

HASHTABLES

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CS 62 – Spring 2021

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Admin

No mentor hours: Friday (3/5) – Tuesday (3/16)

Mentors will reach out regarding learning community meetings.

Office hours today: 3:30-4pm

No office hours next week

No assignment over spring break!

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Sets

An unordered collection

- Things can be added and removes
- Check if things are in the set

```
public interface Set<E> {
    public void put(E key);
    public boolean containsKey(E key);
    public E remove(E key);
    public boolean isEmpty();
    public int size();
}
```

Could we do this with any of our data structures so far?
Big-O runtime of methods?

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Sets

An unordered collection

- Things can be added and removes
- Check if things are in the set

```
public interface Set<E> {
    public void put(E key);
    public boolean containsKey(E key);
    public E remove(E key);
    public boolean isEmpty();
    public int size();
}
```

We'd like to make these operations fast: $O(1)$!

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Integer sets

```
public interface IntegerSet {
    public void put(int key);
    public boolean containsKey(int key);
    public int remove(int key);
    public boolean isEmpty();
    public int size();
}
```


What if we assume that they're integers.

Any ideas how to do this fast?

5

Integer sets

```
public interface IntegerSet {
    public void put(int key);
    public boolean containsKey(int key);
    public int remove(int key);
    public boolean isEmpty();
    public int size();
}
```

Array 

6

Integer sets

```
public interface IntegerSet {
    public void put(int key);
    public boolean containsKey(int key);
    public int remove(int key);
    public boolean isEmpty();
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}
```

put(5) Where should we put it?

Array 

7

Integer sets

```
public interface IntegerSet {
    public void put(int key);
    public boolean containsKey(int key);
    public int remove(int key);
    public boolean isEmpty();
    public int size();
}
```

put(5)

Array 

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Integer sets

```
public interface IntegerSet {
    public void put(int key);
    public boolean containsKey(int key);
    public int remove(int key);
    public boolean isEmpty();
    public int size();
}
```

put(7) Where should we put it?

Array

null	null	null	null	null	5	null	null	null	null	null	null	null	null	null
------	------	------	------	------	---	------	------	------	------	------	------	------	------	------

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Integer sets

```
public interface IntegerSet {
    public void put(int key);
    public boolean containsKey(int key);
    public int remove(int key);
    public boolean isEmpty();
    public int size();
}
```

put(7) Where should we put it?

Array

null	null	null	null	null	5	null	7	null	null	null	null	null	null	null
------	------	------	------	------	---	------	---	------	------	------	------	------	------	------

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Integer sets

```
public interface IntegerSet {
    public void put(int key);
    public boolean containsKey(int key);
    public int remove(int key);
    public boolean isEmpty();
    public int size();
}
```

containsKey(4)?

Array

null	null	null	null	null	5	null	7	null	null	null	null	null	null	null
------	------	------	------	------	---	------	---	------	------	------	------	------	------	------

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Integer sets

```
public interface IntegerSet {
    public void put(int key);
    public boolean containsKey(int key);
    public int remove(int key);
    public boolean isEmpty();
    public int size();
}
```

containsKey(4)? No!

Array

null	null	null	null	null	5	null	7	null	null	null	null	null	null	null
------	------	------	------	------	---	------	---	------	------	------	------	------	------	------

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Integer sets

```
public interface IntegerSet {
    public void put(int key);
    public boolean containsKey(int key);
    public int remove(int key);
    public boolean isEmpty();
    public int size();
}
```

containsKey(7)?

Array

null	null	null	null	null	5	7	null	null	null	null	null	null	null
------	------	------	------	------	---	---	------	------	------	------	------	------	------

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Integer sets

```
public interface IntegerSet {
    public void put(int key);
    public boolean containsKey(int key);
    public int remove(int key);
    public boolean isEmpty();
    public int size();
}
```

containsKey(7)? Yes!

Array

null	null	null	null	null	5	7	null	null	null	null	null	null	null
------	------	------	------	------	---	---	------	------	------	------	------	------	------

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Integer sets

```
public interface IntegerSet {
    public void put(int key);
    public boolean containsKey(int key);
    public int remove(int key);
    public boolean isEmpty();
    public int size();
}
```

remove(5)?

Array

null	null	null	null	null	5	7	null	null	null	null	null	null	null
------	------	------	------	------	---	---	------	------	------	------	------	------	------

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Integer sets

```
public interface IntegerSet {
    public void put(int key);
    public boolean containsKey(int key);
    public int remove(int key);
    public boolean isEmpty();
    public int size();
}
```

remove(5)?

Array

null	null	null	null	null	null	7	null	null	null	null	null	null	null
------	------	------	------	------	------	---	------	------	------	------	------	------	------

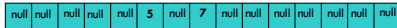
16

Integer sets

```
public interface IntegerSet {
    public void put(int key);
    public boolean containsKey(int key);
    public int remove(int key);
    public boolean isEmpty();
    public int size();
}
```

What is the big-O runtime of the methods? $O(1)$

Array



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Integer sets

```
public interface IntegerSet {
    public void put(int key);
    public boolean containsKey(int key);
    public int remove(int key);
    public boolean isEmpty();
    public int size();
}
```

Any problems?

Array



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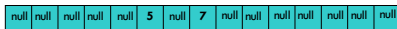
Integer sets

```
public interface IntegerSet {
    public void put(int key);
    public boolean containsKey(int key);
    public int remove(int key);
    public boolean isEmpty();
    public int size();
}
```

Any problems?

- Negative numbers (could fix this by adding a constant)
- The array would have to be huge!
(ints range from -2^{31} to $2^{31}-1$,
 $2^{31} \approx 2$ billion)

Array



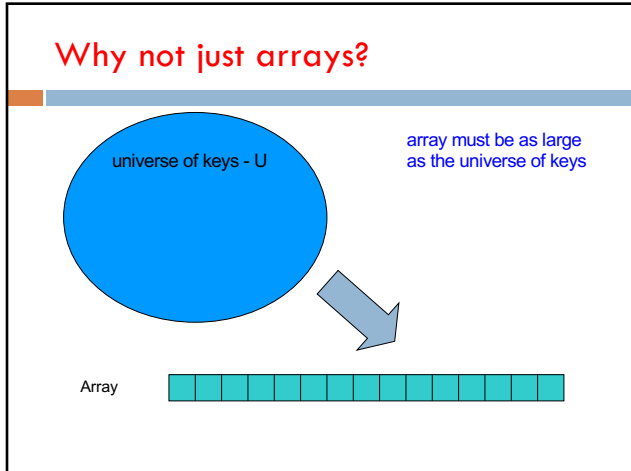
19

Keys

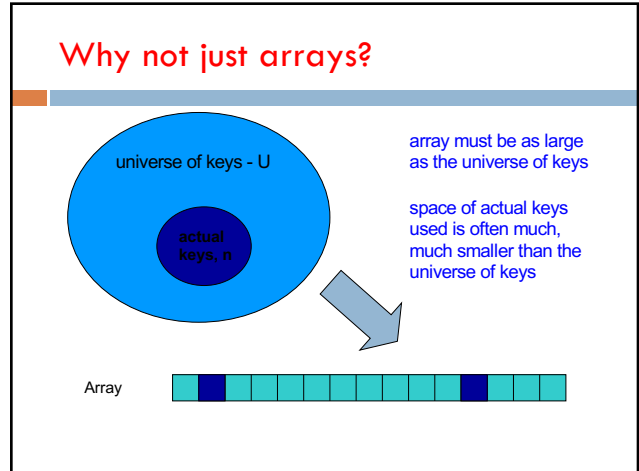
universe of keys - U

For whatever key we're using (e.g., integers), there is a universe of all possible keys

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Why not arrays?

Think of indexing all last names < 10 characters

- ❑ Census listing of all last names
 - <http://www.census.gov/genealogy/names/dist.all.last>
 - 88,799 last names
- ❑ What is the size of our space of keys?
 - 26^{10} = a big number
- ❑ Not feasible!
- ❑ Even if it were, not space efficient

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Hashtables

```
public interface Set<E> {
    public void put(E key);
    public boolean containsKey(E key);
    public E remove(E key);
    public boolean isEmpty();
    public int size();
}
```

Using an array is still a good idea

Key idea: need to translate from the key into an index in the array

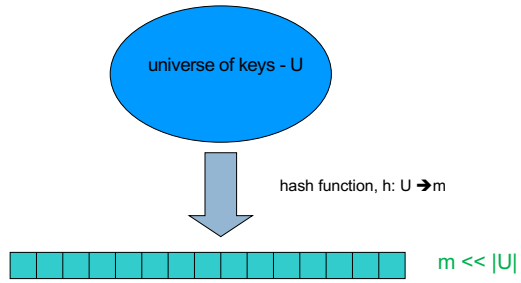
Array

null	null	null	null	5	null	7	null	null	null	null	null	null	m
------	------	------	------	---	------	---	------	------	------	------	------	------	---

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Hash function, h

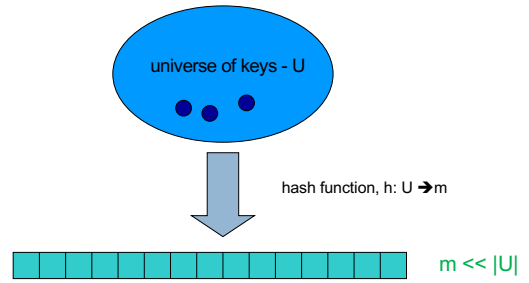
A hash function is a function that maps the universe of keys to a restricted range (e.g., the size of an array)



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Hash function, h

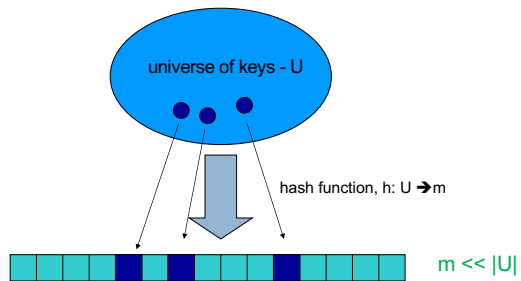
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Hash function, h

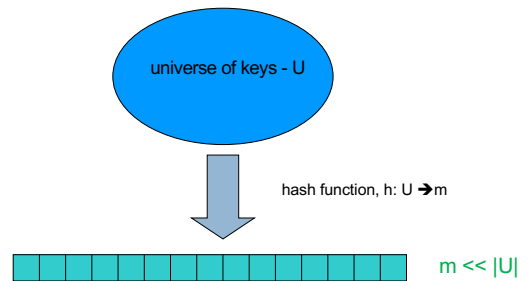
A hash function is a function that maps the universe of keys to a restricted range (e.g., the size of an array)



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Hash function, h

What can happen if $m < |U|$?



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Collisions

If $m < |U|$, then two keys can map to the same position in the hashtable (pidgeonhole principle)

$m \ll |U|$

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Collisions

A collision occurs when $h(x) = h(y)$, but $x \neq y$

A good hash function will minimize the number of collisions

Because the number of hashtable (array) entries is less than the possible keys (i.e. $m < |U|$) collisions are inevitable!

We need to handle collisions!

Collision resolution techniques?

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Collision resolution by chaining

Hashtable consists of an array of linked lists

```
private LinkedList<E>[] table;
```

When a collision occurs, the element is added to linked list at that location

If two entries $x \neq y$ have the same hash value $h(x) = h(y)$, then `table[h(x)]` will contain a linked list with both values

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put

```
public void put(E key) {
    LinkedList<E> entry = table[h(key)];

    if( entry == null ){
        entry = new LinkedList<E>();
        table[h(key)] = entry;
    }

    entry.addFirst(key);
}
```

What does this code do?

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```

public void put(E key) {
    LinkedList<E> entry = table[h(key)];

    if( entry == null ){
        entry = new LinkedList<E>();
        table[h(key)] = entry;
    }

    entry.addFirst(key);
}
    
```

put(■)

33

```

public void put(E key) {
    LinkedList<E> entry = table[h(key)];

    if( entry == null ){
        entry = new LinkedList<E>();
        table[h(key)] = entry;
    }

    entry.addFirst(key);
}
    
```

put(■) h(■) hash function is a mapping from the key to some value < m

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```

public void put(E key) {
    LinkedList<E> entry = table[h(key)];

    if( entry == null ){
        entry = new LinkedList<E>();
        table[h(key)] = entry;
    }

    entry.addFirst(key);
}
    
```

put(■) h(■) hash function is a mapping from the key to some value < m

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```

public void put(E key) {
    LinkedList<E> entry = table[h(key)];

    if( entry == null ){
        entry = new LinkedList<E>();
        table[h(key)] = entry;
    }

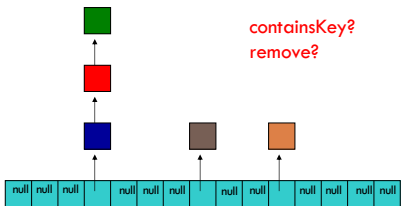
    entry.addFirst(key);
}
    
```

put(■) h(■) hash function is a mapping from the key to some value < m

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More methods

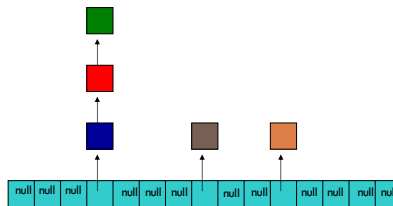
```
public interface Set<E> {
    public void put(E key);
    public boolean containsKey(E key);
    public E remove(E key);
    public boolean isEmpty();
    public int size();
}
```



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containsKey

```
public boolean containsKey(E key){
    LinkedList<E> entry = table[h(key)];
    return entry == null ? false : entry.contains(key);
}
```



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ternary operator

```
public boolean containsKey(E key){
    LinkedList<E> entry = table[h(key)];
    return entry == null ? false : entry.contains(key);
}
```

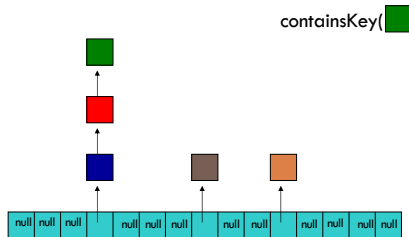
```
public boolean containsKey(E key){
    LinkedList<E> entry = table[h(key)];

    if( entry == null ) {
        return false;
    } else {
        return entry.contains(key);
    }
}
```

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containsKey

```
public boolean containsKey(E key){
    LinkedList<E> entry = table[h(key)];
    return entry == null ? false : entry.contains(key);
}
```



40

containsKey

```

public boolean containsKey(E key){
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```

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containsKey

```

public boolean containsKey(E key){
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}

```

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containsKey

```

public boolean containsKey(E key){
    LinkedList<E> entry = table[h(key)];
    return entry == null ? false : entry.contains(key);
}

```

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containsKey

```

public boolean containsKey(E key){
    LinkedList<E> entry = table[h(key)];
    return entry == null ? false : entry.contains(key);
}

```

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remove

```

public E remove(E key) {
    LinkedList<E> entry = table[h(key)];
    if( entry != null && entry.remove(key) ){
        return key;
    }else{
        return null;
    }
}

```

45

remove

```

public E remove(E key) {
    LinkedList<E> entry = table[h(key)];
    if( entry != null && entry.remove(key) ){
        return key;
    }else{
        return null;
    }
}

```

containsKey(■)

46

remove

```

public E remove(E key) {
    LinkedList<E> entry = table[h(key)];
    if( entry != null && entry.remove(key) ){
        return key;
    }else{
        return null;
    }
}

```

containsKey(■)

h(■)

47

remove

```

public E remove(E key) {
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    if( entry != null && entry.remove(key) ){
        return key;
    }else{
        return null;
    }
}

```

containsKey(■)

h(■)

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remove

```

public E remove(E key) {
    LinkedList<E> entry = table[h(key)];
    if( entry != null && entry.remove(key) ){
        return key;
    }else{
        return null;
    }
}
    
```

containsKey(■)
h(■)

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remove

```

public E remove(E key) {
    LinkedList<E> entry = table[h(key)];
    if( entry != null && entry.remove(key) ){
        return key;
    }else{
        return null;
    }
}
    
```

containsKey(■)
h(■)

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Running time?

put

containsKey

remove

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Running time?

put: $O(1)$

containsKey: $O(\text{length of linked list})$

remove: $O(\text{length of linked list})$

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Length of the chain

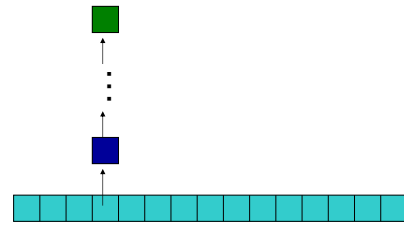
Worst case?

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Length of the chain

Worst case?

- All elements hash to the same location
- $h(k) = 4$
- n



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