# BINARY TREES

David Kauchak
CS 62 – Spring 2021

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## Admin

Learning community reorganization

https://docs.google.com/spreadsheets/d/1xvZL4y_FTq1h8nhYFmwCYBL_qv2neLM-n8KqkNRdadA/edit#gid=0

Advisor declaration + pre-pre enrollment

Town hall this afternoon (4:10pm)

Office hours today: 3:30-4pm

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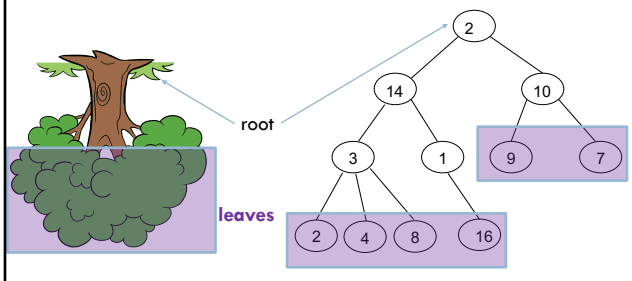
## Trees

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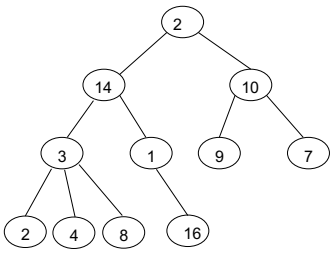
## Trees

- A set of nodes based on a parent-child relationship
  - Each node has one parent
  - Root has no parent

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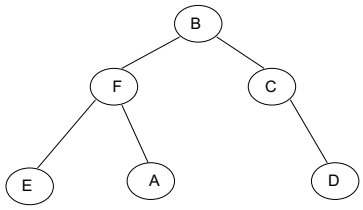
## Another example

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## Another example: the lone wolf


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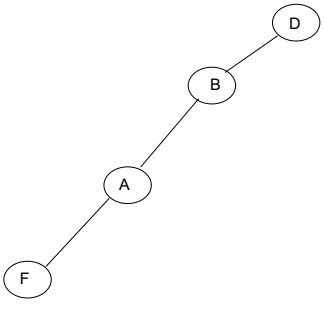
## Another example: the twig

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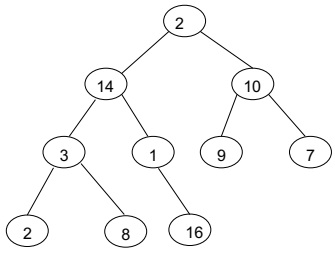
## Binary tree

Each parent has at most 2 children

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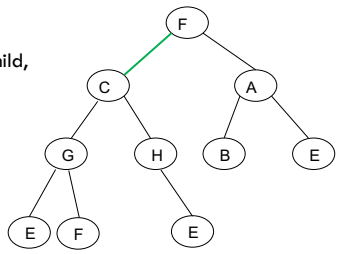
## Tree terminology

Edge:
- one of the lines
- Defined by a parent and child, **(C,F)**

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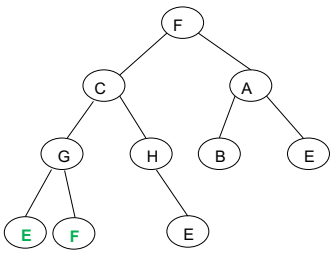
## Tree terminology

Sibling: two nodes that share the same parent

E and F are siblings

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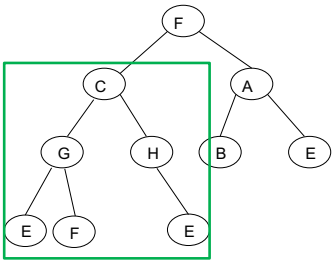
## Tree terminology

Subtree: a node and all of the nodes below it

subtree rooted at C

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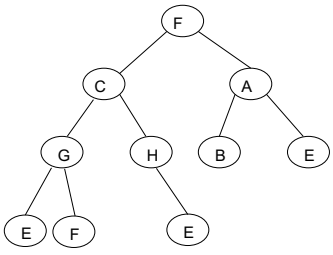
## Tree terminology

Leaf: node without any children

Internal node: non-leaf node

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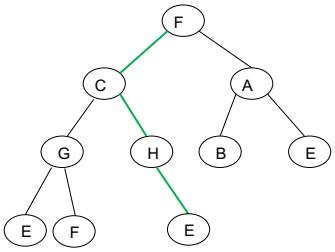
## Tree terminology

Simple path: a series of distinct nodes with edges between successive nodes

F-C-H-E

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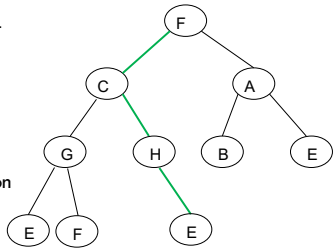
## Tree terminology

Simple path: a series of distinct nodes with edges between successive nodes

F-C-H-E

Path length: number of edges on path

How long is this path?

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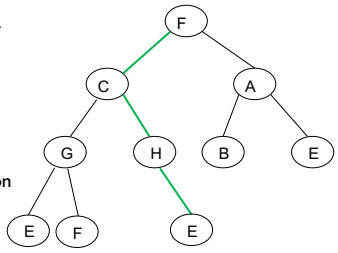
## Tree terminology

Simple path: a series of distinct nodes with edges between successive nodes

F-C-H-E

Path length: number of edges on path

3, (F,C), (C,H), (H,E)

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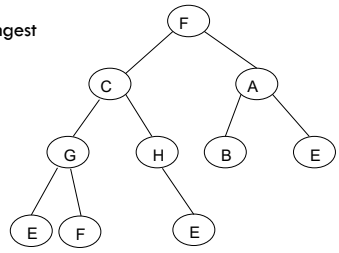
## Tree terminology

Height of a node: length of longest path from the node to a leaf

What is the height of C?

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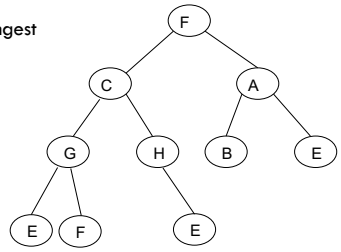
2

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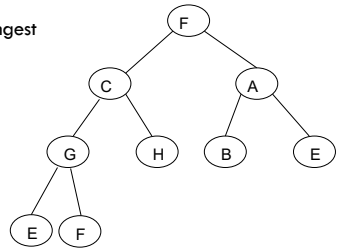


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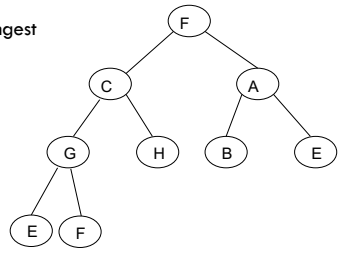


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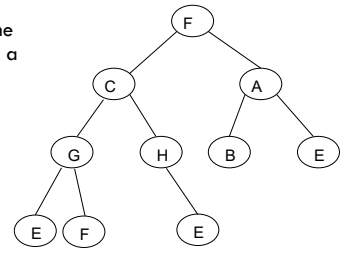
## Tree terminology

Height of the tree: height of the root (longest path from root to a leaf)

What is the height of the tree?

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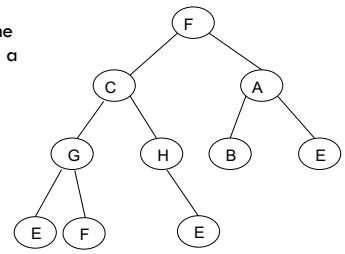


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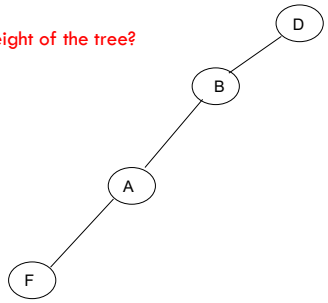
## Tree terminology

What is the height of the tree?

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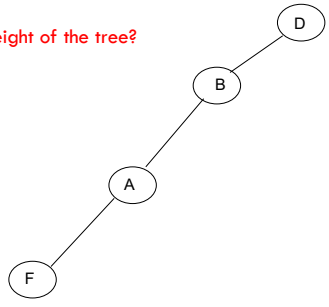
## Tree terminology

What is the height of the tree?

3

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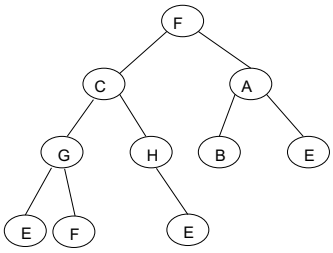
## Tree terminology

Degree of node: number of children

Degree of tree (arity): max degree of any of the nodes

What is the degree of the tree?

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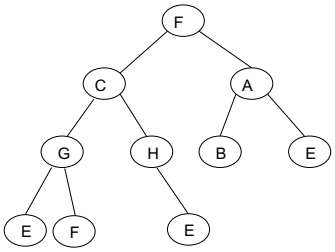


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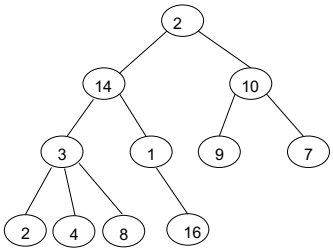


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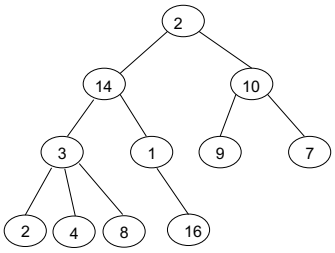


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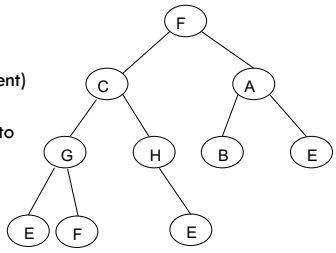
## Tree terminology

Level/depth of node:
- Root is 0
- Level(child) = 1 + level(parent)

(also: length of path from root to node)

What is the depth of G?

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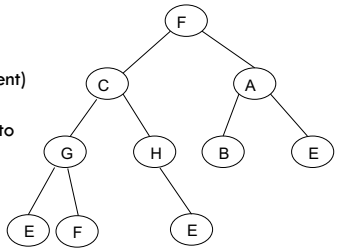


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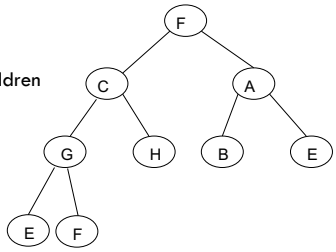
## Tree terminology

Height of node:
- Leaf is 0
- h(node) = max height of children

What is the height of C?

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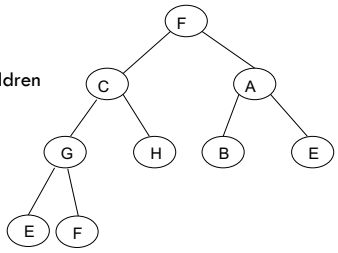


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## Tree terminology (almost there!)

Full tree: a binary tree where every node has 0 or 2 children

Complete: All levels except the last are completely filled and all nodes on the last level are on the left

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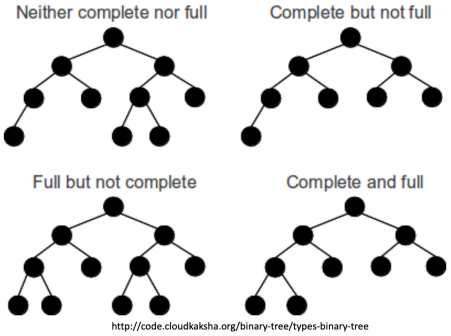
## Full + Complete

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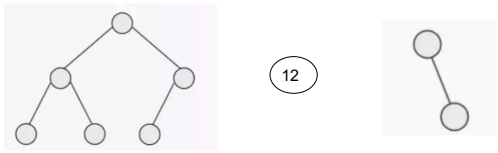
## Full + Complete?

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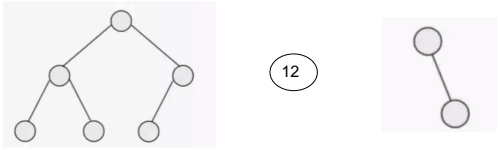
## Full + Complete?




Complete  Full + Complete  Neither

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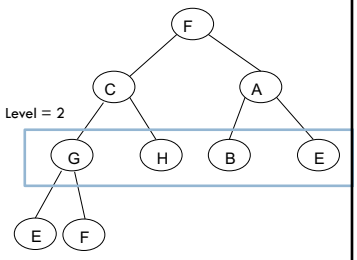
## Nodes in a binary tree

What is the most nodes we can have at a level k?

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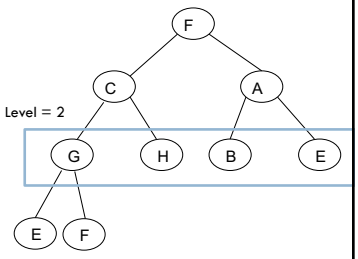
## Nodes in a binary tree

What is the most nodes we can have at a level k?

At most $2^k$ nodes (when every node above it has two children)

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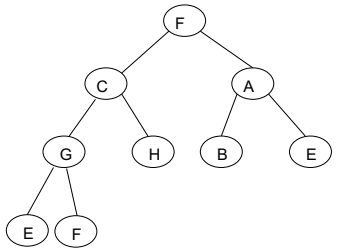
## Nodes in a binary tree

What is the most nodes we can have in a tree of height *h*?

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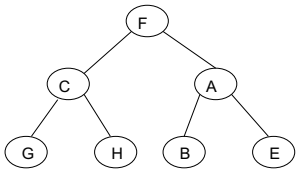


$1 + 2 + 4 + 8 + \cdots + 2^h = 2^{h+1} - 1$

When the tree is full and complete!

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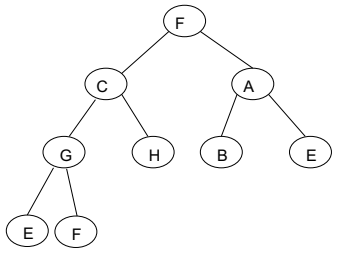
## Nodes in a binary tree

What is the smallest number of nodes we can have in a tree of height *h*?

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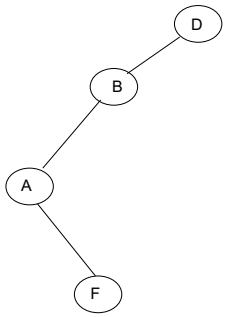


h+1 (the twig!)

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## Nodes in a binary tree

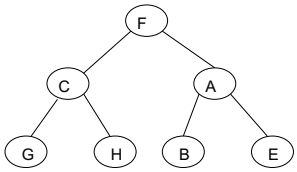
$n = 2^{h+1} - 1$

$n + 1 = 2^{h+1}$

$2^{h+1} = n + 1$

$h + 1 = \log(n + 1)$

$h = \log(n + 1) - 1$

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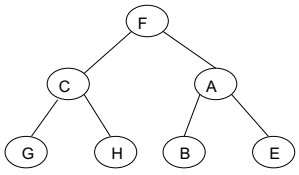
## Nodes in a binary tree




$\log(n + 1) - 1 \leq h \leq n - 1$

Height is somewhere between log(n) of nodes and n nodes

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