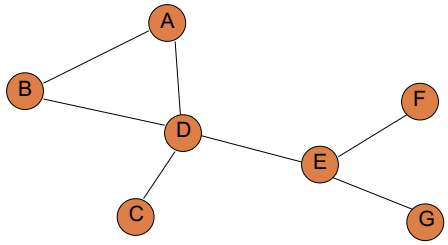
Graphs

A graph is a set of vertices V and a set of edges (u,v) ∈ E where u,v ∈ V



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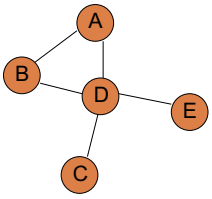
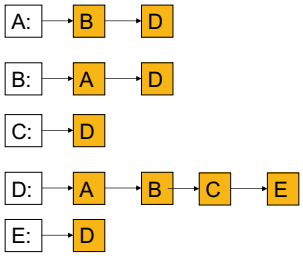
Representing graphs

Adjacency list – Each vertex u ∈ V contains an adjacency list of the set of vertices v such that there exists an edge (u,v) ∈ E





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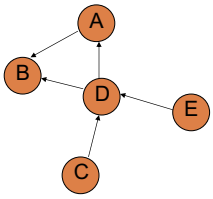
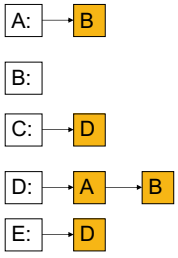
Representing graphs

Adjacency list – Each vertex u ∈ V contains an adjacency list of the set of vertices v such that there exists an edge (u,v) ∈ E





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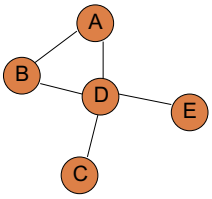
Representing graphs

Adjacency matrix – A |V|x|V| matrix A such that:

$$a_{ij} = \begin{cases} 1 & \text{if } (i,j) \in E \\ 0 & \text{otherwise} \end{cases}$$



	A	B	C	D	E
A	0	1	0	1	0
B	1	0	0	1	0
C	0	0	0	1	0
D	1	1	1	0	1
E	0	0	0	1	0

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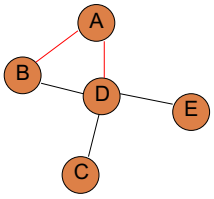
Representing graphs

Adjacency matrix – A |V|x|V| matrix A such that:

$$a_{ij} = \begin{cases} 1 & \text{if } (i,j) \in E \\ 0 & \text{otherwise} \end{cases}$$



	A	B	C	D	E
A	0	**1**	0	**1**	0
B	1	0	0	1	0
C	0	0	0	1	0
D	1	1	1	0	1
E	0	0	0	1	0

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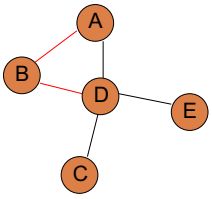
Representing graphs

Adjacency matrix – A |V|x|V| matrix A such that:

$$a_{ij} = \begin{cases} 1 & \text{if } (i,j) \in E \\ 0 & \text{otherwise} \end{cases}$$



	A	B	C	D	E
A	0	1	0	1	0
B	**1**	0	0	**1**	0
C	0	0	0	1	0
D	1	1	1	0	1
E	0	0	0	1	0

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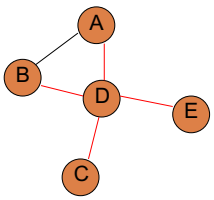
Representing graphs

Adjacency matrix – A |V|x|V| matrix A such that:

$$a_{ij} = \begin{cases} 1 & \text{if } (i,j) \in E \\ 0 & \text{otherwise} \end{cases}$$



	A	B	C	D	E
A	0	1	0	1	0
B	1	0	0	1	0
C	0	0	0	1	0
D	**1**	**1**	**1**	0	**1**
E	0	0	0	1	0

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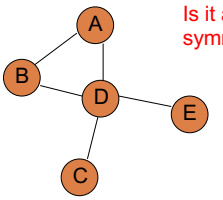
Representing graphs

Adjacency matrix – A |V|x|V| matrix A such that:

$$a_{ij} = \begin{cases} 1 & \text{if } (i,j) \in E \\ 0 & \text{otherwise} \end{cases}$$



Is it always symmetric?

	A	B	C	D	E
A	0	1	0	1	0
B	1	0	0	1	0
C	0	0	0	1	0
D	1	1	1	0	1
E	0	0	0	1	0

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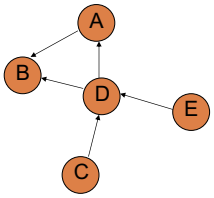
Representing graphs

Adjacency matrix – A |V|x|V| matrix A such that:

$$a_{ij} = \begin{cases} 1 & \text{if } (i,j) \in E \\ 0 & \text{otherwise} \end{cases}$$



	A	B	C	D	E
A	0	1	0	0	0
B	0	0	0	0	0
C	0	0	0	1	0
D	1	1	0	0	0
E	0	0	0	1	0

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Adjacency list vs. adjacency matrix

Adjacency list | Adjacency matrix

Pros/Cons?

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Adjacency list vs. adjacency matrix

Adjacency list

- Sparse graphs (e.g. web)
- Space efficient
- Must traverse the adjacency list to discover is an edge exists

Adjacency matrix

- Dense graphs
- Constant time lookup to discover if an edge exists
- Simple to implement
- For non-weighted graphs, only requires boolean matrix

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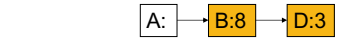
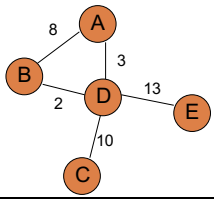
Weighted graphs

Adjacency list

- store the weight as an additional field in the list





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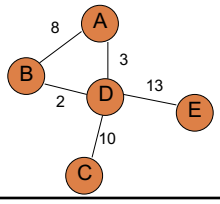
Weighted graphs

Adjacency matrix

$$a_{ij} = \begin{cases} weight & \text{if } (i,j) \in E \\ 0 & \text{otherwise} \end{cases}$$



	A	B	C	D	E
A	0	8	0	3	0
B	8	0	0	2	0
C	0	0	0	10	0
D	3	2	10	0	13
E	0	0	0	13	0

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