HEAPS

David Kauchak
CS 62 – Spring 2021

1

Admin

Pre-pre enrollment

OnDiskSort

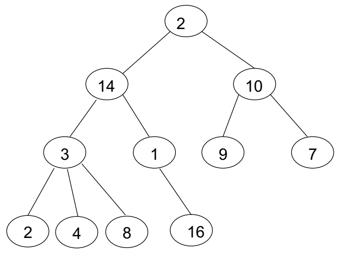
Lab tomorrow

2

Trees

A set of nodes based on a parent-child relationship

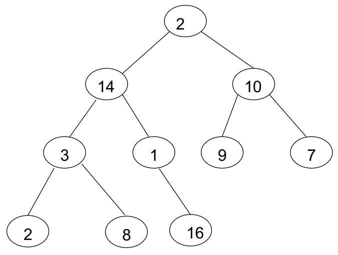
- Each node has one parent
- Root has no parent



3

Binary tree

Each parent has at most 2 children



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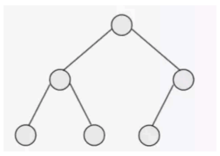
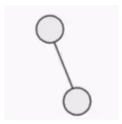
Full + Complete?



12

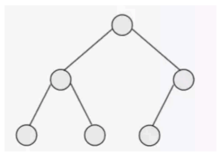


Full tree: a binary tree where every node has 0 or 2 children

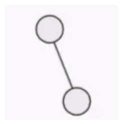
Complete: All levels except the last are completely filled and all nodes on the last level are on the left

5

Full + Complete?



12



Complete

Full + Complete

Neither

6

Implementing a binary tree

```
public class BinaryTree<E> {
    private Node root;

    private class Node {
        private E value;
        private Node left;
        private Node right;

        public Node(Node left, Node right, E value) {
            this.left = left;
            this.right = right;
            this.value = value;
        }
    }
```

7

Recursive data structure!

```
public class BinaryTree<E> {
    private Node root;

    private class Node {
        private E value;
        private Node left;
        private Node right;

        public Node(Node left, Node right, E value) {
            this.left = left;
            this.right = right;
            this.value = value;
        }
    }
```

```
public class LinkedList<E> {
    private Node head;

    private class Node {
        private E value;
        private Node next;

        public Node(Node next, E value) {
            this.next = next;
            this.value = value;
        }
    }
```

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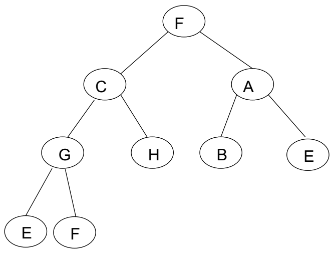
Tree traversals

```
Inorder(node):
    Inorder(left)
    visit node (e.g., print)
    Inorder(right)
```

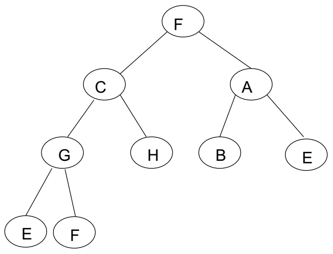


What would be printed out?

9

Tree traversals

```
Inorder(node):
    Inorder(left)
    visit node (e.g., print)
    Inorder(right)
```

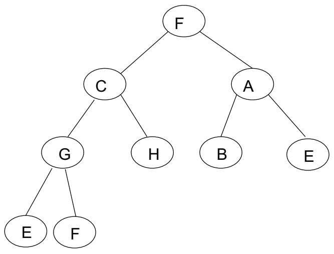


E G F C H F B A E

10

Tree traversals

```
Postorder(node):
    Postorder(left)
    Postorder(right)
    visit node (e.g., print)
```

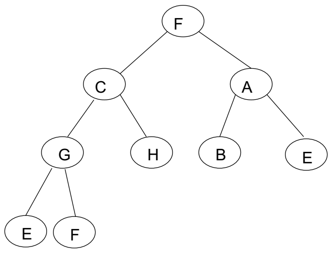


What would be printed out?

11

Tree traversals

```
Postorder(node):
    Postorder(left)
    Postorder(right)
    visit node (e.g., print)
```

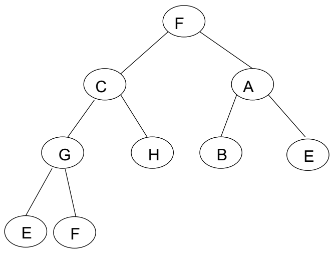


E F G H C B E A F

12

Tree traversals

```
Preorder(node):
    visit node (e.g., print)
    Preorder(left)
    Preorder(right)
```



What would be printed out?

13

Tree traversals

```
Preorder(node):
    visit node (e.g., print)
    Preorder(left)
    Preorder(right)
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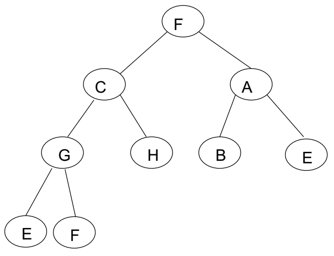
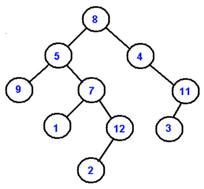


F C G E F H A B E

14

in-order, post-order, pre-order



```
Inorder(node):
    Inorder(left)
    visit node (e.g., print)
    Inorder(right)
```

```
Postorder(node):
    Postorder(left)
    Postorder(right)
    visit node (e.g., print)
```

```
Preorder(node):
    visit node (e.g., print)
    Preorder(left)
    Preorder(right)
```

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Binary heap

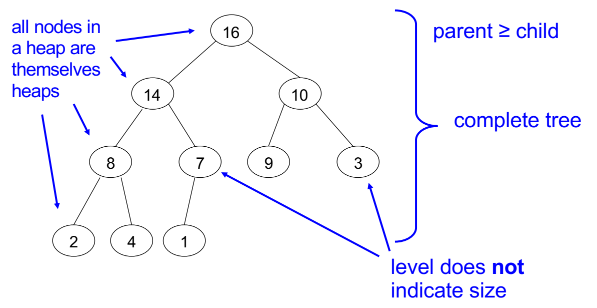
A binary tree where the value of a parent is greater than or equal to the value of its children

Additional restriction: the tree must be **complete**!

Max heap vs. min heap

16

Binary heap - references


all nodes in a heap are themselves heaps
16
14
10
8
7
9
3
2
4
1
parent ≥ child
complete tree
level does **not** indicate size

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Binary heap - array

```
PARENT($i$)
    return $\lfloor i/2 \rfloor$

LEFT($i$)
    return $2i$

RIGHT($i$)
    return $2i + 1$
```

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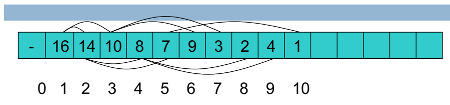
Binary heap - array



Note 0 is empty!

```
PARENT($i$)
    return $\lfloor i/2 \rfloor$

LEFT($i$)
    return $2i$

RIGHT($i$)
    return $2i + 1$
```

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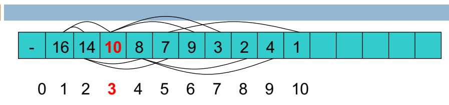
Binary heap - array



```
PARENT($i$)
    return $\lfloor i/2 \rfloor$

LEFT($i$)
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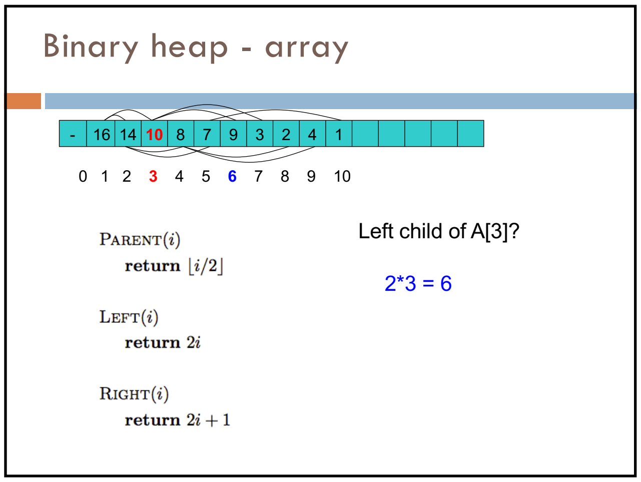
RIGHT($i$)
    return $2i + 1$
```

Left child of A[3]?

20



Binary heap - array

-	16	14	10	8	7	9	3	2	4	1					
0	1	2	3	4	5	6	7	8	9	10					

```
PARENT($i$)
    return $\lfloor i/2 \rfloor$

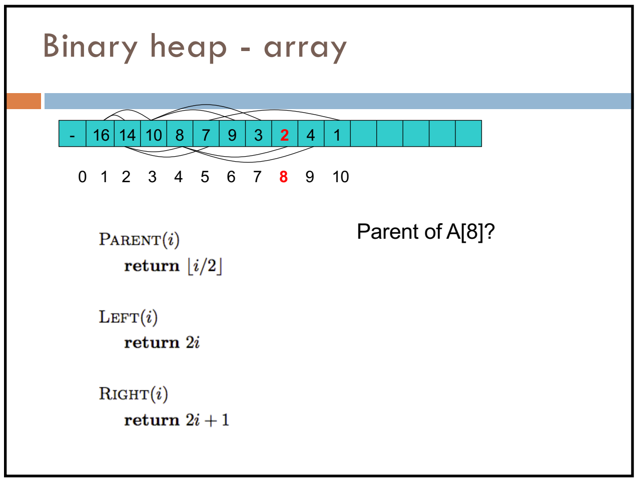
LEFT($i$)
    return $2i$

RIGHT($i$)
    return $2i + 1$
```

Left child of A[3]?

2*3 = 6

21



Binary heap - array

-	16	14	10	8	7	9	3	2	4	1					
0	1	2	3	4	5	6	7	8	9	10					

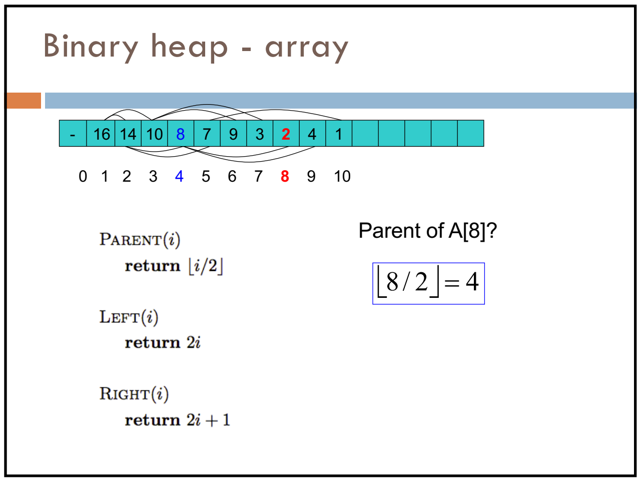
```
PARENT($i$)
    return $\lfloor i/2 \rfloor$

LEFT($i$)
    return $2i$

RIGHT($i$)
    return $2i + 1$
```

Parent of A[8]?

22



Binary heap - array

-	16	14	10	8	7	9	3	2	4	1					
0	1	2	3	4	5	6	7	8	9	10					

```
PARENT($i$)
    return $\lfloor i/2 \rfloor$

LEFT($i$)
    return $2i$

RIGHT($i$)
    return $2i + 1$
```

Parent of A[8]?

$\boxed{\lfloor 8/2 \rfloor = 4}$

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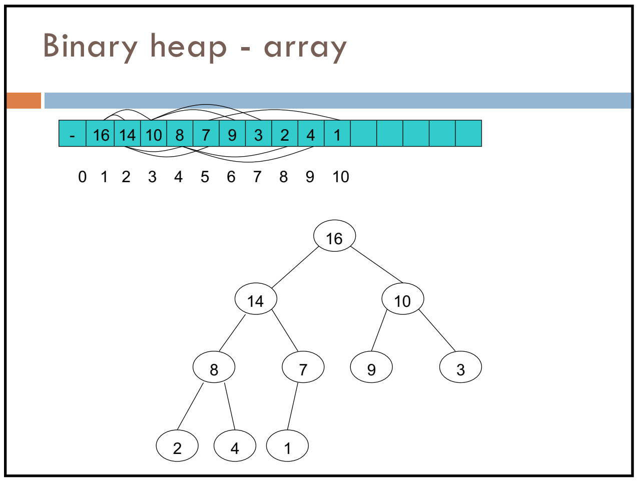

Binary heap - array

-	16	14	10	8	7	9	3	2	4	1					
0	1	2	3	4	5	6	7	8	9	10					

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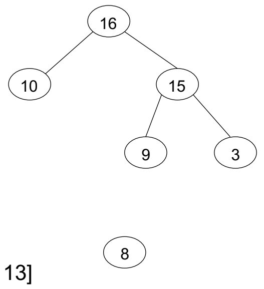
Identify the valid heaps

[-, 15, 12, 3, 11, 10, 2, 1, 7, 8]



[-, 20, 18, 10, 17, 16, 15, 9, 14, 13]

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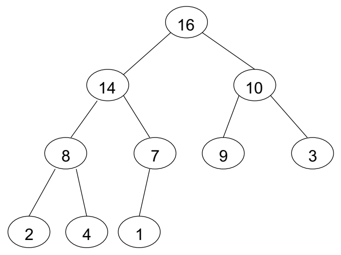
What are heaps good for?



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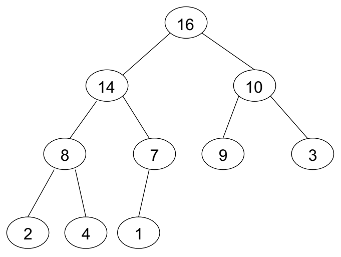
What are heaps good for?

What's the largest value in this heap?



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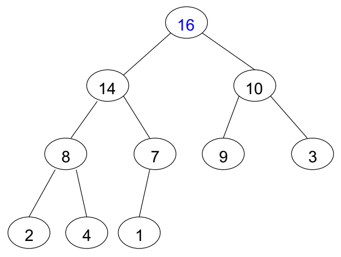
What are heaps good for?

What's the largest value in this heap?

Heaps are good at min/max operations (depending on min/max ordering)!



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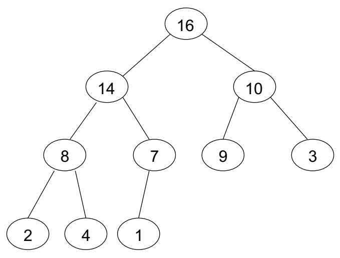
What are heaps good for?

What's the 2nd largest value? The 3rd? The 4th?



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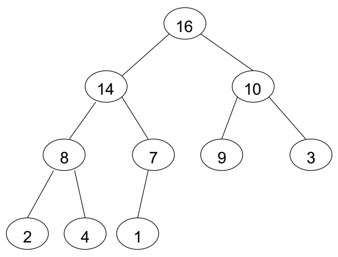
ExtractMax

Return and remove the largest element in the set. The rest of the data should stay as a heap



30

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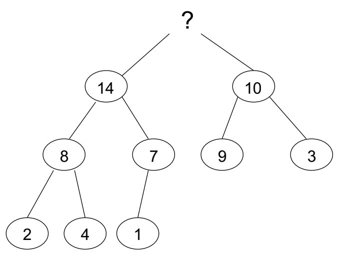

31

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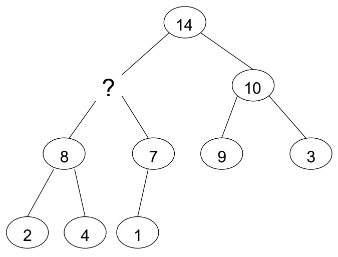

32

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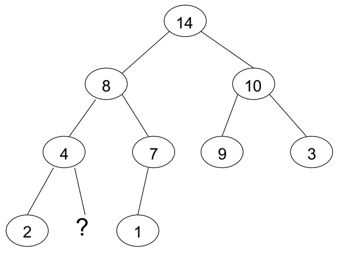
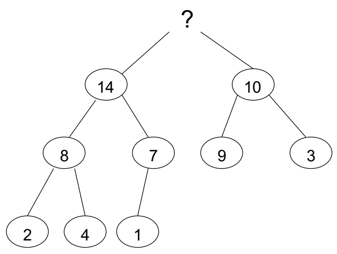


Now what? Is this a heap?

33

ExtractMax

Return and remove the largest element in the set. The rest of the data should stay as a heap



34

ExtractMax

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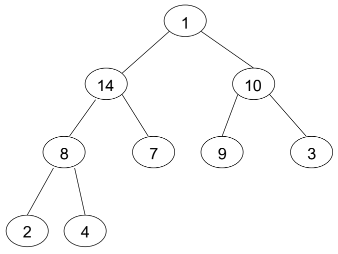

35

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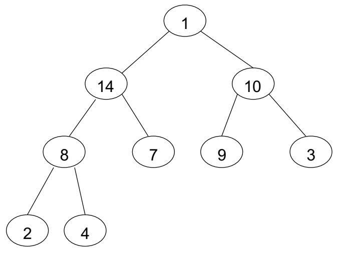
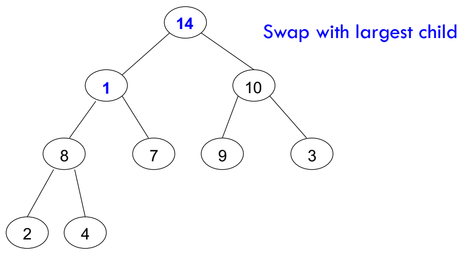


Now what?

36

ExtractMax

Return and remove the largest element in the set. The rest of the data should stay as a heap



37

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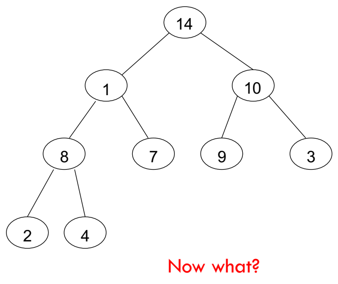

38

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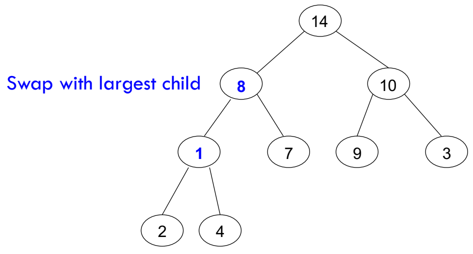

39

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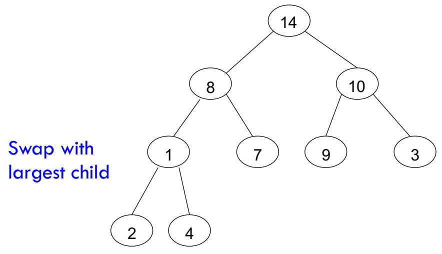

40

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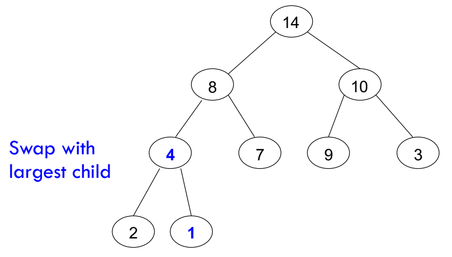

41

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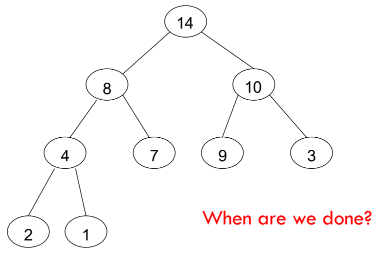

42

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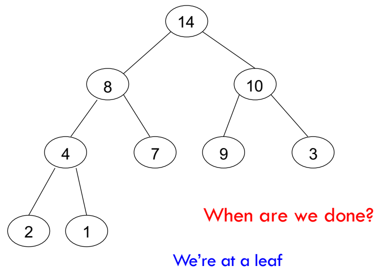

43

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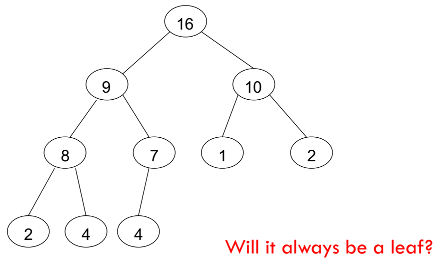

44

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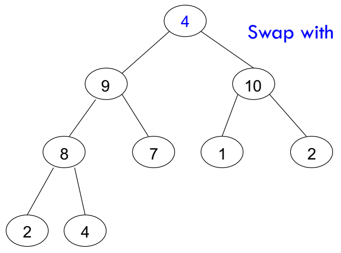
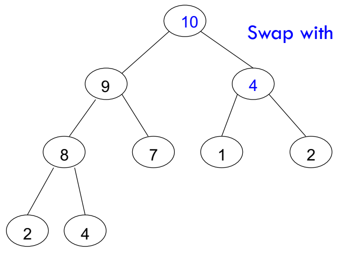


Swap with largest child

45

ExtractMax

Return and remove the largest element in the set. The rest of the data should stay as a heap

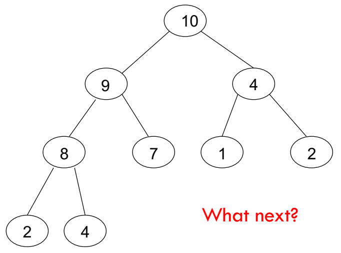


Swap with largest child

46

ExtractMax

Return and remove the largest element in the set. The rest of the data should stay as a heap



What next?

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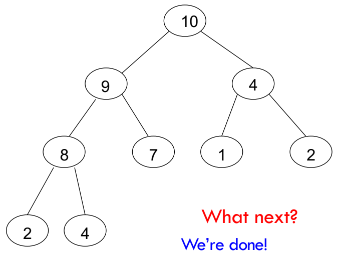
ExtractMax

Return and remove the largest element in the set. The rest of the data should stay as a heap



What next?

We're done!
Also stop if the node is larger than the two children

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sink/heapify/demote

```
private void sink(int i) {
    // if we're not a leaf
    if( left(i) < heap.size() ) {
        // find the largest child
        int maxIndex = maxChildIndex(i);

        E current = heap.get(i);
        E maxChild = heap.get(maxIndex);

        if( maxChild.compareTo(current) > 0 ) {
            swap(i, maxIndex);
            sink(maxIndex);
        }
    }
}
```

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sink runtime

```
private void sink(int i) {
    // if we're not a leaf
    if( left(i) < heap.size() ) {
        // find the largest child
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        E current = heap.get(i);
        E maxChild = heap.get(maxIndex);

        if( maxChild.compareTo(current) > 0 ) {
            swap(i, maxIndex);
            sink(maxIndex);
        }
    }
}
```

What is the worst case runtime?

50

sink runtime

```
private void sink(int i) {
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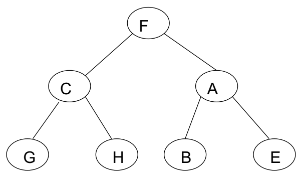
        if( maxChild.compareTo(current) > 0 ) {
            swap(i, maxIndex);
            sink(maxIndex);
        }
    }
}
```

What is the worst case runtime? O(height of tree)

51

Nodes in a binary tree

What is the tallest you can make a complete tree, using the fewest nodes?



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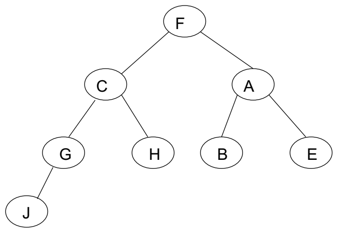
Nodes in a binary tree

What is the tallest you can make a complete tree, using the fewest nodes?

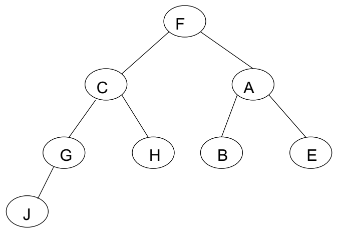


$1 + 2 + 4 + 8 + \cdots + 2^{h-1} + 1 = 2^h$

53

Nodes in a binary tree

What is the tallest you can make a complete tree, using the fewest nodes?



$n = 2^h \Rightarrow h = \log(n)$

54

sink runtime

```
private void sink(int i) {
    // if we're not a leaf
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        // find the largest child
        int maxIndex = maxChildIndex(i);

        E current = heap.get(i);
        E maxChild = heap.get(maxIndex);

        if( maxChild.compareTo(current) > 0 ) {
            swap(i, maxIndex);
            sink(maxIndex);
        }
    }
}
```

What is the worst case runtime? O(log n)

55

ExtractMax

```
public E extractMax() {
    E maxVal = data.get(1);
    data.set(1, data.get(data.size()-1));
    data.remove(data.size()-1);
    sink(1);
    return maxVal;
}
```

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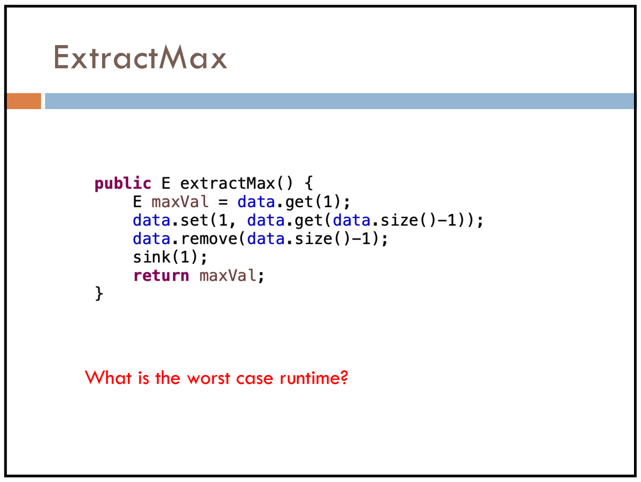

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}
```

What is the worst case runtime?

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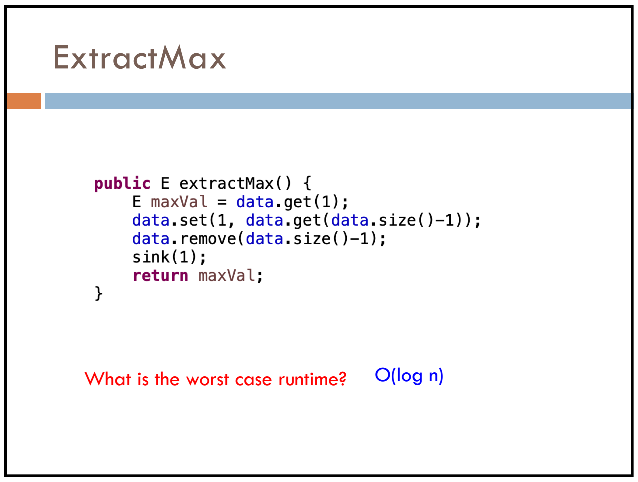

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```

What is the worst case runtime? O(log n)

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