

# Adversarial Search 2

CS51A  
David Kauchak  
Spring 2019

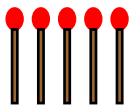
*Some material borrowed from :*  
Sara Owsley Sood and others

## Admin

Assignment 10

Pre-registration

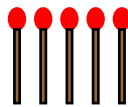
## Baby Nim



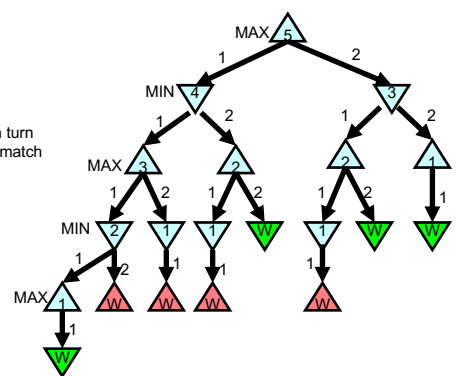
Take 1 or 2 at each turn  
Goal: take the last match

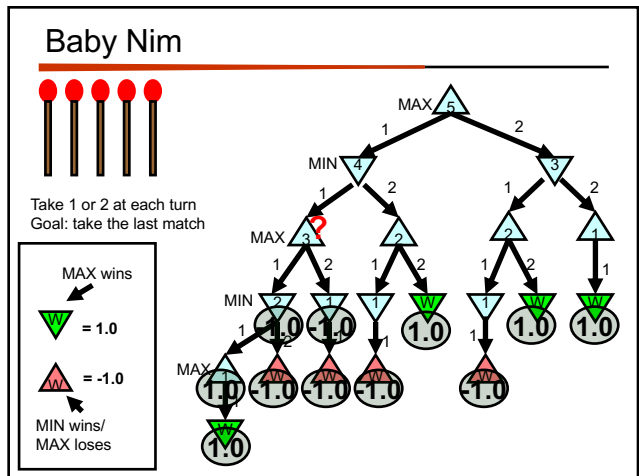
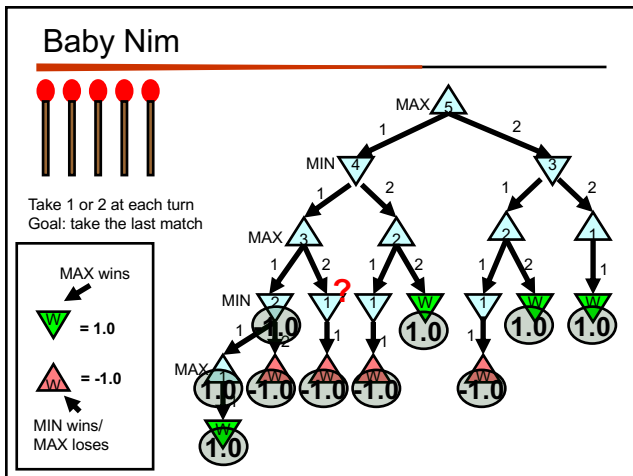
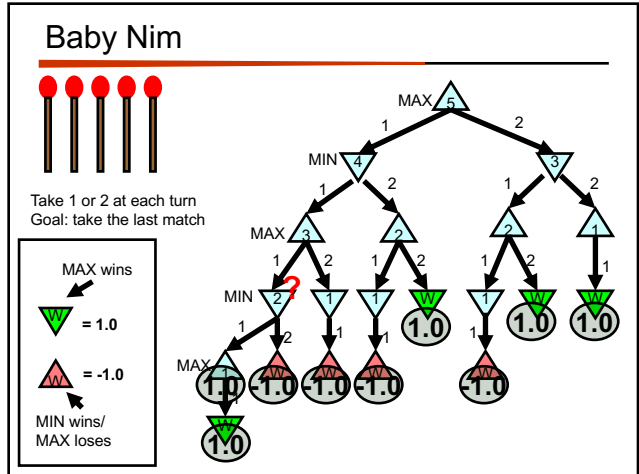
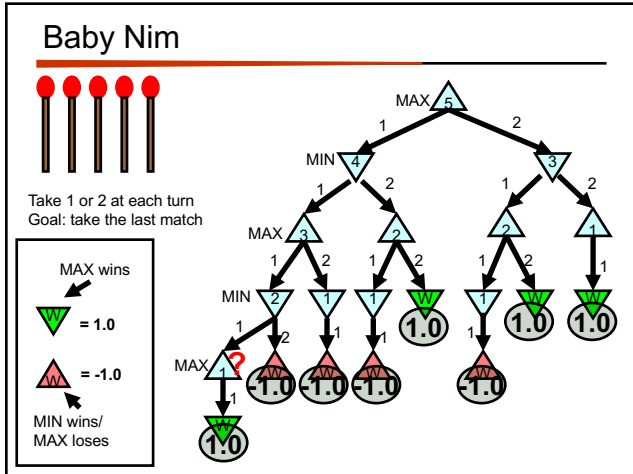
What move should I take?

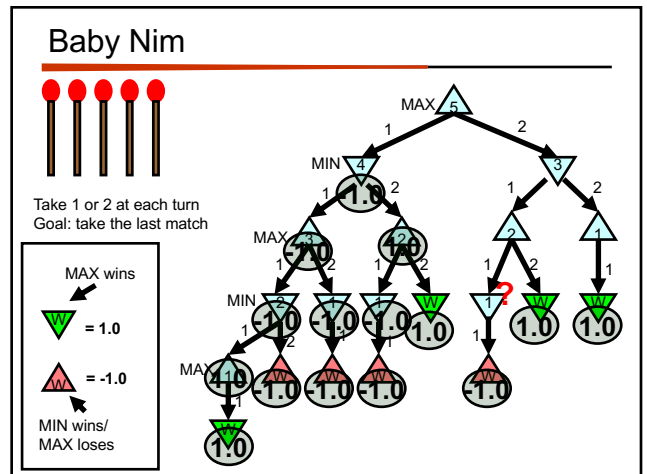
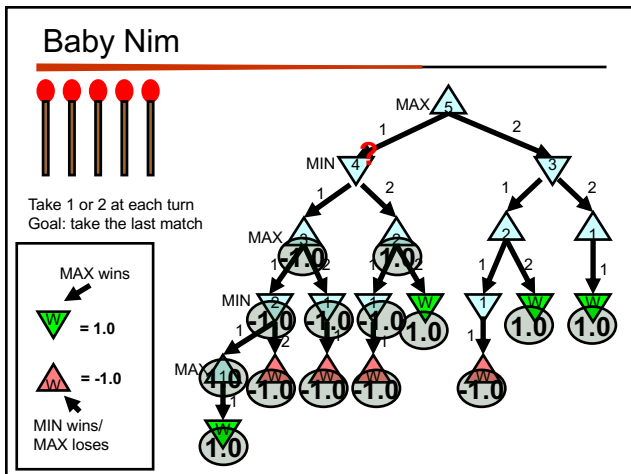
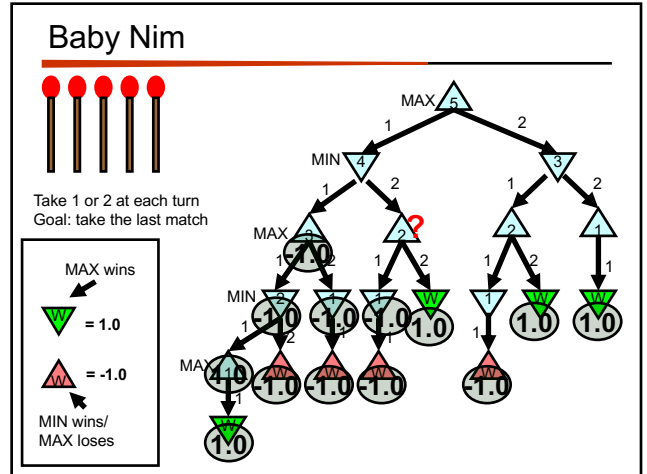
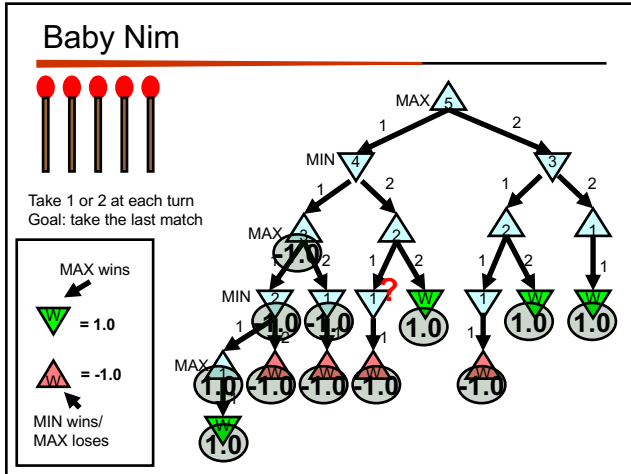
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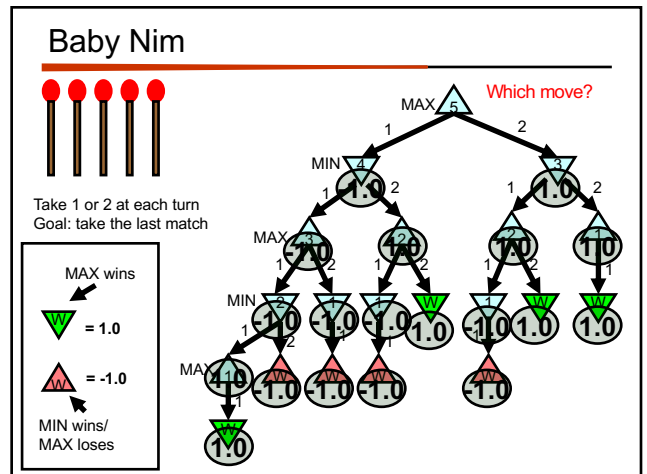
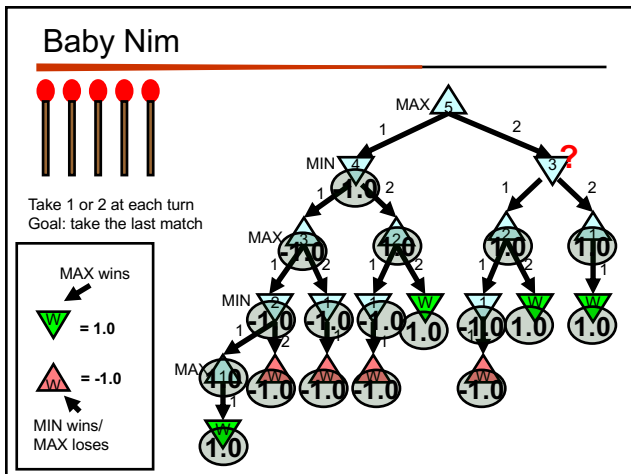
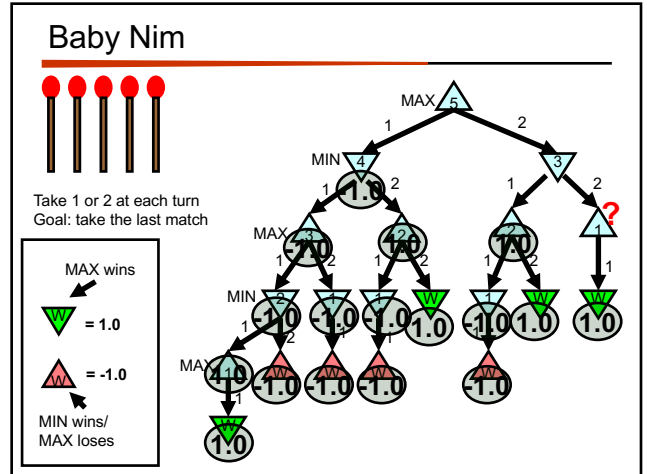
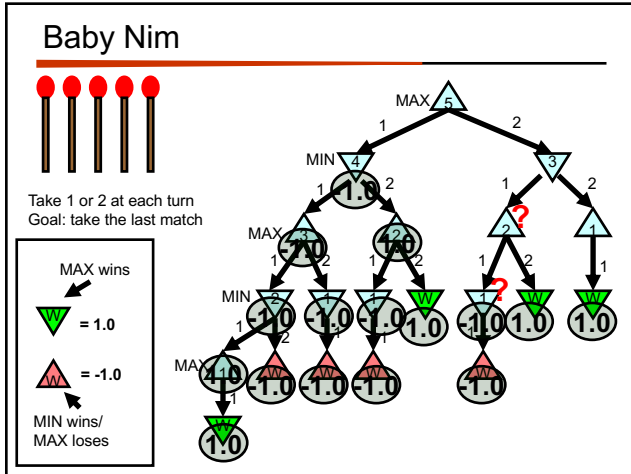


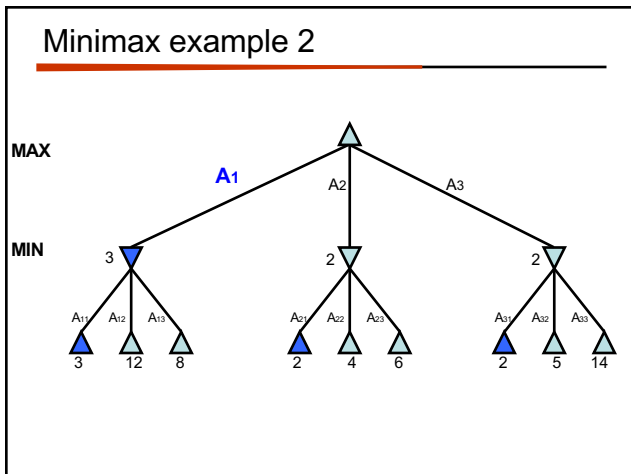
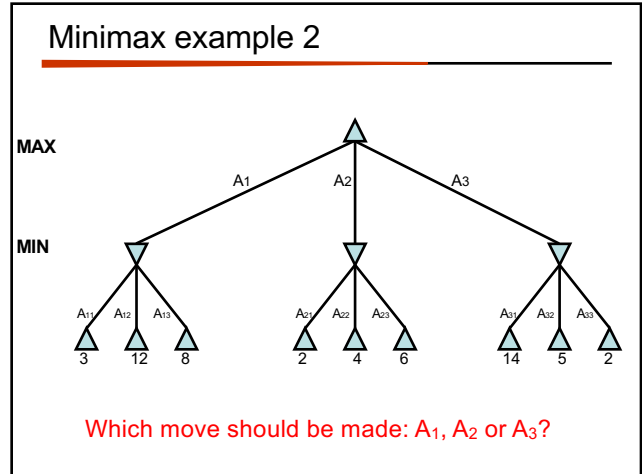
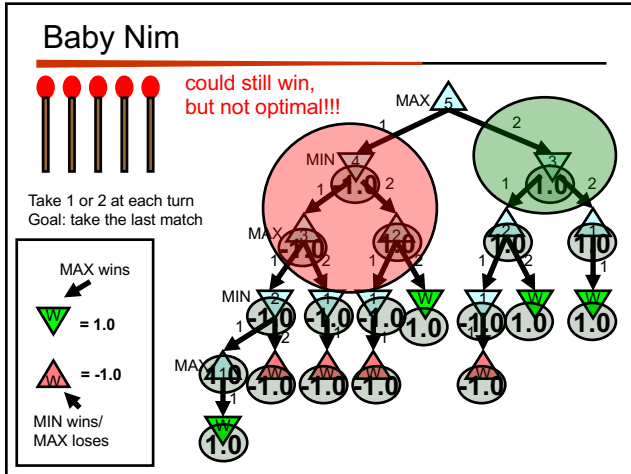
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### Properties of minimax

Minimax is optimal!

Are we done?

## Games State Space Sizes

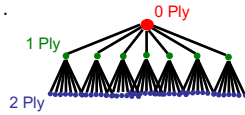
On average, there are ~35 possible moves that a chess player can make from any board configuration...



18 Ply!!

17005

Hydra at home in the United Arab Emirates...

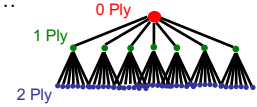


Branching Factor Estimates for different two-player games

|              |     |
|--------------|-----|
| Tic-tac-toe  | 4   |
| Connect Four | 7   |
| Checkers     | 10  |
| Othello      | 30  |
| Chess        | 35  |
| Go           | 300 |

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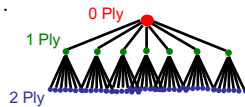
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Boundaries for qualitatively different games...

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Can search entire space

"solved" games

CHINOOK (2007) →

computer-dominated

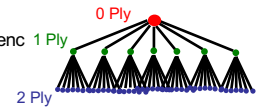
Can't ☹

Is this true? human-dominated

## Games State Space Sizes

AlphaGo (created by Google), in April 2016 beat one of the best Go players:

<http://www.nytimes.com/2016/04/05/science/google-alpha-go-artificial-intelligence.html>



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Can't ☹

What do we do?

## Alpha-Beta pruning

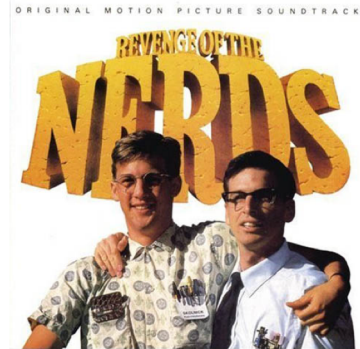
An optimal pruning strategy

- only prunes paths that are suboptimal (i.e. wouldn't be chosen by an optimal playing player)
- returns the *same* result as minimax, but faster



Name the movie ☺

## Alpha-Beta pruning

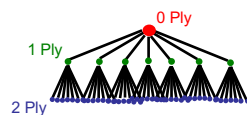


## Games State Space Sizes

Pruning helps get a bit deeper

For many games, still can't search the entire tree

Now what?



| Branching Factor Estimates for different two-player games |     |
|---|-----|
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computer-dominated

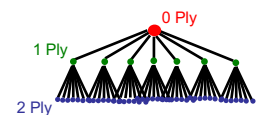
## Games State Space Sizes

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For many games, still can't search the entire tree

Go as deep as you can:

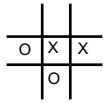
- *estimate* the score/quality of the state (called an evaluation function)
- use that instead of the real score



| Branching Factor Estimates for different two-player games |     |
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computer-dominated

## Tic Tac Toe evaluation functions



Ideas?

## Example Tic Tac Toe EVAL

**Tic Tac Toe**  
Assume MAX is using "X"

$EVAL(state) =$

if state is win for MAX:

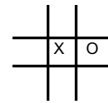
+  $\infty$

if state is win for MIN:

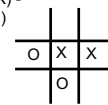
-  $\infty$

else:

(number of rows, columns and diagonals available to MAX) -  
(number of rows, columns and diagonals available to MIN)

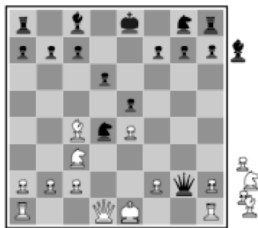


$$= 6 - 4 = 2$$



$$= 4 - 3 = 1$$

## Chess evaluation functions



Ideas?

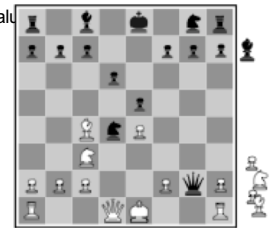
## Chess EVAL

Assume each piece has the following value

pawn = 1;  
knight = 3;  
bishop = 3;  
rook = 5;  
queen = 9;

$EVAL(state) =$

sum of the value of white pieces -  
sum of the value of black pieces



$$= 31 - 36 = -5$$

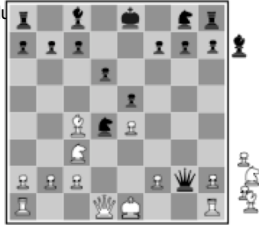


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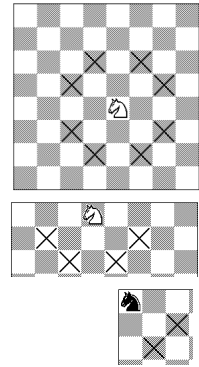
Any problems with this?

## Chess EVAL

Ignores actual positions!

Actual heuristic functions are often a weighted combination of features

$$EVAL(s) = w_1 f_1(s) + w_2 f_2(s) + w_3 f_3(s) + \dots$$



## Chess EVAL

$$EVAL(s) = w_1 f_1(s) + w_2 f_2(s) + w_3 f_3(s) + \dots$$

number of pawns  
 number of attacked knights  
 1 if king has knighted, 0 otherwise

A feature can be any numerical information about the board

- as general as the number of pawns
- to specific board configurations

Deep Blue: 8000 features!

## history/end-game tables

History

- keep track of the quality of moves from previous games
- use these instead of search

end-game tables

- do a reverse search of certain game configurations, for example all board configurations with king, rook and king
- tells you what to do in **any** configuration meeting this criterion
- if you ever see one of these during search, you lookup exactly what to do

## end-game tables

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Devastatingly good

Allows much deeper branching

- for example, if the end-game table encodes a 20-move finish and we can search up to 14
- can search up to depth 34

Stiller (1996) explored all end-games with 5 pieces

- one case check-mate required 262 moves!

Knoval (2006) explored all end-games with 6 pieces

- one case check-mate required 517 moves!

Traditional rules of chess require a capture or pawn move within 50 or it's a stalemate

## Opening moves

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At the very beginning, we're the farthest possible from any goal state

People are good with opening moves

Tons of books, etc. on opening moves

Most chess programs use a database of opening moves rather than search

## Nim

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K piles of coins

On your turn you must take one or more coins from one pile

Player that takes the last coin wins

Example:

<https://www.goobix.com/games/nim/>