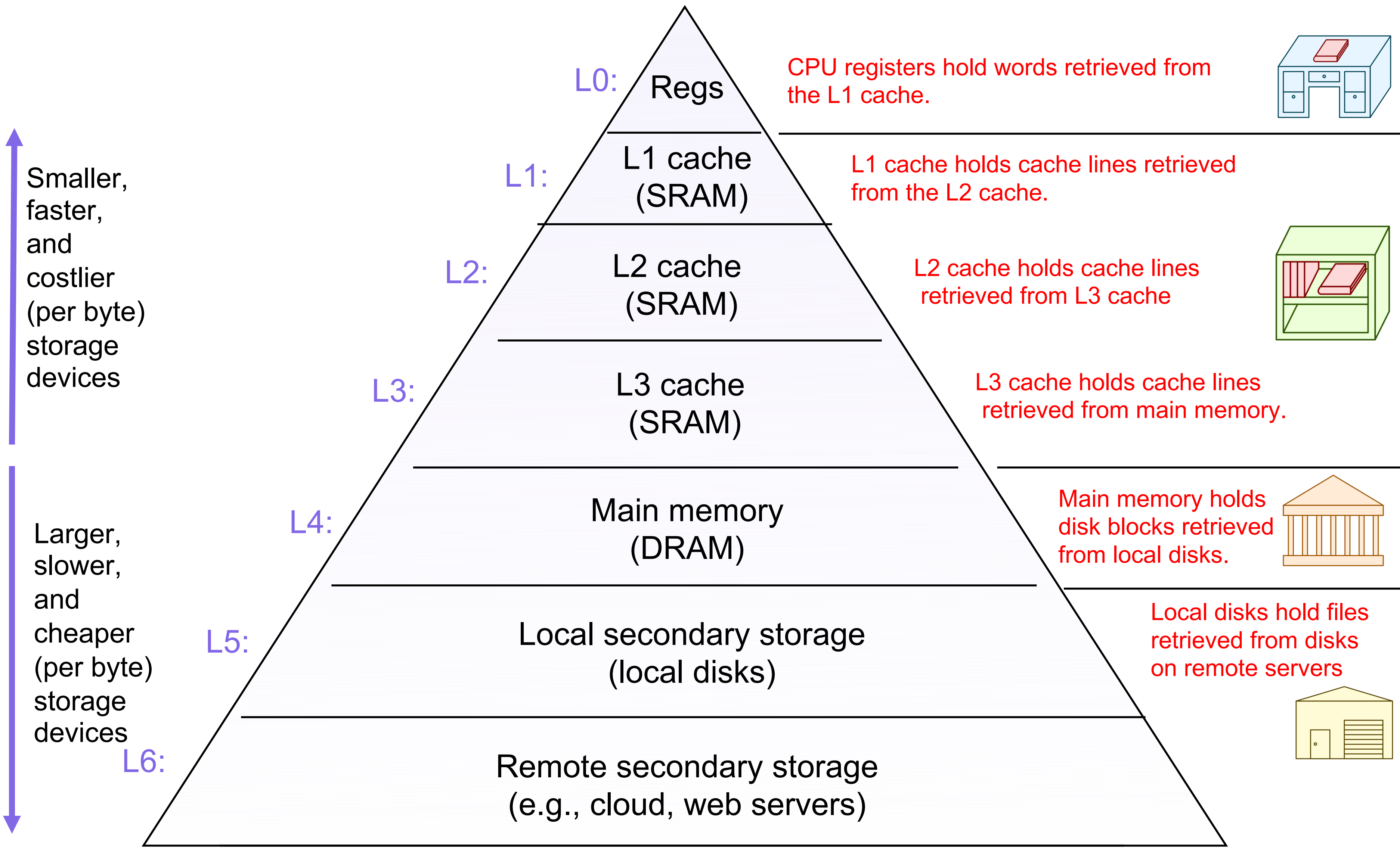
Lecture 16: Optimization with Caches

CS 105 Spring 2021

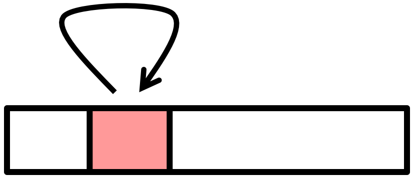
Review: Memory Hierarchy



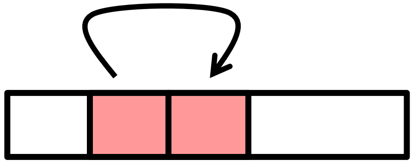
Review: Principle of Locality

Programs tend to use data and instructions with addresses near or equal to those they have used recently

- Temporal locality:
  - Recently referenced items are likely to be referenced again in the near future

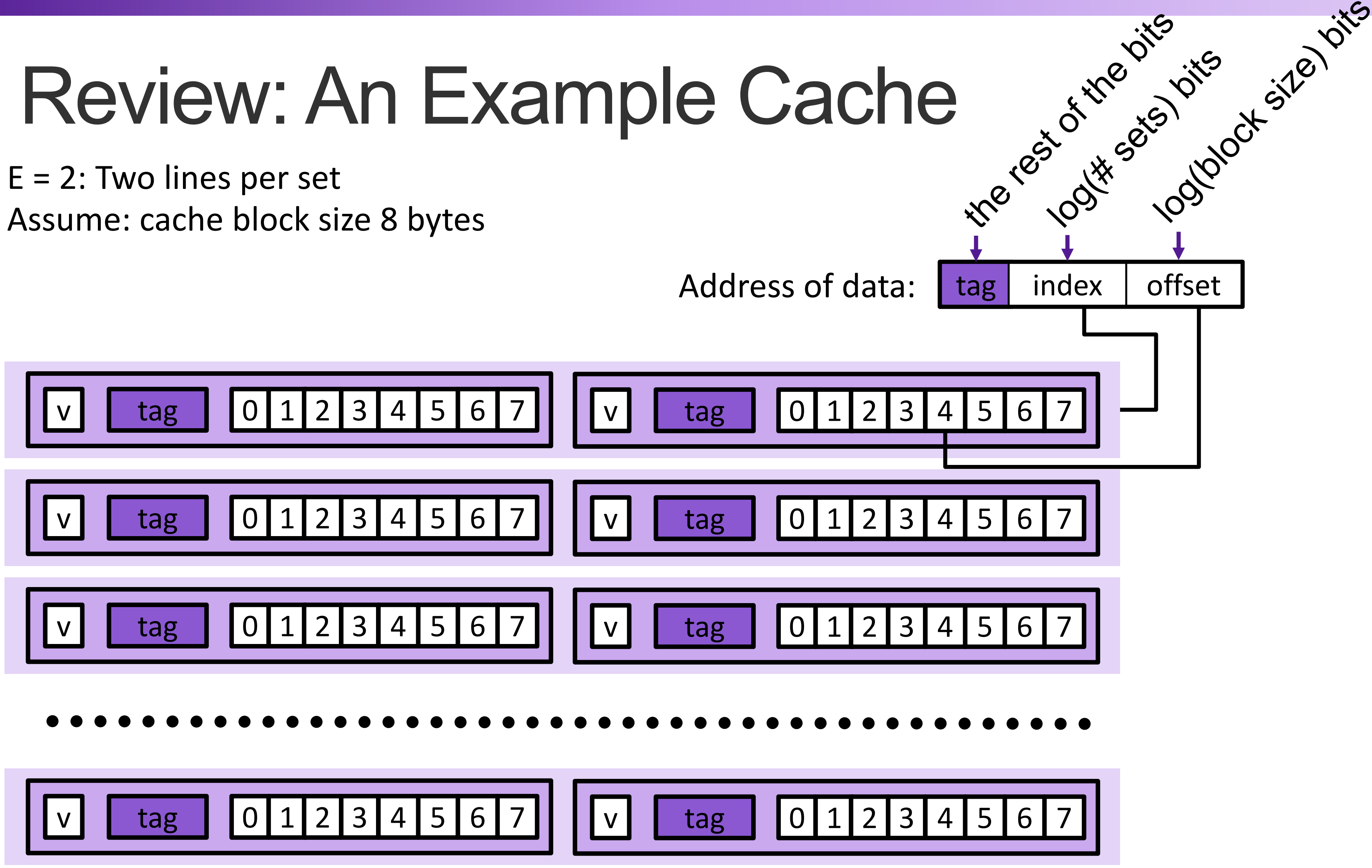


- Spatial locality:
  - Items with nearby addresses tend to be referenced close together in time

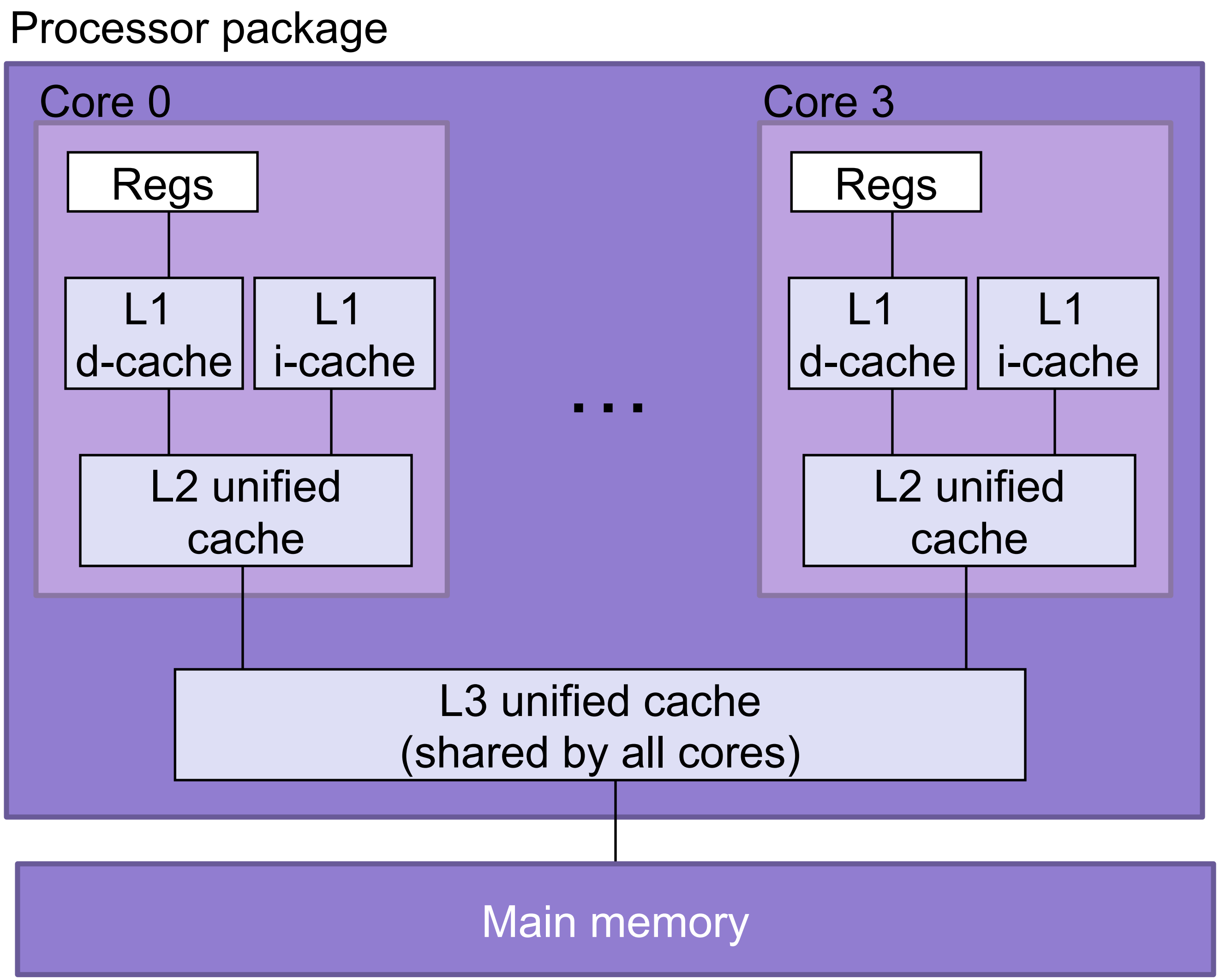


Review: An Example Cache

E = 2: Two lines per set
Assume: cache block size 8 bytes



Typical Intel Core i7 Hierarchy



L1 i-cache and d-cache:
32 KB, 8-way,
Access: 4 cycles

L2 unified cache:
256 KB, 8-way,
Access: 10 cycles

L3 unified cache:
8 MB, 16-way,
Access: 40-75 cycles

Block size: 64 bytes for all caches.

Cache Performance Metrics

- Miss Rate
  - Fraction of memory references not found in cache (misses / accesses)
  - Typically 3-10% for L1
  - can be quite small (e.g., < 1%) for L2, depending on size, etc.
- Hit Time
  - Time to deliver a line in the cache to the processor
    - includes time to determine whether the line is in the cache
  - Typically 4 clock cycles for L1, 10 clock cycles for L2
- Miss Penalty
  - Additional time required because of a miss
    - typically 50-200 cycles for main memory (Trend: increasing!)

Memory Performance with Caching

- **Read throughput (aka read bandwidth):** Number of bytes read from memory per second (MB/s)

- **Memory mountain:** Measured read throughput as a function of spatial and temporal locality.
  - Compact way to characterize memory system performance.

Memory Mountain Test Function

Call `test()` with many combinations of `elems` and `stride`.

`For each elems and stride:`

`1. Call test() once to warm up the caches.`

`2. Call test() again and measure the read throughput(MB/s)`

```
long data[MAXELEMS];  /* Global array to traverse */

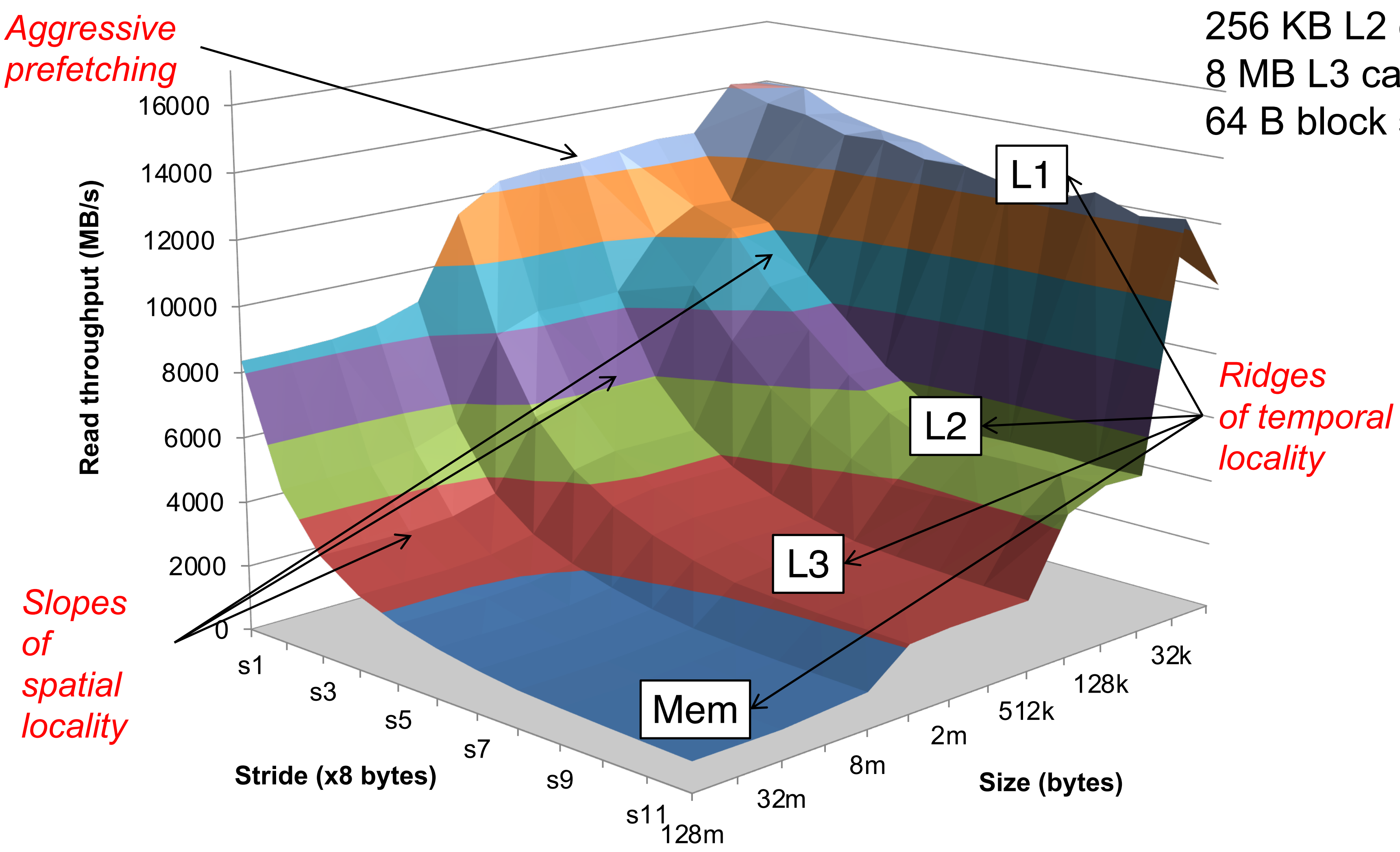
/* test - Iterate over first "elems" elements of
 *        array “data” with stride of "stride", using
 *        using 4x4 loop unrolling.
 */
int test(int elems, int stride) {
    long i, sx2=stride*2, sx3=stride*3, sx4=stride*4;
    long acc0 = 0, acc1 = 0, acc2 = 0, acc3 = 0;
    long length = elems, limit = length - sx4;

    /* Combine 4 elements at a time */
    for (i = 0; i < limit; i += sx4) {
        acc0 = acc0 + data[i];
        acc1 = acc1 + data[i+stride];
        acc2 = acc2 + data[i+sx2];
        acc3 = acc3 + data[i+sx3];
    }

    /* Finish any remaining elements */
    for (; i < length; i++) {
        acc0 = acc0 + data[i];
    }
    return ((acc0 + acc1) + (acc2 + acc3));
}
```

The Memory Mountain

Core i7 Haswell
2.1 GHz
32 KB L1 d-cache
256 KB L2 cache
8 MB L3 cache
64 B block size



Exercise 1: Locality

- Which of the following functions is better in terms of locality with respect to array src?

```
void copyij(int src[2048][2048],
            int dst[2048][2048])
{
  int i,j;
  for (i = 0; i < 2048; i++)
    for (j = 0; j < 2048; j++)
      dst[i][j] = src[i][j];
}
```

```
void copyji(int src[2048][2048],
            int dst[2048][2048])
{
  int i,j;
  for (j = 0; j < 2048; j++)
    for (i = 0; i < 2048; i++)
      dst[i][j] = src[i][j];
}
```

Exercise 1: Locality

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    for (j = 0; j < 2048; j++)
      dst[i][j] = src[i][j];
}
```

```
void copyji(int src[2048][2048],
            int dst[2048][2048])
{
  int i,j;
  for (j = 0; j < 2048; j++)
    for (i = 0; i < 2048; i++)
      dst[i][j] = src[i][j];
}
```

4.3ms

81.8ms

2.0 GHz Intel Core i7 Haswell

Writing Cache-Friendly Code

- Make the common case go fast
  - Focus on the inner loops of the core functions

- Minimize the misses in the inner loops
  - Repeated references to variables are good (**temporal locality**)
  - Stride-1 reference patterns are good (**spatial locality**)

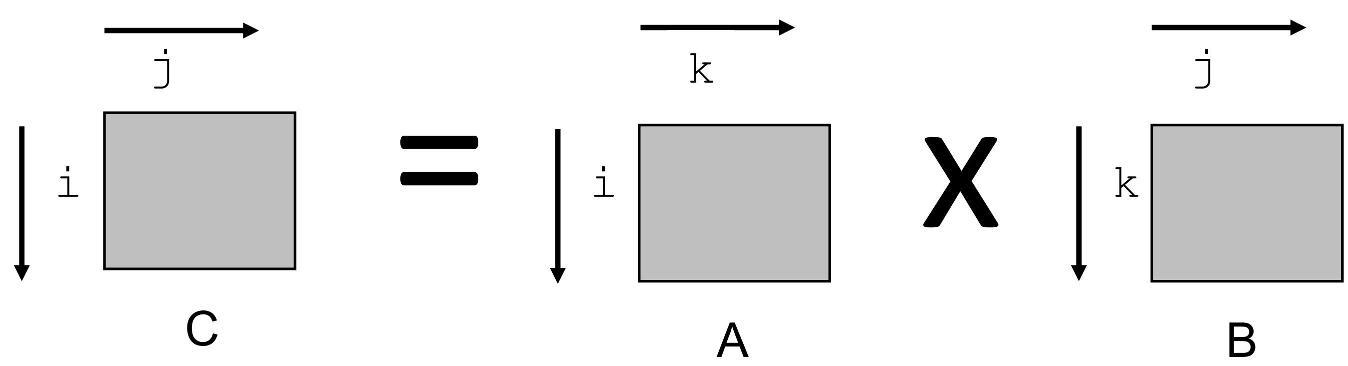
Example: Matrix Multiplication

- Multiply N x N matrices
- Matrix elements are doubles (8 bytes)
- $O(N^3)$ total operations
- N reads per source element
- N values summed per destination

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++){
      sum += a[i][k] * b[k][j];
    }
    c[i][j] = sum;
  }
}
```

Miss Rate Analysis for Matrix Multiply

- Assume:
  - Block size = 32B (big enough for four doubles)
  - Matrix dimension (N) is very large
    - Approximate 1/N as 0.0
  - Cache is not even big enough to hold multiple rows
- Analysis Method:
  - Look at access pattern of inner loop



Layout of C Arrays in Memory (review)

- C arrays allocated in row-major order
  - each row in contiguous memory locations

- Stepping through columns in one row:
  - accesses successive elements
  - if data block size (B) > sizeof(a_{ij}) bytes, exploit spatial locality
    - miss rate = sizeof(a_{ij}) / B

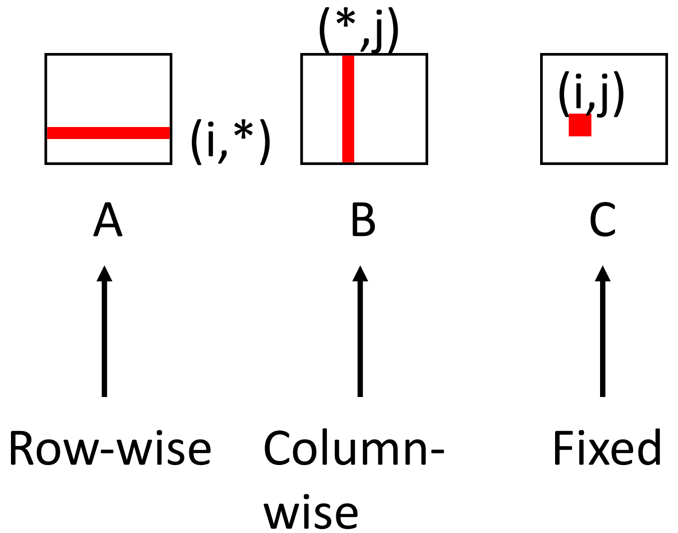
- Stepping through rows in one column:
  - accesses distant elements
  - no spatial locality!
    - miss rate = 1 (i.e. 100%)

Matrix Multiplication (ijk)

(jik is similar)

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

Inner loop:



Misses per inner loop iteration:

A	B	C
0.25	1.0	0.0

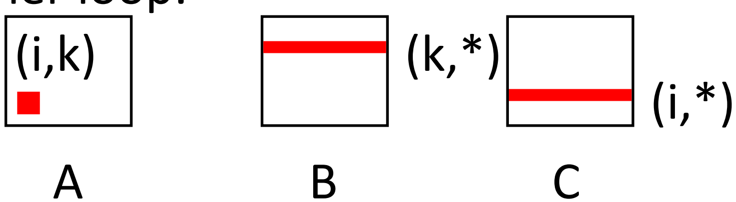
2 loads, no stores per inner loop iteration

Exercise 2: Matrix Multiplication

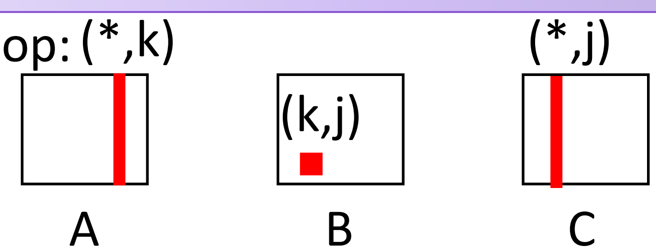
```
/* kij */
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

```
/* jki */
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```

Inner loop:



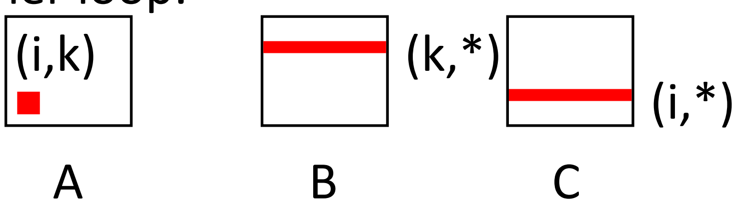
Inner loop:



Exercise 2: Matrix Multiplication

```
/* kij */
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

Inner loop:



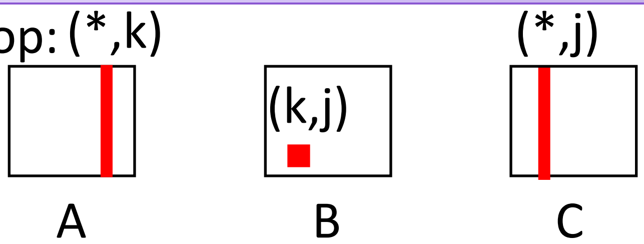
2 loads, 1 store per inner loop iteration

<u>Misses per inner loop iteration:</u>

<u>A</u>	<u>B</u>	<u>C</u>
0.0	0.25	0.25

```
/* jki */
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```

Inner loop:



2 loads, 1 store per inner loop iteration

<u>Misses per inner loop iteration:</u>

<u>A</u>	<u>B</u>	<u>C</u>
1.0	0.0	1.0

Summary of Matrix Multiplication

```
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
   sum = 0.0;
   for (k=0; k<n; k++)
     sum += a[i][k] * b[k][j];
   c[i][j] = sum;
 }
}
```

ijk (& jik):

- 2 loads, 0 stores
- misses/iter = 1.25

```
for (k=0; k<n; k++) {
 for (i=0; i<n; i++) {
  r = a[i][k];
  for (j=0; j<n; j++)
   c[i][j] += r * b[k][j];
 }
}
```

kij (& ikj):

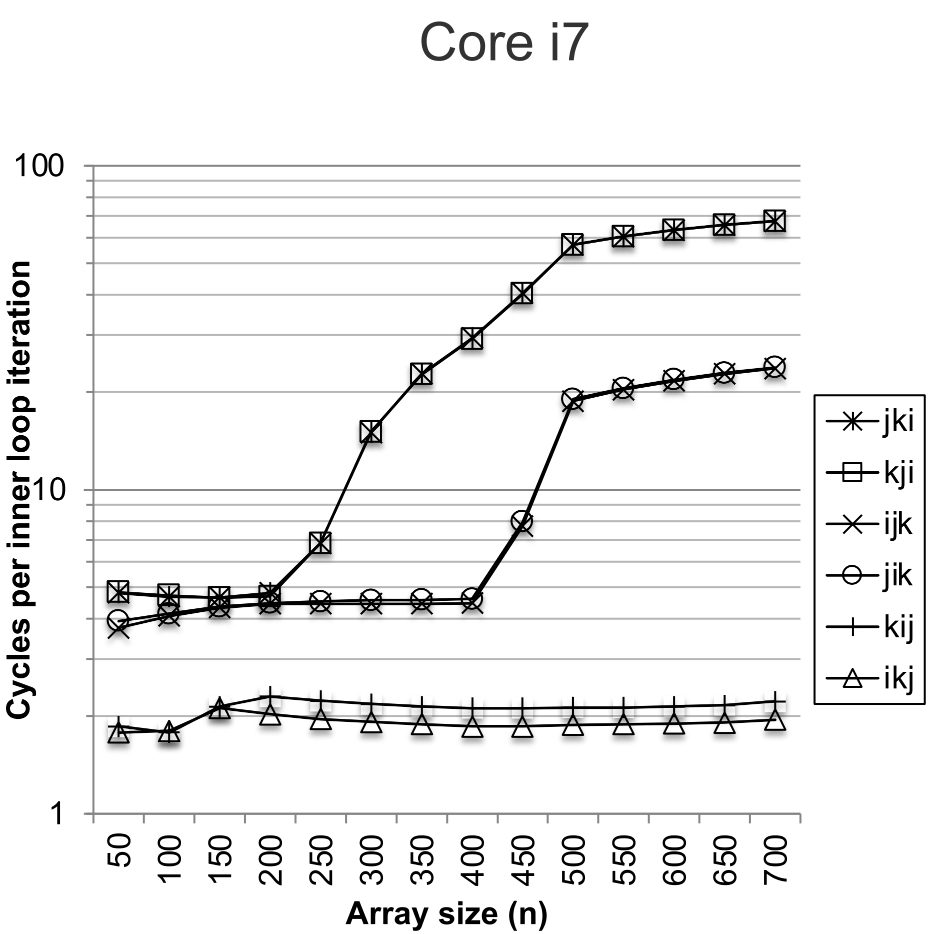
- 2 loads, 1 store
- misses/iter = 0.5

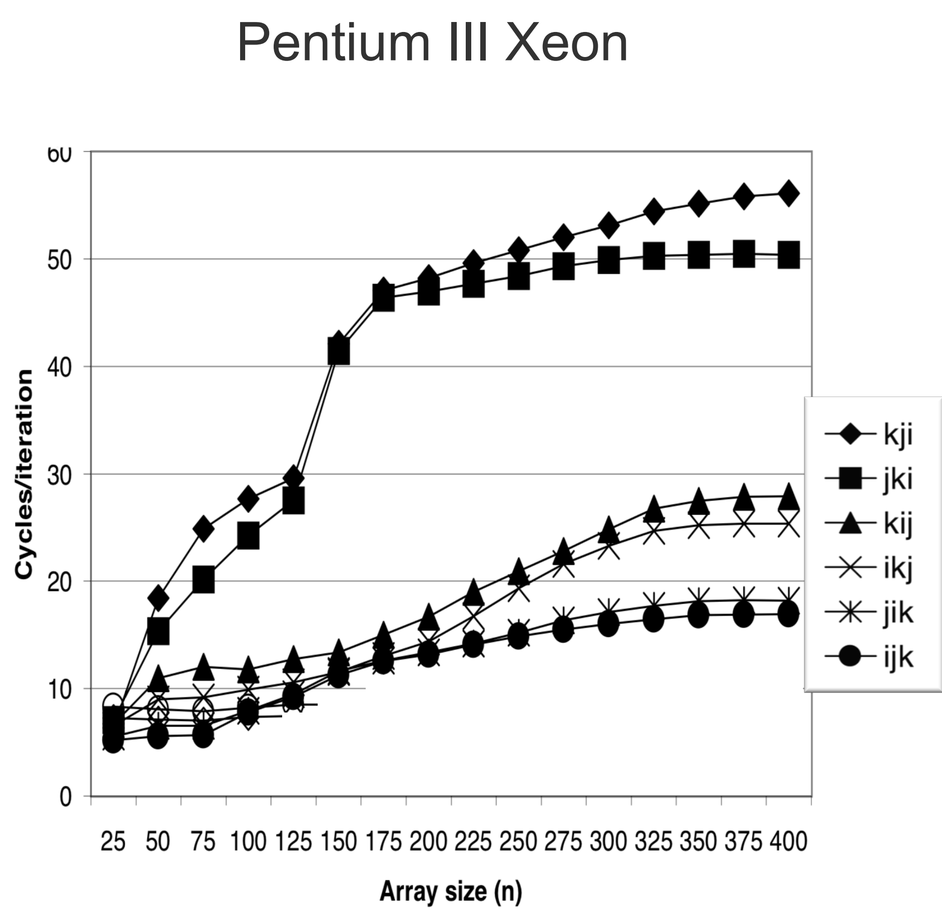
```
for (j=0; j<n; j++) {
 for (k=0; k<n; k++) {
   r = b[k][j];
   for (i=0; i<n; i++)
    c[i][j] += a[i][k] * r;
 }
}
```

jki (& kji):

- 2 loads, 1 store
- misses/iter = 2.0

Matrix Multiply Performance

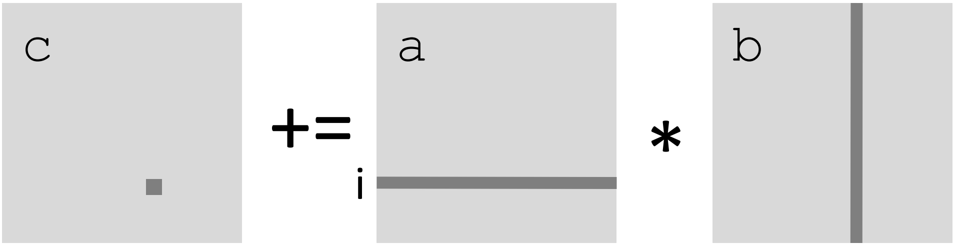




Can we do better?

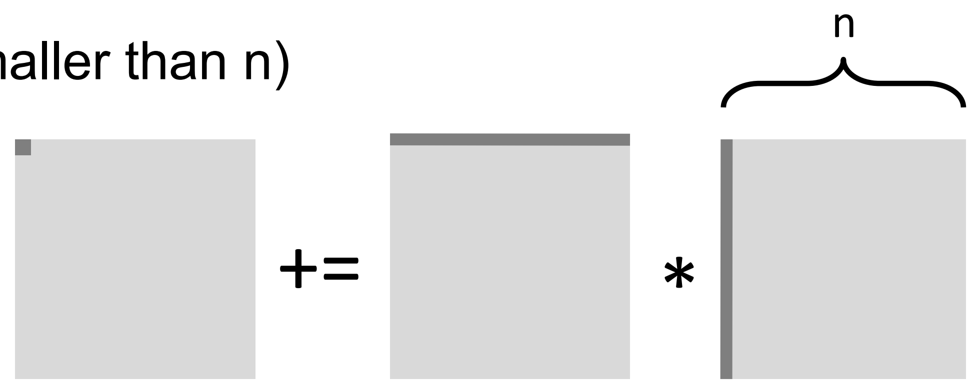
```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b  */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i++)
        for (j = 0; j < n; j++)
            for (k = 0; k < n; k++)
                c[i*n + j] += a[i*n + k] * b[k*n + j];
}
```



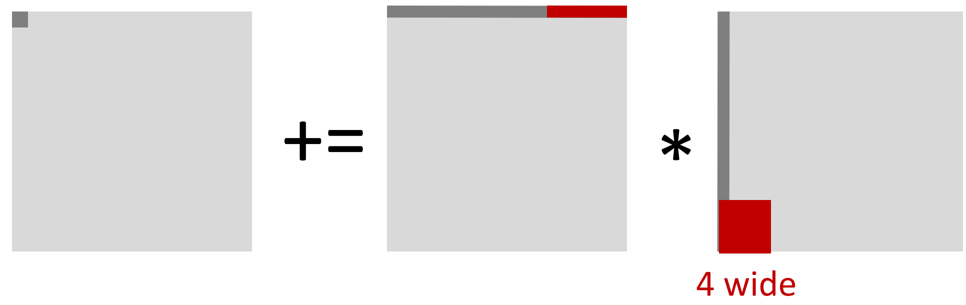
Cache Miss Analysis

- Assume:
  - Matrix elements are doubles
  - Cache block = 4 doubles
  - Cache size C << n (much smaller than n)



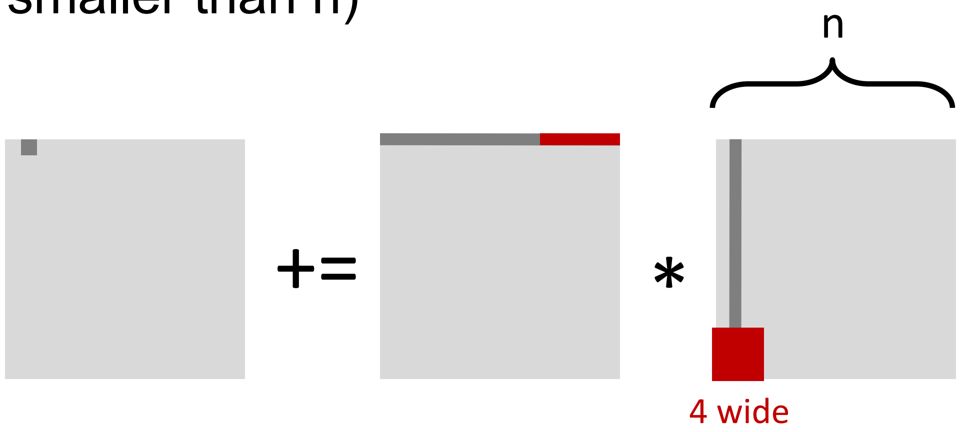
- First iteration:
  - n/4 + n = 5n/4 misses

  - Afterwards in cache: (schematic)



Cache Miss Analysis

- Assume:
  - Matrix elements are doubles
  - Cache block = 4 doubles
  - Cache size C << n (much smaller than n)
- Second iteration:
  - n/4 + n = 5n/4 misses

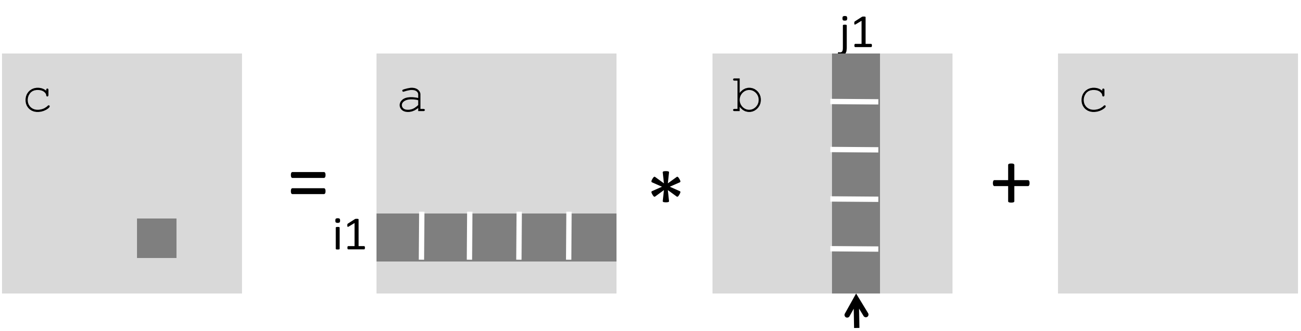


- Total misses:
  - $5n/4 * n^2 = (5/4) * n^3$

Blocked Matrix Multiplication

```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b  */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i+=B)
        for (j = 0; j < n; j+=B)
            for (k = 0; k < n; k+=B)
                /* B x B mini matrix multiplications */
                for (i1 = i; i1 < i+B; i++)
                    for (j1 = j; j1 < j+B; j++)
                        for (k1 = k; k1 < k+B; k++)
                            c[i1*n+j1] += a[i1*n + k1]*b[k1*n + j1];
}
```

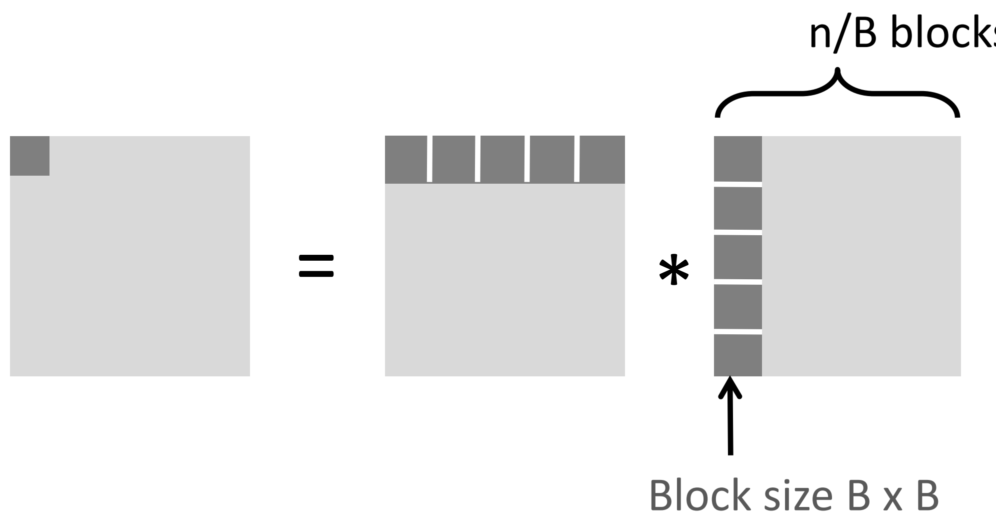


Block size B x B

Cache Miss Analysis

- Assume:
  - Cache block = 4 doubles
  - Cache size C << n (much smaller than n)
  - Three blocks ■ fit into cache: $3B^2 < C$

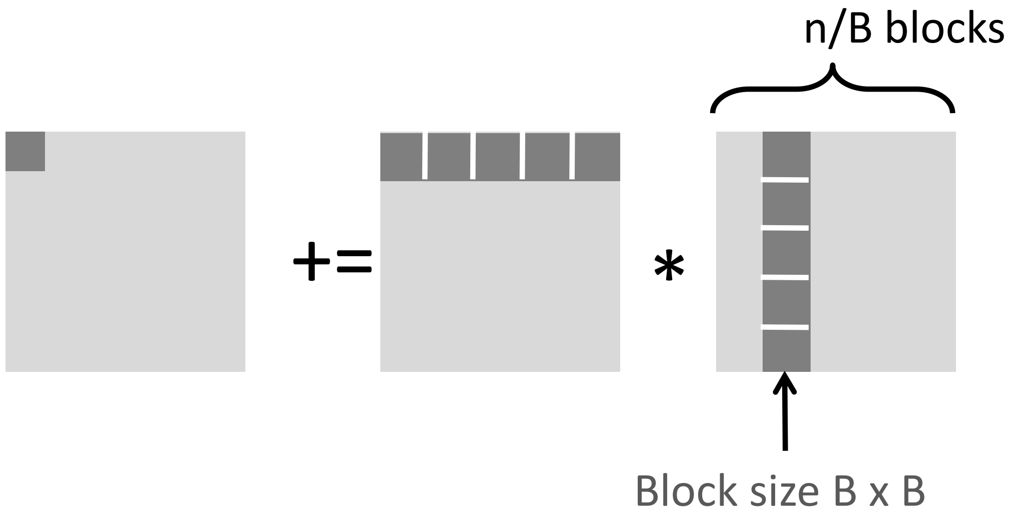
- First (block) iteration:
  - $B^2/4$ misses for each block
  - $2n/B * B^2/4 = nB/2$ (omitting matrix c)



  - Afterwards in cache (schematic)

Cache Miss Analysis

- Assume:
  - Cache block = 4 doubles
  - Cache size C << n (much smaller than n)
  - Three blocks fit into cache: $3B^2 < C$
- Second (block) iteration:
  - Same as first iteration
  - $2n/B * B^2/4 = nB/2$



- Total misses:
  - $nB/2 * (n/B)^2 = n^3/(2B)$

Blocking Summary

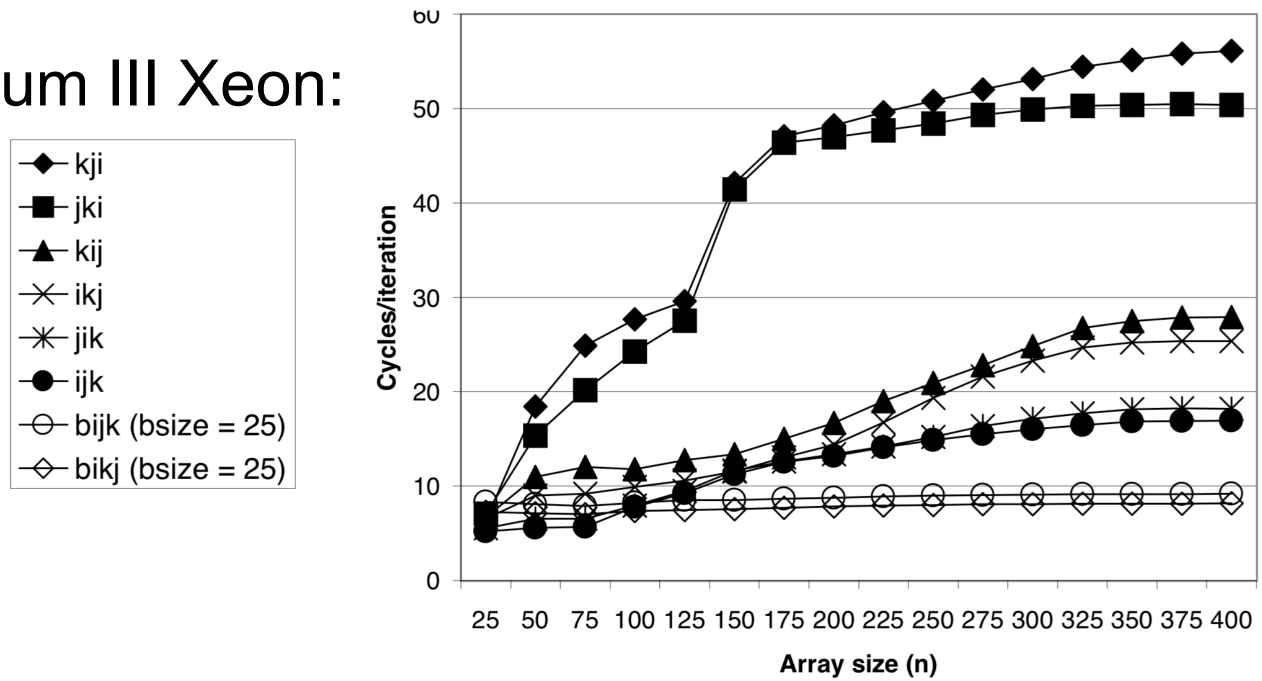
- No blocking: (5/4) * n^3
- Blocking: n^3 / (2B)

- Suggest largest possible block size B, but limit $3B^2 < C$!

- Reason for dramatic difference:
  - Matrix multiplication has inherent temporal locality:
    - Input data: $3n^2$, computation $2n^3$
    - Every array elements used O(n) times!
  - But program has to be written properly

A reality check

- This analysis only holds on some machines!
- Intel Core i7 does aggressive pre-fetching for one-stride programs, so blocking doesn't actually improve performance
- But on a Pentium III Xeon:



And that's the end of Part 1

Exercise 3: Feedback

1. Rate how well you think this recorded lecture worked
    1. Better than an in-person class
    2. About as well as an in-person class
    3. Less well than an in-person class, but you still learned something
    4. Total waste of time, you didn't learn anything
2. How much time did you spend on this video lecture (including time spent on exercises)?
3. Do you have any questions that you would like me to address in this week's problem session?
4. Do you have any other comments or feedback?