CS151 - Written Problem 4
Solutions

1. Exercise 13.8

   a. p(toothache) = 0.108 + 0.012 + 0.016 + 0.064 = 0.2
   b. **P**(Cavity) = $< 0.2, 0.8 >$
   p(cavity) = 0.108 + 0.012 + 0.072 + 0.008 = 0.2, P($\sim$cavity) = 1-0.2=0.8
   c. **P**(Toothache | cavity) = <(.108+.012)/0.2, (0.072+0.008)/0.2> = <0.6, 0.4>
   d. p(cavity *or* catch) = 0.108+0.012+0.016+0.064+0.072+0.144=0.416
   P(Cavity | toothache *or* catch) =
   <(0.108+0.012+0.072)/0.416, (0.016+0.064+0.144)/0.416> =
   <0.4615, 0.5384>

2. Exercise 13.21

   This problem is like the cancer problem and you need to use Bayes rule to calculate this probability:

   p(taxiIsBlue | taxiLooksBlue) = α p(taxiLooksBlue | taxiIsBlue) p(taxiIsBlue)

   You know p(taxiLooksBlue | taxiIsBlue) = 0.75, but you do not know the prior probability that the taxi is blue, so you cant calculate the posterior.

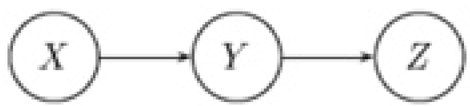
   Once you're given that 9 out of 10 taxis in Athens are green, then you know that p(taxiIsBlue) = 0.1, so you can calculate:

p(taxiIsBlue | taxiLooksBlue = α * 0.75 * 0.1 = α * 0.075
p(~taxiIsBlue | taxiLooksBlue) = α * 0.25 * 0.9 = α * 0.225

p(taxiIsBlue | taxiLooksBlue) = 0.075 / (0.075+0.225) = 0.25
p(~taxiIsBlue | taxiLooksBlue) = 1-0.25 = 0.75

The next two problems are taken from
http://www-nlp.stanford.edu/~grenager/cs121//handouts/hw2.pdf

3. In this problem were going to prove the conditional independence properties of the following Bayesian network:



(a) What are the conditional probability distributions (CPDs) that are represented in this Bayesian network?

$P(Z|Y)$
$P(Y|X)$

(b) Write down the joint probability distribution over X, Y , and Z as represented by this Bayesian network. This expression should be written in terms of the CPDs you enumerated in a. (plus any unconditional distributions).

$P(X, Y, Z) = P(X)P(Y|X)P(Z|Y)$

(c) Now write down an expression in terms of these for $P(X, Z)$, the marginal probability of X and Z (hint: sum the variable Y "out" from the joint distribution you wrote above).

$P(X, Z) = P(X)\sum_y P(y|X)P(Z|y)$

(d) Based on the expression in c., and the definition of independence, are X and Z independent?

No. This would only be the case if
$\sum_y P(y|X)P(Z|y) = P(Z)$

Notice as well that this expression still has a dependence on X.

(e) Write down an expression for $P(X, Z|Y)$, again in terms of these simplified probability distributions

$$P(X, Y, Z) = P(X, Z|Y)P(Y) \qquad \text{(using the chain rule)}$$

$$\begin{aligned} P(X, Z|Y) &= P(X, Y, Z)/P(Y) && \text{(with some math)} \\ &= P(X)P(Y|X)P(Z|Y)/P(Y) && \text{(from part b. above)} \end{aligned}$$

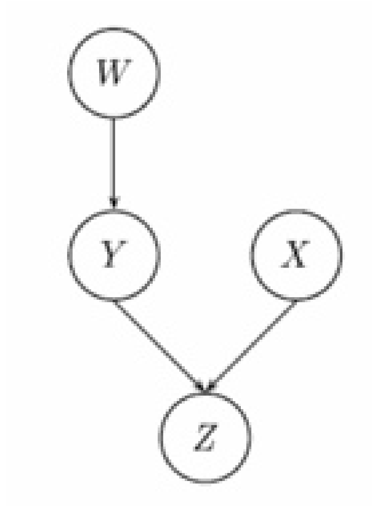
and using Bayes' rule we know that:
$P(Y|X) = P(X|Y)P(Y)/P(X)$

which gives us: $P(X, Z|Y) = P(X|Y)P(Z|Y)$

(f) Based on this expression, and the definition of conditional independence, are X and Z conditionally independent given Y?

Yes, the equation above is the definition of conditional independence between X and Z given Y. The only way that Z is influenced by X is through Y, so conditioned on Y, X and Z are independent.

4. In this question we examine the conditional independence assumptions encoded in the Bayesian network graph topology. Consider the following Bayesian network:



(a) Write down all the independencies not conditioned on other variables that are enforced by this Bayesian network, using the notation $A \perp\!\!\!\perp B$ to mean that A is independent of B.

$X \perp\!\!\!\perp W$
$X \perp\!\!\!\perp Y$
$W, Y \perp\!\!\!\perp X$

(b) Write down three independencies which do not necessarily hold in this Bayesian network.

Z is not independent of X
Z is not independent of Y
Z is not independent of W
Y is not independent of W

(c) Write down all the conditional independencies that are enforced by this Bayesian network that are not superseded by unconditional independencies, using the notation $A \perp\!\!\!\perp B|C$ to mean that A is conditionally independent of B given C. For example, you wouldn't include $Y \perp\!\!\!\perp X|W$, but this is superseded by $Y \perp\!\!\!\perp ofX$.

$W \perp\!\!\!\perp Z|Y$

(d) Write down three conditional independencies which do not necessarily hold in this Bayesian network.

$Y \perp\!\!\!\perp Z|W$
$W \perp\!\!\!\perp Y|Z$
$Y \perp\!\!\!\perp X|Z$ (notice here that by conditioning on Z, we actually made two independent variables dependent.