

CS062

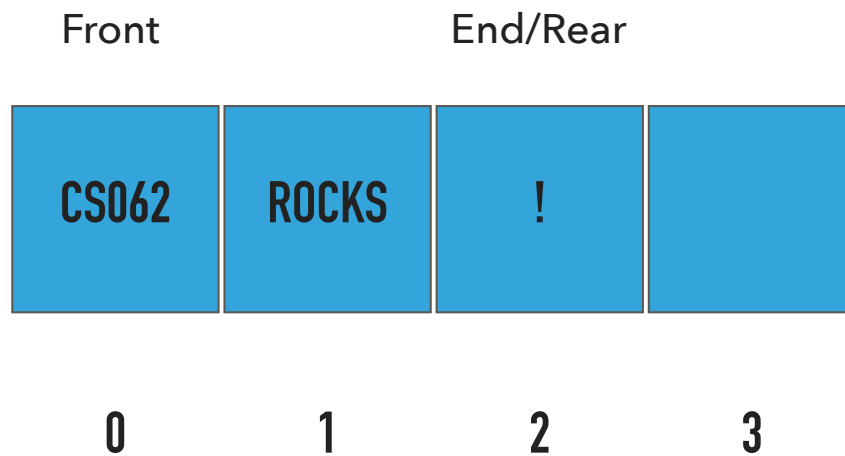
DATA STRUCTURES AND ADVANCED PROGRAMMING

7: Singly Linked Lists



Tom Yeh
he/him/his

`add(Item item)`: Appends the item to the end of the ArrayList



Capacity = 4

Number of items = 3

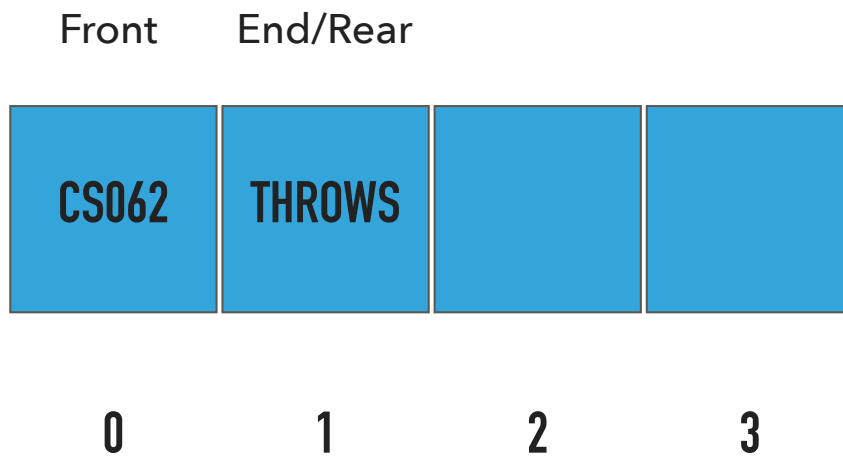
**DOUBLE CAPACITY SINCE IT'S FULL
AND THEN ADD NEW ITEM
INCREASE NUMBER OF ITEMS**

```
al.add("!");
```

What should happen?

```
al.add(1, "THROWS");
```

`remove()`: Retrieves and removes item from the end of ArrayList



Capacity = 4

Number of items = 2

`al.remove();`

**REMOVE ITEM FROM THE END
HALVE CAPACITY WHEN 1/4 FULL**

What should happen?

`al.remove(0);`

Asymptotic analysis simplifies analyzing worst-case performance

- ▶ Sorting them by increasing rate of growth:
 - ▶ $O(1)$, $O(\log n)$, $O(n)$, $O(n \log n)$, $O(n^2)$, $O(n^3)$, $O(2^n)$, $O(n!)$

How to interpret Big O

- ▶ $O(1)$ or "order one": running time does not change as size of the problem changes, that is running time stays constant and independent of problem size.
- ▶ $O(\log n)$ or "order log n": running time increases as problem size grows. Whenever problem size doubles, running time increases by a constant.
- ▶ $O(n)$ or "order n": time increases proportionally to the the rate of growth of the size of the problem, that is in a linear rate. Double the problem size, you get double running time.
- ▶ $O(n^2)$ or "order n squared": Double the problem size you get quadruple running time.

Lecture 6: Resizable Arrays

- ▶ Background
- ▶ ArrayList
- ▶ Java Collections
- ▶ Theory of Algorithms
- ▶ Running Time of ArrayList operations

Worst-case performance of `add()` is $O(n)$

- **Cost model:** 1 for insertion, n for copying n items to a new array.
- **Worst-case:** If `ArrayList` is full, `add()` will need to **call `resize`** to create a new array of double the size, **copy all items**, **insert new one**.
- Total cost: $n + 1 = O(n)$.
- Realistically, this won't be happening often and worst-case analysis can be too strict. We will use **amortized time analysis** instead.

Amortized analysis

- **Amortized cost per operation:** for a sequence of n operations, it is the total cost of operations divided by n .
 - Simplest form of amortized analysis called aggregate method. More complicated methods exist, such as accounting (banking) and potential (physicist's).

Amortized analysis for n `add()` operations

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Insertion Cost	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Copying Cost	0	1	2	0	4	0	0	0	8	0	0	0	0	0	0	0	16
Total Cost	1	2	3	1	5	1	1	1	9	1	1	1	1	1	1	1	17

- As the ArrayList increases, doubling happens *half as often* but costs *twice as much*.
- $O(\text{total cost}) = \sum (\text{"cost of insertions"}) + \sum (\text{"cost of copying"})$
- $\sum (\text{"cost of insertions"}) = n.$
- $\sum (\text{"cost of copying"}) = 1 + 2 + 2^2 + \dots + 2^{\log_2(n-1)} \leq 2n.$
- $O(\text{total cost}) \leq 3n$, therefore amortized cost is $\leq \frac{3n}{n} = 3 = O(1)$, but "lumpy".

Amortized analysis for n `add()` operations when increasing ArrayList by 1.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Insertion Cost	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Copying Cost	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Total Cost	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17

- ▶ $\sum (\text{"cost of insertions"}) = n.$
- ▶ $\sum (\text{"cost of copying"}) = 1 + 2 + 3 + \dots + n - 1 = n(n - 1)/2.$
- ▶ $O(\text{total cost}) = n + n(n - 1)/2 = n(n + 1)/2$, therefore amortized cost is $(n + 1)/2$ or $O(n).$
- ▶ Same idea when increasing ArrayList size by a constant.
 - ▶ This is why in the lab yesterday, we saw that doubling was the fastest and linear(1) the slowest

Readings:

- ▶ Oracle's guides:
 - ▶ Collections: <https://docs.oracle.com/javase/tutorial/collections/intro/index.html>
 - ▶ ArrayLists: <https://docs.oracle.com/javase/8/docs/api/java/util/ArrayList.html>
- ▶ Textbook:
 - ▶ Chapter 1.3 (Page 136-137)
 - ▶ Chapter 1.4 (pages 197-199)
- ▶ Textbook Website:
 - ▶ Resizable arrays: <https://algs4.cs.princeton.edu/13stacks/>
 - ▶ Analysis of Algorithms: <https://algs4.cs.princeton.edu/14analysis/>

Practice Problems:

- ▶ 1.4.1, 1.4.5 - 1.4.7, 1.4.32, 1.4.35-1.4.36.

Readings:

- ▶ Oracle's guides:
 - ▶ Collections: <https://docs.oracle.com/javase/tutorial/collections/intro/index.html>
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Practice Problems:

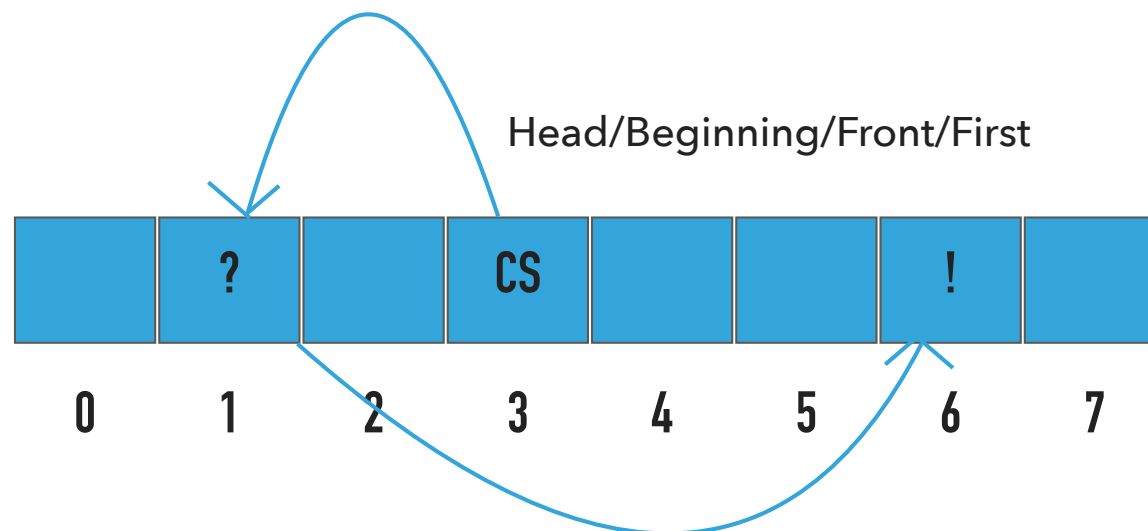
- ▶ 1.4.1, 1.4.5 - 1.4.7, 1.4.32, 1.4.35-1.4.36.

Lecture 7: Singly Linked Lists

- ▶ Singly Linked Lists

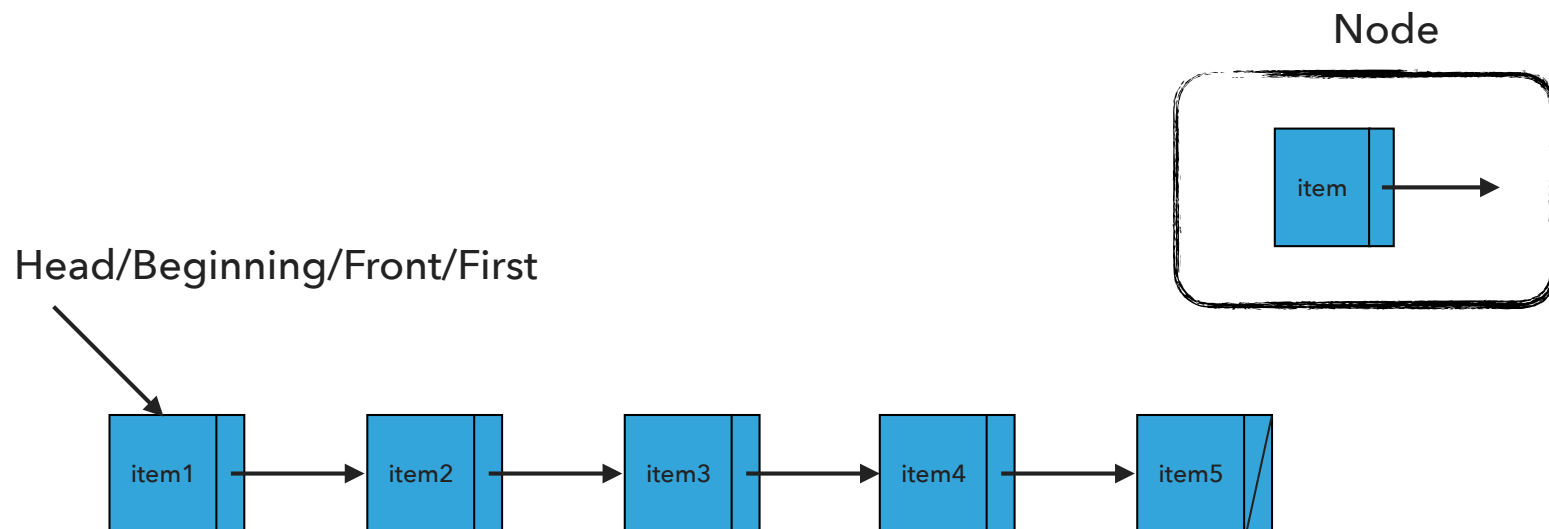
Singly Linked Lists

- ▶ Dynamic linear data structures.
- ▶ In contrast to sequential data structures, linked data structures use **pointers/links/references** from one object to another.



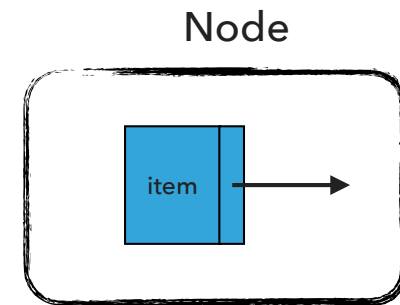
Recursive Definition of Singly Linked Lists

- ▶ A singly linked list is either empty (null) or a **node** having a reference to a singly linked list.
- ▶ **Node**: is a data type that holds any kind of data and a reference to a node.



Node

```
private class Node {  
    Item item;  
    Node next;  
}
```



Standard Operations

- ▶ `SinglyLinkedList()`: Constructs an empty singly linked list.
- ▶ `isEmpty()`: Returns true if the singly linked list does not contain any item.
- ▶ `size()`: Returns the number of items in the singly linked list.
- ▶ `Item get(int index)`: Returns the item at the specified index.
- ▶ `add(Item item)`: Inserts the specified item at the head of the singly linked list.
- ▶ `add(int index, Item item)`: Inserts the specified item at the specified index.
- ▶ `Item remove()`: Removes and returns the head of the singly linked list.
- ▶ `Item remove(int index)`: Removes and returns the item at the specified index.

`SinglyLinkedList()`: Constructs an empty SLL

`first = ?`

`n = ?`

What should happen?

```
SinglyLinkedList<String> sll = new SinglyLinkedList<String>();
```

`SinglyLinkedList()`: Constructs an empty SLL

```
SinglyLinkedList<String> sll = new SinglyLinkedList<String>();
```

`first = null`

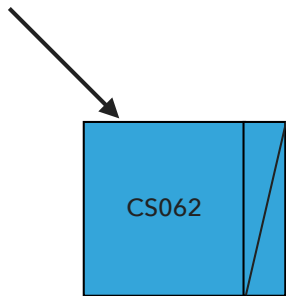
`n = 0`

What should happen?

```
sll.add("CS062");
```

`add(Item item)`: Inserts the specified item at the head of the singly linked list

Head/Beginning/Front/First



```
sll.add("CS062")
```

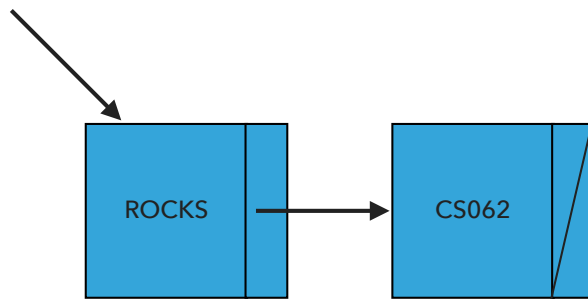
```
n=1
```

What should happen?

```
sll.add("ROCKS");
```

`add(Item item)`: Inserts the specified item at the head of the singly linked list

Head/Beginning/Front/First



```
sll.add("ROCKS")
```

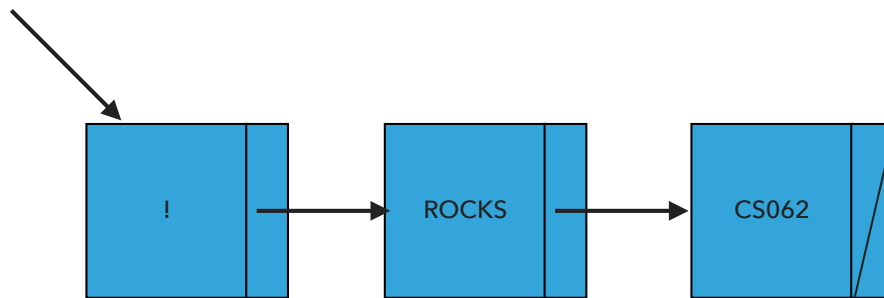
`n=2`

What should happen?

```
sll.add("!");
```

`add(Item item)`: Inserts the specified item at the head of the singly linked list

Head/Beginning/Front/First



```
sll.add("!")
```

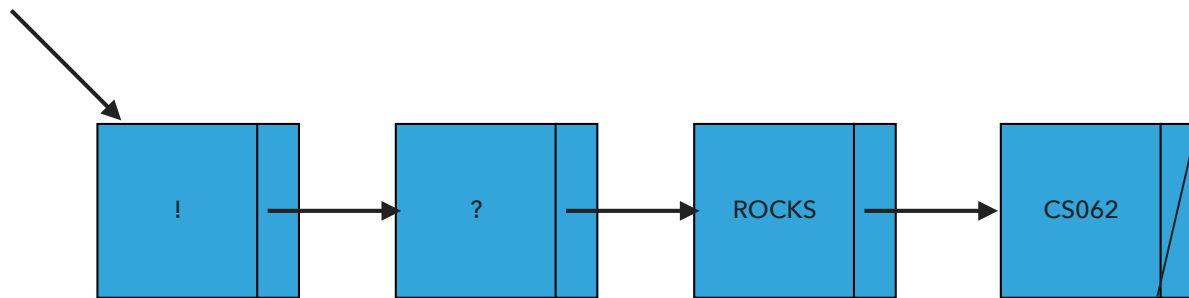
`n=3`

What should happen?

```
sll.add(1, "?");
```

`add(int index, Item item):` Adds item at the specified index

Head/Beginning/Front/First



`sll.add(1, "?")`

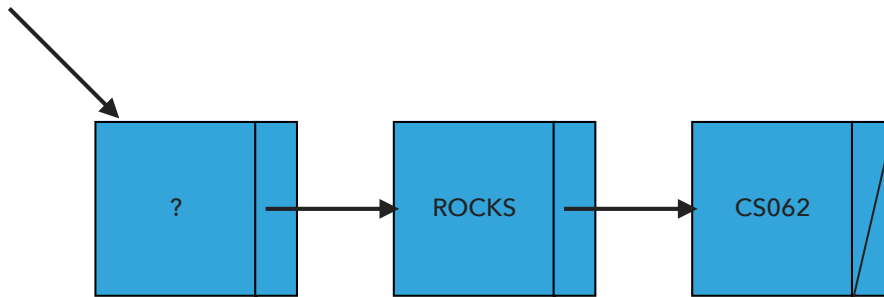
`n=4`

What should happen?

`sll.remove();`

`remove()`: Retrieves and removes the head of the singly linked list

Head/Beginning/Front/First



`sll.remove()`

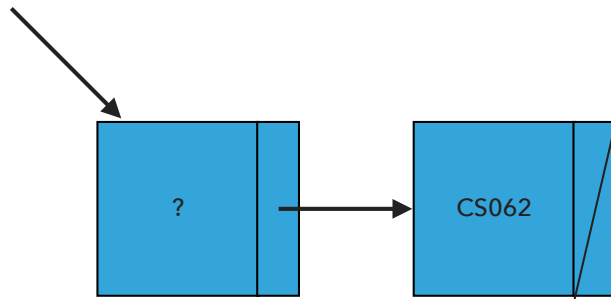
`n=3`

What should happen?

`sll.remove(1);`

`remove(int index)`:Retrieves and removes the item at the specified index

Head/Beginning/Front/First



`sll.remove(1)`

`n=2`

Our own implementation of Singly Linked Lists

- ▶ We will follow the textbook style.
 - ▶ It does not offer a class for this so we will build our own.
- ▶ We will work with generics because we don't want to offer multiple implementations.
- ▶ We will use an inner class Node and we will keep track of how many elements we have in our singly linked list.

Instance variables and inner class

```
public class SinglyLinkedList<Item> implements Iterable<Item> {  
    private Node first; // head of the singly linked list  
    private int n; // number of nodes in the singly linked list  
  
    /**  
     * This nested class defines the nodes in the singly linked list with a  
value  
     * and pointer to the next node they are connected.  
     */  
    private class Node {  
        Item item;  
        Node next;  
    }  
}
```

Check if is empty and how many items

```
/**
 * Returns true if the singly linked list does not contain any item.
 *
 * @return true if the singly linked list does not contain any item
 */
public boolean isEmpty() {
    return first == null; // return size() == 0;
}

/**
 * Returns the number of items in the singly linked list.
 *
 * @return the number of items in the singly linked list
 */
public int size() {
    return n;
}
```

Check if index is ≥ 0 and $< n$

```
/**
 * A helper method to check if an index is in range  $0 \leq \text{index} < n$ 
 *
 * @param index
 *         the index to check
 */
private void rangeCheck(int index) {
    if (index > n || index < 0)
        throw new IndexOutOfBoundsException("Index " + index + " out of bounds");
}
```

Retrieve item from specified index

```
/**
 * Returns item at the specified index.
 *
 * @param index
 *         the index of the item to be returned
 * @return the item at specified index
 */
public Item get(int index) {
    // check whether index is valid
    rangeCheck(index);
    // set a temporary pointer to the head
    Node finger = first;
    // search for index-th element or end of list
    while (index > 0) {
        finger = finger.next;
        index--;
    }
    // return the item stored in the node that the temporary pointer points to
    return finger.item;
}
```

Insert item at head of singly linked list

```
/**
 * Inserts the specified item at the head of the singly linked list.
 *
 * @param item
 *         the item to be inserted
 */
public void add(Item item) {
    // Create a pointer to head
    Node oldfirst = first;

    // Make a new node that will hold the item and assign it to head.
    first = new Node();
    first.item = item;
    // fix pointers
    first.next = oldfirst;
    // increase number of nodes
    n++;
}
```

Insert item at a specified index

```
/**
 * Inserts the specified item at the specified index.
 *
 * @param index
 *         the index to insert the node
 * @param item
 *         the item to insert
 */
public void add(int index, Item item) {
    // check that index is within range
    rangeCheck(index);
    // if index is 0, then call one-argument add
    if (index == 0) {
        add(item);
    } else {
        // make two pointers, previous and finger. Point previous to null and finger to head
        Node previous = null;
        Node finger = first;
        // search for index-th position by pointing previous to finger and advancing finger
        while (index > 0) {
            previous = finger;
            finger = finger.next;
            index--;
        }
        // create new node to insert in correct position. Set its pointers and contents
        Node current = new Node();
        current.next = finger;
        current.item = item;
        // make previous point to newly created node.
        previous.next = current;
        // increase number of nodes
        n++;
    }
}
```

Retrieve and remove head

```
/**
 * Retrieves and removes the head of the singly linked list.
 *
 * @return the head of the singly linked list.
 */
public Item remove() {
    // Make a temporary pointer to head
    Node temp = first;
    // Move head one to the right
    first = first.next;
    // Decrease number of nodes
    n--;
    // Return item held in the temporary pointer
    return temp.item;
}
```

Retrieve and remove element from a specific index

```
/**
 * Retrieves and removes the item at the specified index.
 *
 * @param index
 *         the index of the item to be removed
 * @return the item previously at the specified index
 */
public Item remove(int index) {
    // check that index is within range
    rangeCheck(index);
    // if index is 0, then call remove
    if (index == 0) {
        return remove();
    } else {
        // make two pointers, previous and finger. Point previous to null and finger to head
        Node previous = null;
        Node finger = first;
        // search for index-th position by pointing previous to finger and advancing finger
        while (index > 0) {
            previous = finger;
            finger = finger.next;
            index--;
        }
        // make previous point to finger's next
        previous.next = finger.next;
        // reduce number of items
        n--;
        // return finger's item
        return finger.item;
    }
}
```

`add()` in singly linked lists is $O(1)$ for worst case

```
public void add(Item item) {  
    // Save the old node  
    Node oldfirst = first;  
  
    // Make a new node and assign it to head. Fix pointers.  
    first = new Node();  
    first.item = item;  
    first.next = oldfirst;  
  
    n++; // increase number of nodes in singly linked list.  
}
```

get() in singly linked lists is $O(n)$ for worst case

```
public Item get(int index) {  
    rangeCheck(index);  
  
    Node finger = first;  
    // search for index-th element or end of list  
    while (index > 0) {  
        finger = finger.next;  
        index--;  
    }  
    return finger.item;  
}
```

`add(int index, Item item)` in singly linked lists is $O(n)$ for worst case

```
public void add(int index, Item item) {
    rangeCheck(index);

    if (index == 0) {
        add(item);
    } else {

        Node previous = null;
        Node finger = first;
        // search for index-th position
        while (index > 0) {
            previous = finger;
            finger = finger.next;
            index--;
        }
        // create new value to insert in correct position.
        Node current = new Node();
        current.next = finger;
        current.item = item;
        // make previous value point to new value.
        previous.next = current;

        n++;
    }
}
```

remove() in singly linked lists is $O(1)$ for worst case

```
public Item remove() {  
    Node temp = first;  
    // Fix pointers.  
    first = first.next;  
  
    n--;  
  
    return temp.item;  
}
```

`remove(int index)` in singly linked lists is $O(n)$ for worst case

```
public Item remove(int index) {
    rangeCheck(index);

    if (index == 0) {
        return remove();
    } else {
        Node previous = null;
        Node finger = first;
        // search for value indexed, keep track of previous
        while (index > 0) {
            previous = finger;
            finger = finger.next;
            index--;
        }
        previous.next = finger.next;

        n--;
        // finger's value is old value, return it
        return finger.item;
    }
}
```

Lecture 7: Singly Linked Lists

- ▶ Singly Linked Lists

Readings:

- ▶ Textbook:
 - ▶ Chapter 1.3 (Page 142-146)
- ▶ Textbook Website:
 - ▶ Linked Lists: <https://algs4.cs.princeton.edu/13stacks/>

Practice Problems:

- ▶ 1.3.18-1.3.27