BALANCED SEARCH TREES

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CS 62 – Spring 2021

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Admin

Last day for "normal" mentor hours, Friday (5/7)

More on mentor hours next week

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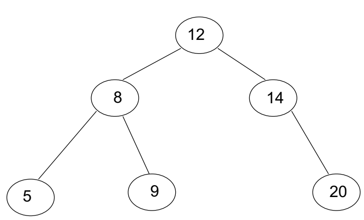
Binary Search Trees

BST – A binary tree where each each node has a key, and every node's key is:

- Larger than all keys in its left subtree. (everything left is smaller)
- Smaller than all keys in its right subtree. (everything right is larger)



3

Operations

Search – Does the key exist in the tree

Insert – Insert the key into tree

Delete – Delete the key from the tree

4

Height of the tree

Most of the operations take time
O(height)

We said trees built from random data have height
O(log n), which is asymptotically tight

Two problems:
- We can't always insure random data
- What happens when we delete nodes and insert others after building a tree?

Worst case height for binary search trees is O(n) ☹

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Balanced trees

Make sure that the trees remain balanced!
- Red-black trees
- AVL trees
- 2-3 trees
- 2-3-4 trees
- B-trees
- …

Height is guaranteed to be O(log n)

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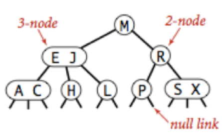
2-3 trees



Anatomy of a 2-3 search tree

2-node: one key and two children (left and right)
- everything in left is smaller than key
- everything right is greater than (or equal to) key

3-node: two keys (k_1, k_2) and three children, left, middle and right
- $k_1 < k_2$
- everything in left is less than k_1
- everything in middle is between k_1 and k_2 (greater than or equal to k_1 and less than k_2)
- everything in right is greater than (or equal to) k_2

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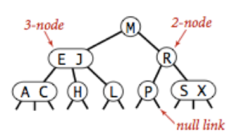
Search

How do we search for a key?



Anatomy of a 2-3 search tree

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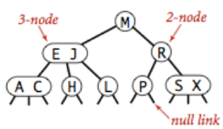
Search

Almost identical to BST search

Only difference: sometimes we have two keys



Anatomy of a 2-3 search tree

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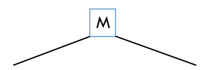
Search

Search(H)

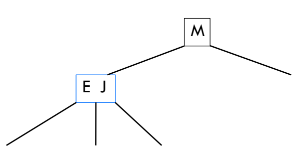


Which child?

10

Search

Search(H)

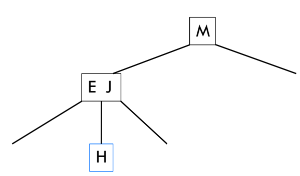


Which child?

11

Search

Search(H)



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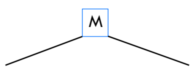
Search

Search(B)

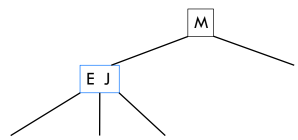


Which child?

13

Search

Search(B)

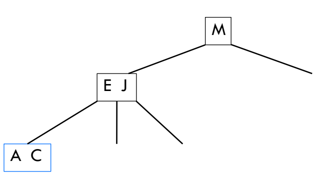


Which child?

14

Search

Search(B)

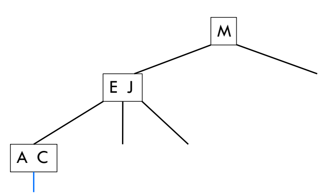


Which child?

15

Search

Search(B)



Not found!

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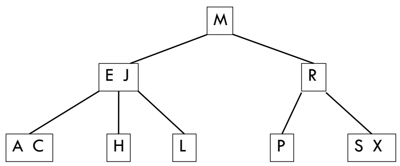
Search



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Insertion

Like BST, insert always happens at a leaf

If the leaf is a 2-node, just insert it directly

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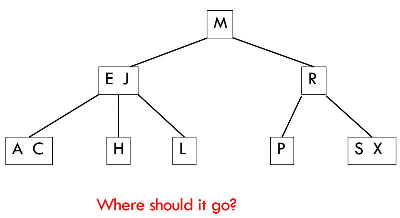
Insertion

If the leaf is a 2-node, just insert it directly

Insert(F)



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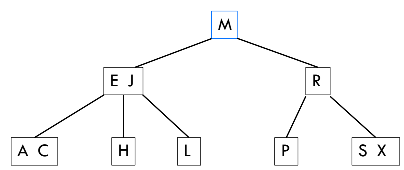
Insertion

If the leaf is a 2-node, just insert it directly

Insert(F)



20

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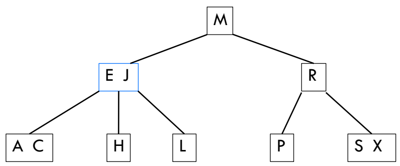

21

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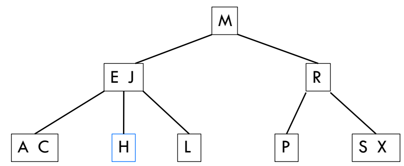
Insert(F)

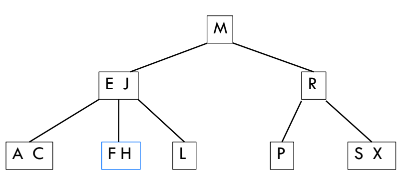


22

Insertion

If the leaf is a 2-node, just insert it directly

Insert(F)



23

Insertion

Like BST, insert always happens at a leaf

If the leaf is a 2-node, just insert it directly

If the leaf is a 3-node:
- We now have three values at this leaf
- Send the middle value up a node
- Make new 2-nodes out of the smallest and largest

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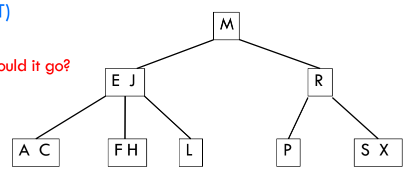
Insertion

If the leaf is a 3-node:

- We now have three values at this leaf
- Send the middle value up a node
- Make new 2-nodes out of the smallest and largest

Insert(T)

Where should it go?



25

Insertion

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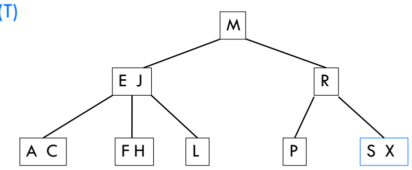
Insert(T)



26

Insertion

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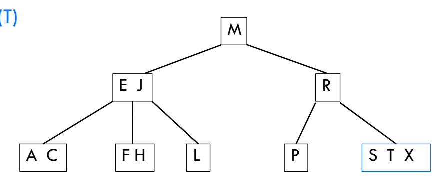
Insert(T)



27

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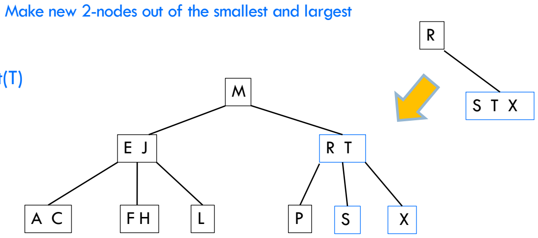
Insert(T)



28

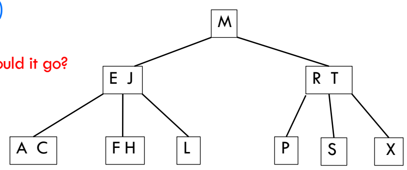
Insertion

If the leaf is a 3-node:

- We now have three values at this leaf
- Send the middle value up a node
- Make new 2-nodes out of the smallest and largest

Insert(I)

Where should it go?



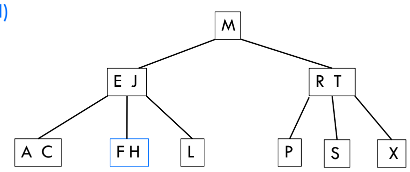
29

Insertion

If the leaf is a 3-node:

- We now have three values at this leaf
- Send the middle value up a node
- Make new 2-nodes out of the smallest and largest

Insert(I)



30

Insertion

If the leaf is a 3-node:

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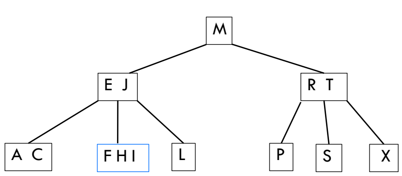
Insert(I)



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Insertion

If the leaf is a 3-node:

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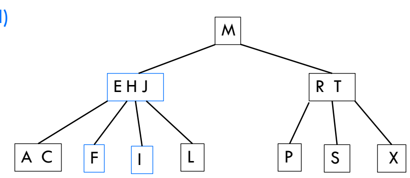
Insert(I)



32

Insertion

If the leaf is a 3-node:

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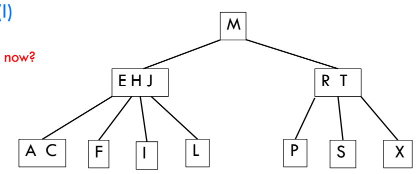
Insert(I)

What now?



33

Insertion

If the leaf is a 3-node:

- We now have three values at this leaf
- Send the middle value up a node
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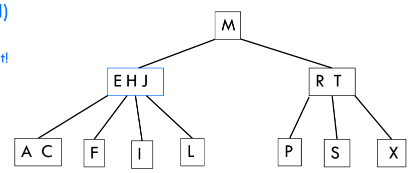
Insert(I)

Repeat!



34

Insertion

If the leaf is a 3-node:

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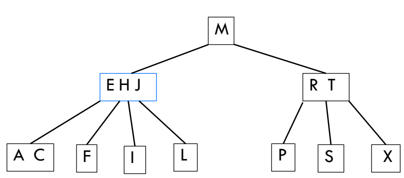
Insert(I)



35

Insertion

If the leaf is a 3-node:

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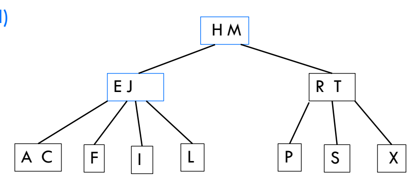
Insert(I)



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Insertion

If the leaf is a 3-node:

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- Send the middle value up a node
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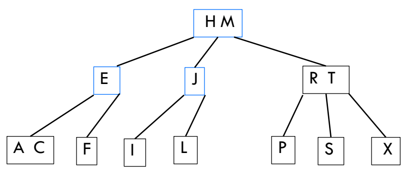
Insert(I)



37

Insertion

If the leaf is a 2-node, just insert it directly

If the leaf is a 3-node:

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When will the height of the tree change?

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Insertion

If the leaf is a 2-node, just insert it directly

If the leaf is a 3-node:

- We now have three values at this leaf
- Send the middle value up a node
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Only when the root is a 3-node and we insert into a path that is all 3-nodes!

Effect: The tree can hold quite a few values before having to increase the height

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Practice

Draw the 2-3 tree that results when you insert the keys:
E A S Y Q U T I O N in that order in an initially empty tree.

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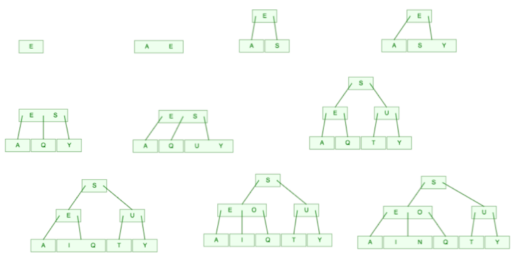
Practice

Draw the 2-3 tree that results when you insert the keys:
E A S Y Q U T I O N in that order in an initially empty tree.



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Running time

Worst case height: O(log n)

What does that mean?

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Running time

Worst case height: O(log n)

Insert, search and delete are all O(log n)

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2-3 search trees in practice

A pain to implement

Overhead can often make slower than standard BST

Other balanced trees exist that provide the same worst case guarantee, but are faster (e.g, red-black trees)

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Red-black tree high-level

https://www.cs.usfca.edu/~galles/visualization/RedBlack.html

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