

GRAPHS

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CS 62 – Spring 2021

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Admin

Midterm 2 next Wednesday

- 2 hours starting at the beginning of lab (email me if that time doesn't work well)
- Material: from stack & queues up through binary search trees
- May bring two pages of notes
- Sample problems posted
- Review on Tuesday

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Graphs

A mathematical model consisting of a set of nodes/vertices and edges

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Graphs

A graph is a set of vertices V and a set of edges $(u,v) \in E$ where $u,v \in V$

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Graphs

$V = \{A, B, C, D, E, F, G\}$
 $E = \{(A,B), (A,D), (B,D), (C,D), (D,E), (E,F), (E,G)\}$

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When do we see graphs in real life problems?

- Transportation networks (flights, roads, etc.)
- Communication networks
- Web
- Social networks
- Circuit design
- Bayesian networks

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Graphs

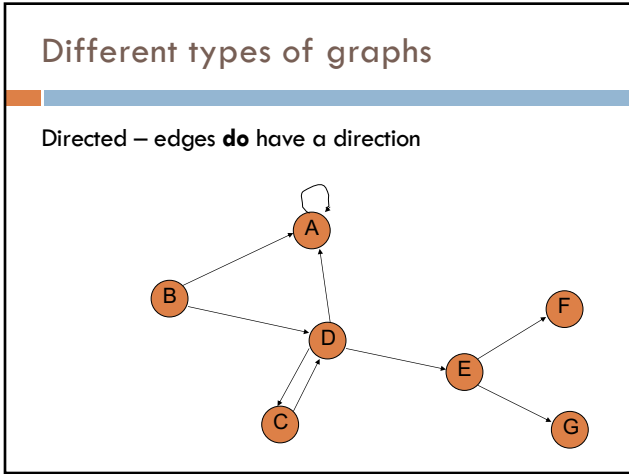
How do graphs differ?
 What are graph characteristics we might care about?

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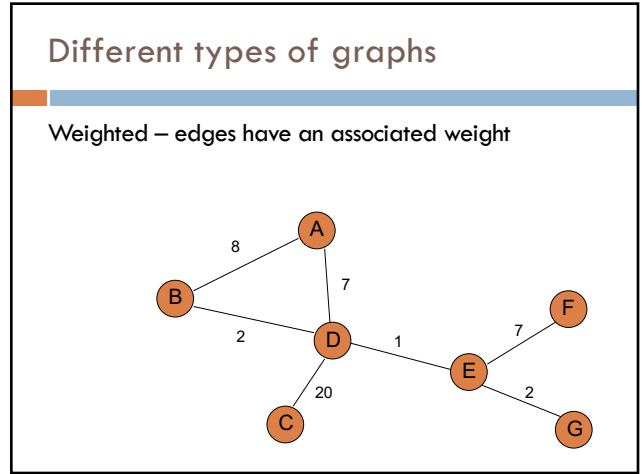
Different types of graphs

Undirected – edges do not have a direction

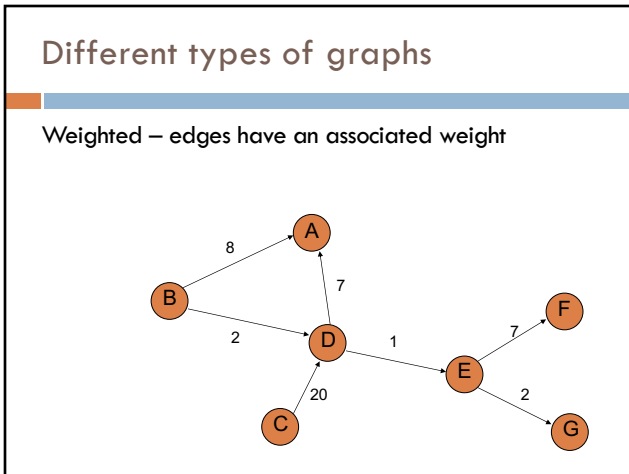
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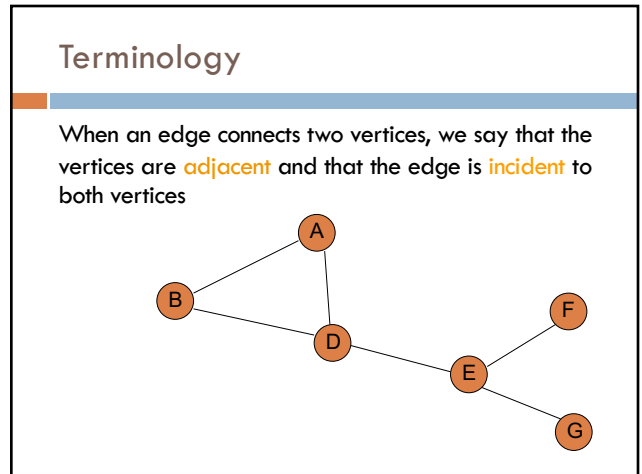
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Terminology

When an edge connects two vertices, we say that the vertices are **adjacent** and that the edge is **incident** to both vertices

A and D are adjacent
 (A, D) is incident to A and D

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Terminology

The **degree** of a vertex is the number of edges incident to it

What is the degree of A?
 What is the degree of D?

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Terminology

The **degree** of a vertex is the number of edges incident to it

What is the degree of A?
 What is the degree of D?

15

Terminology

The **degree** of a vertex is the number of edges incident to it

What is the degree of A?
 What is the degree of D?

16

Terminology

The **degree** of a vertex is the number of edges incident to it

What is the degree of A?
What is the degree of D?

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Terminology

Path – A path is a sequence of vertices p_1, p_2, \dots, p_k where there exists an edge $(p_i, p_{i+1}) \in E$ and no edge is repeated

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Terminology

Path – A path is a sequence of vertices p_1, p_2, \dots, p_k where there exists an edge $(p_i, p_{i+1}) \in E$ and no edge is repeated

{A, B, D, E, F}

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Terminology

Path – A path is a sequence of vertices p_1, p_2, \dots, p_k where there exists an edge $(p_i, p_{i+1}) \in E$ and no edge is repeated

{C, D}

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Terminology

Path – A path is a sequence of vertices p_1, p_2, \dots, p_k where there exists an edge $(p_i, p_{i+1}) \in E$ and no edge is repeated

A simple path contains no repeated vertices (often this is implied)

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Terminology

Cycle – A path where the first and last node are the same

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Terminology

Cycle – A path where the first and last node are the same

Edges: (A,B), (A,D), (B,D)
Path: B, A, D, B

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Terminology

Cycle – A path where the first and last node are the same

not a cycle

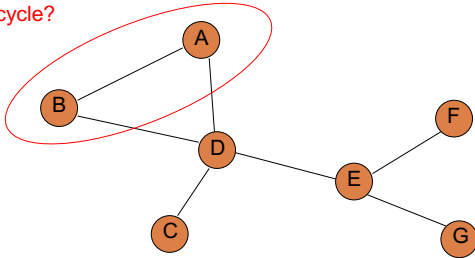
Why not?

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Terminology

Cycle – A path where the first and last node are the same

Is this a cycle?



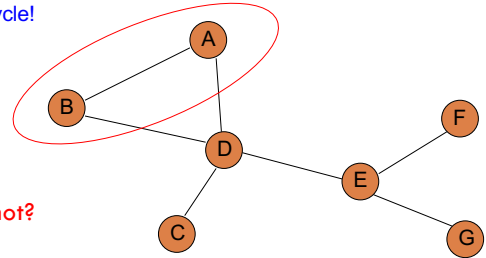
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Terminology

Cycle – A path where the first and last node are the same

Not a cycle!

Why not?

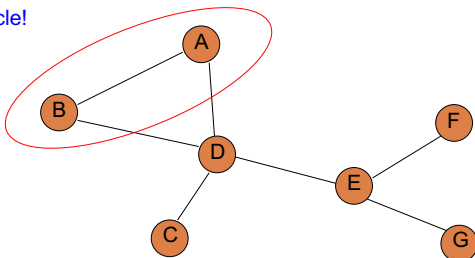


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Path – A path is a sequence of vertices p_1, p_2, \dots, p_k where there exists an edge $(p_i, p_{i+1}) \in E$ and no edge is repeated

Cycle – A path where the first and last node are the same

Not a cycle!

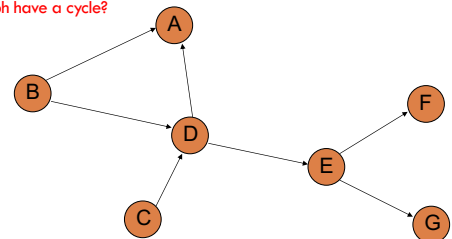


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Terminology

Cycle – A path where the first and last node are the same

Does this graph have a cycle?



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Terminology

Cycle – A path where the first and last node are the same

not a cycle

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Terminology

Cycle – A path p_1, p_2, \dots, p_k where $p_1 = p_k$

This would be a cycle

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Terminology

Connected – every pair of vertices is connected by a path

Is this graph connected?

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Terminology

Connected – every pair of vertices is connected by a path

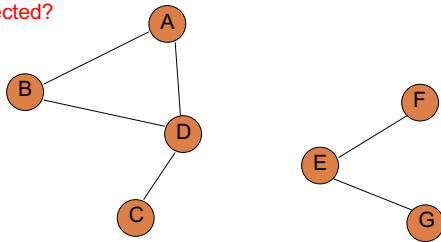
connected

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Terminology

Connected – every pair of vertices is connected by a path

Is this graph
connected?

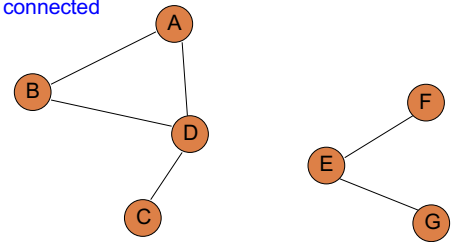


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Terminology

Connected – every pair of vertices is connected by a path

not connected

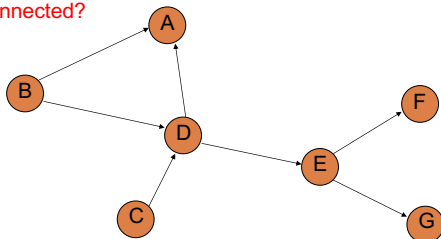


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Terminology

Strongly connected (directed graphs) –
Every two vertices are reachable by a path

Is this graph
strongly connected?

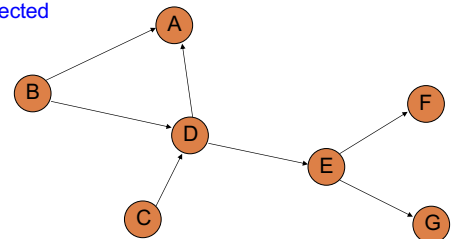


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Terminology

Strongly connected (directed graphs) –
Every two vertices are reachable by a path

not strongly
connected



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Terminology

Strongly connected (directed graphs) –
Every two vertices are reachable by a path

Is this graph strongly connected?

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Terminology

Strongly connected (directed graphs) –
Every two vertices are reachable by a path

not strongly connected

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Terminology

Strongly connected (directed graphs) –
Every two vertices are reachable by a path

Is this graph strongly connected?

39

Terminology

Strongly connected (directed graphs) –
Every two vertices are reachable by a path

strongly connected

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Graphs aren't new...

Have we seen graphs in this class already?

Trees!

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Different types of graphs

What is a tree (in our terminology)?

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Different types of graphs

Tree – connected, undirected graph without any cycles

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Different types of graphs

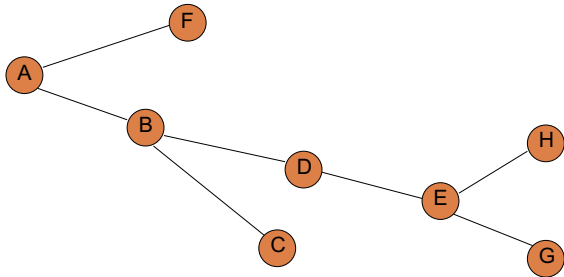
Tree – connected, undirected graph without any cycles

need to specify a root

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Different types of graphs

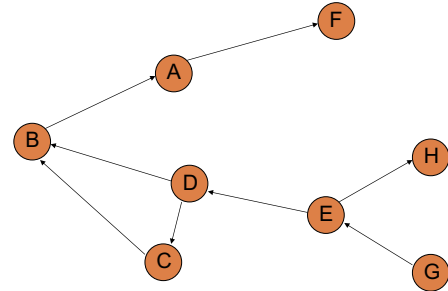
Tree – connected, undirected graph without any cycles



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Different types of graphs

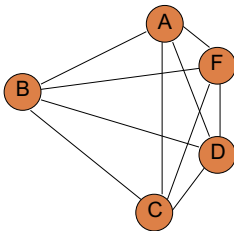
DAG – directed, acyclic graph



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Different types of graphs

Complete graph – an edge exists between every node



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Graph questions?

Does it have a cycle?

Is it connected? Strongly connected?

Is there a path from a to b?

What is the shortest path from a to b? In number of edges? In sum of the edge weights?

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