

GRAPHS: SEARCH

David Kauchak
CS 62 – Spring 2020

Admin



Assignment 8

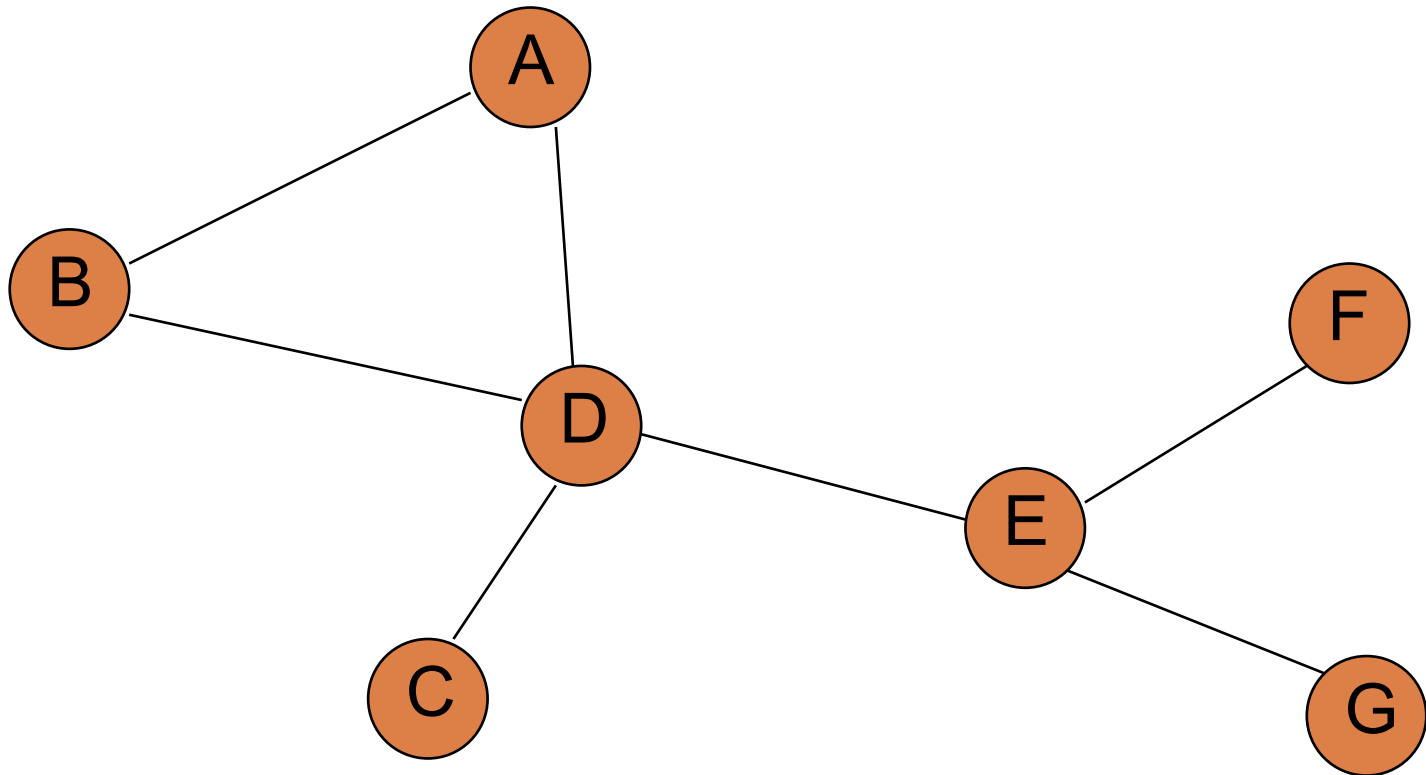
Lab tomorrow:

- ▣ Look at Java code examples of graph representations and graph algorithms
- ▣ Work session for assignment

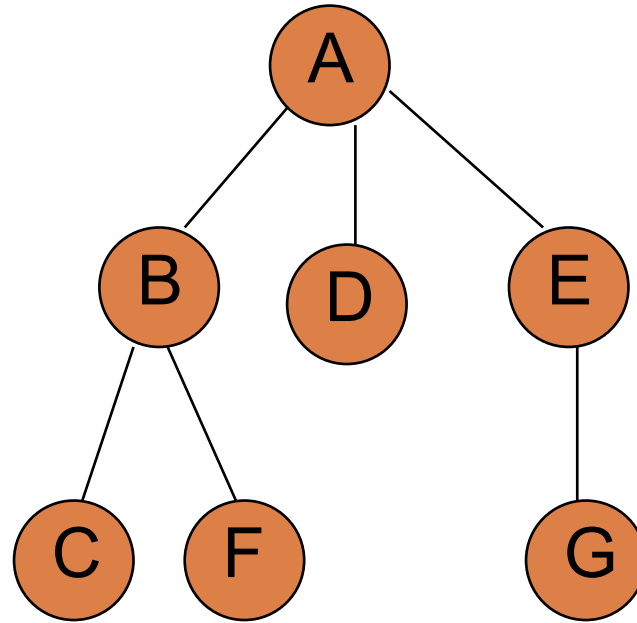
Final

Graphs

A graph is a set of vertices V and a set of edges $(u,v) \in E$ where $u,v \in V$

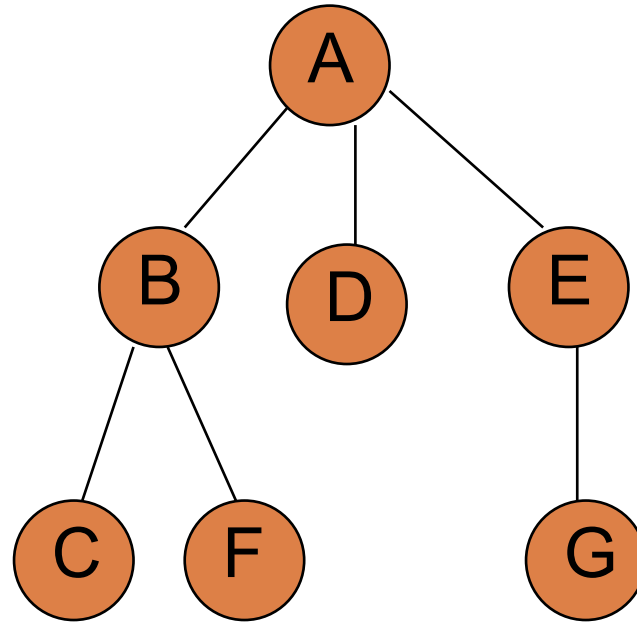


Searching a tree



How can we print out all of the vertices in a tree?

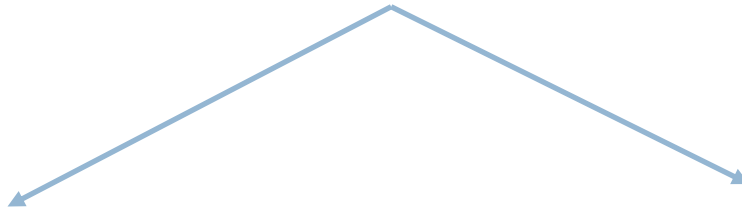
Searching a tree



We could do any of the traversals we saw before:
pre-order, in-order, post-order

A flash from the past

```
public interface Linear<E> {  
    public void add(E item);  
    public E remove();  
    public E peek();  
    public boolean empty();  
}
```



Stack:

- LIFO
- Add to the back
- Remove from the **back**

Queue:

- FIFO
- Add to the back
- Remove from the **front**

Searching a tree

```
treeBFS( start )
```

```
  q = new Queue()
```

```
  q.add(start)
```

```
  treeSearch(q)
```

```
treeDFS( start )
```

```
  s = new Stack()
```

```
  s.add(start)
```

```
  treeSearch(s)
```

```
treeSearch( toVisit )
```

```
  while !toVisit.empty()
```

```
    v = toVisit.remove()
```

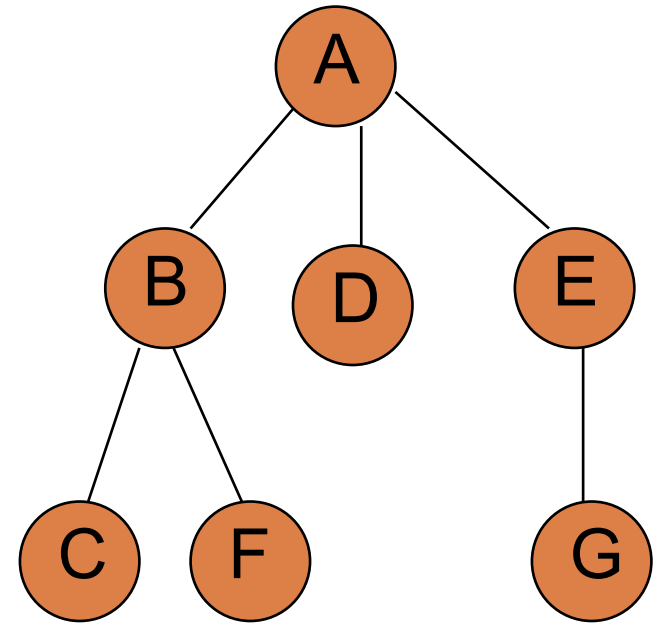
```
    // visit v, e.g., print it out
```

```
    for c in v.getChildren()
```

```
      toVisit.add(c)
```

Tree BFS

```
treeBFS( start )  
  q = new Queue()  
  q.add(start)  
  treeSearch(q)
```

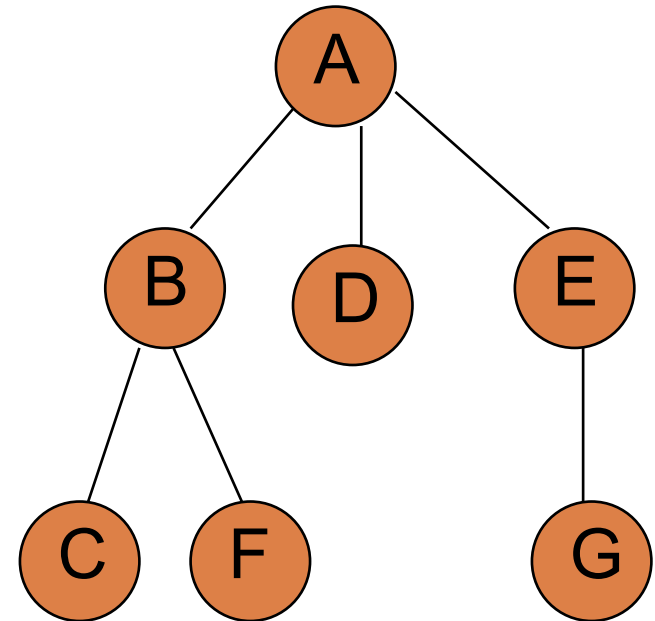


q:

Visited:

Tree BFS

```
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  q = new Queue()  
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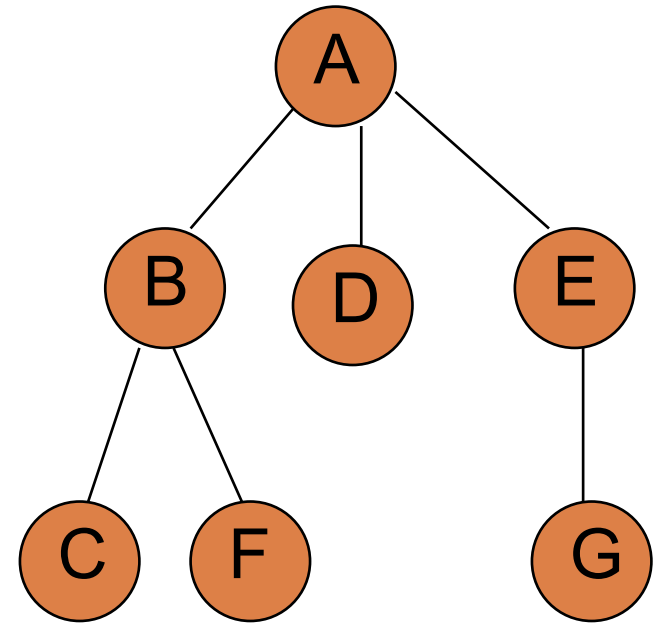


q: **A**

Visited:

Tree BFS

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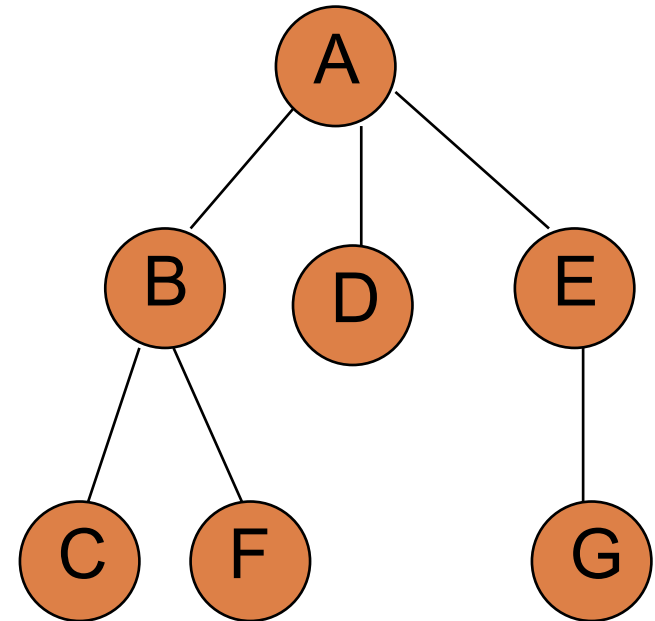


toVisit-queue: A
printed:

What order will the nodes get printed out?
Assume children are traversed left to right.

Tree BFS

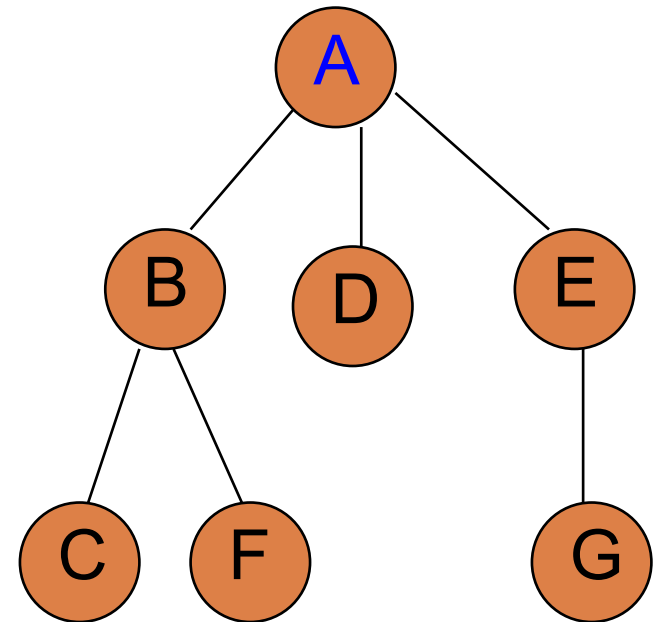
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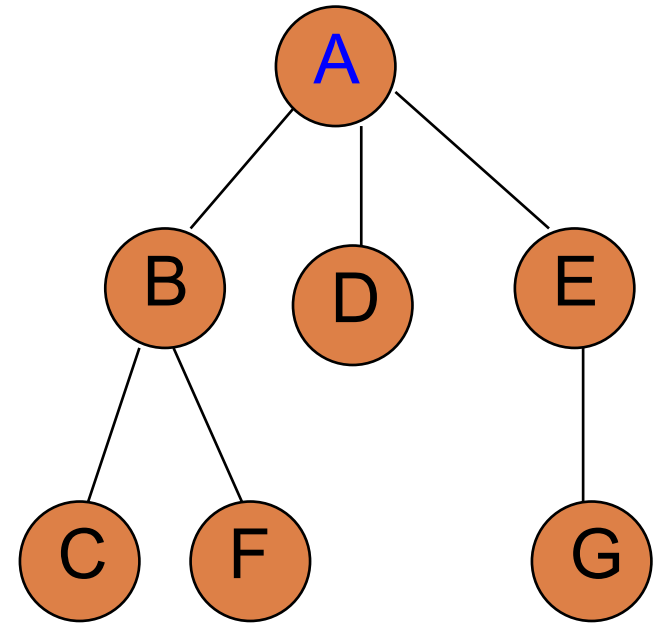


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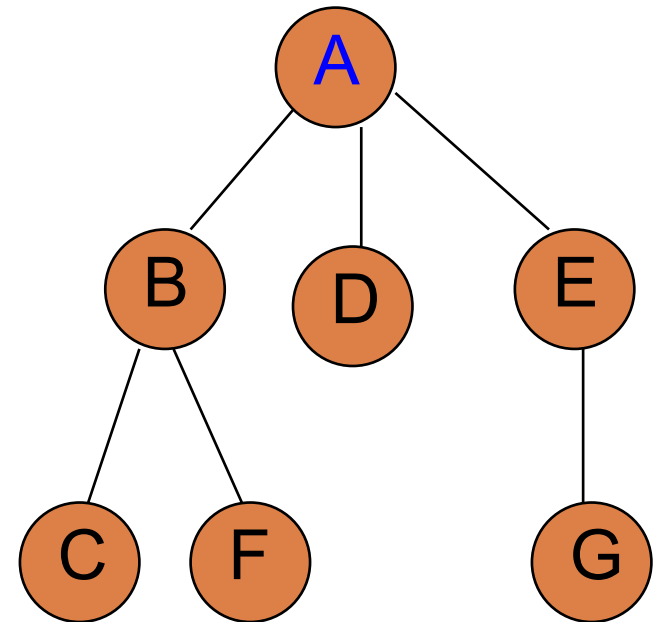


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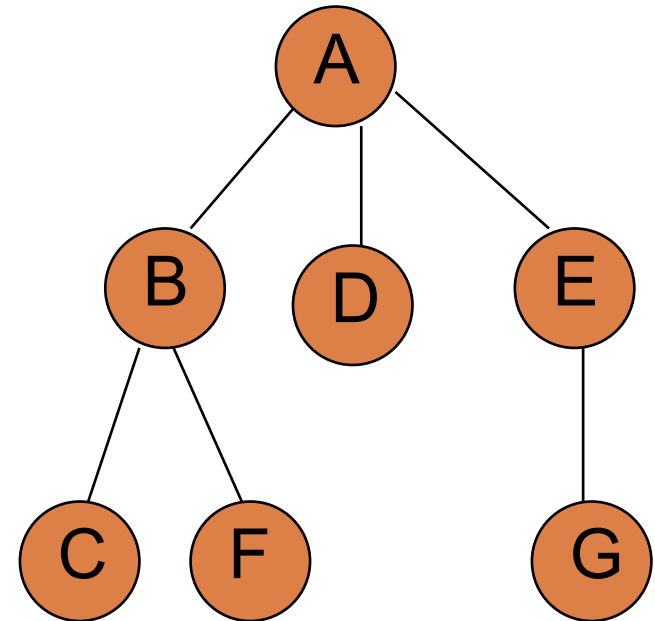


toVisit-queue: B D E

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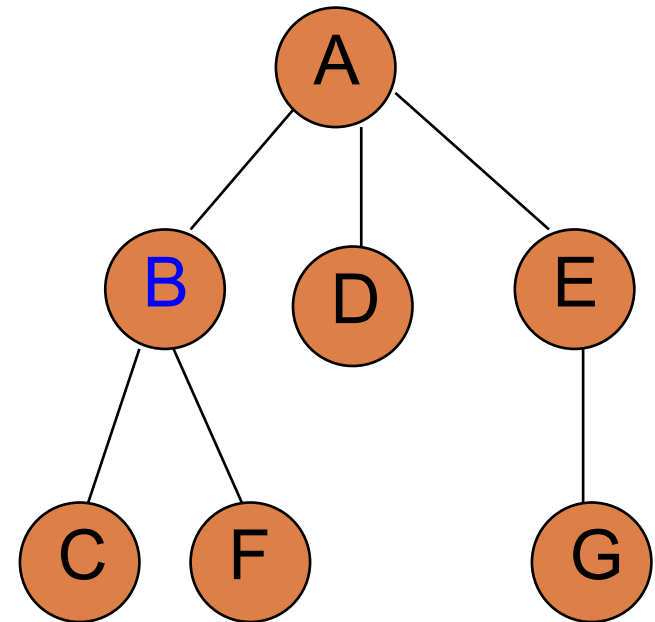


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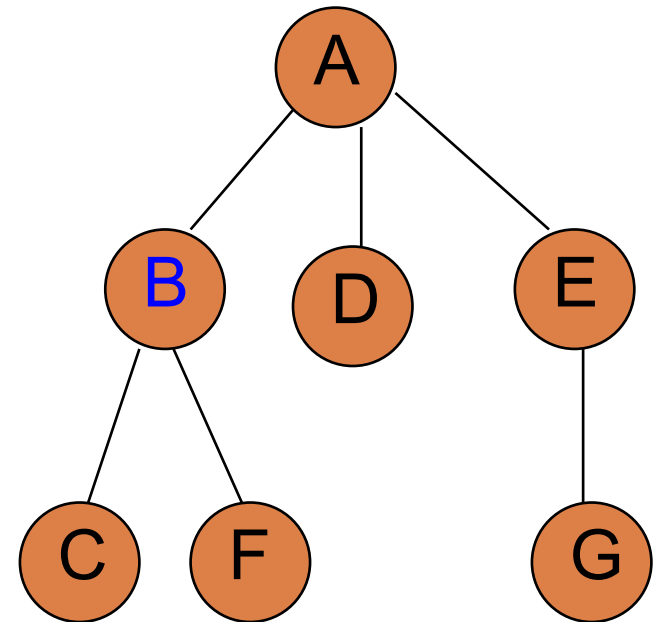


toVisit-queue: D E

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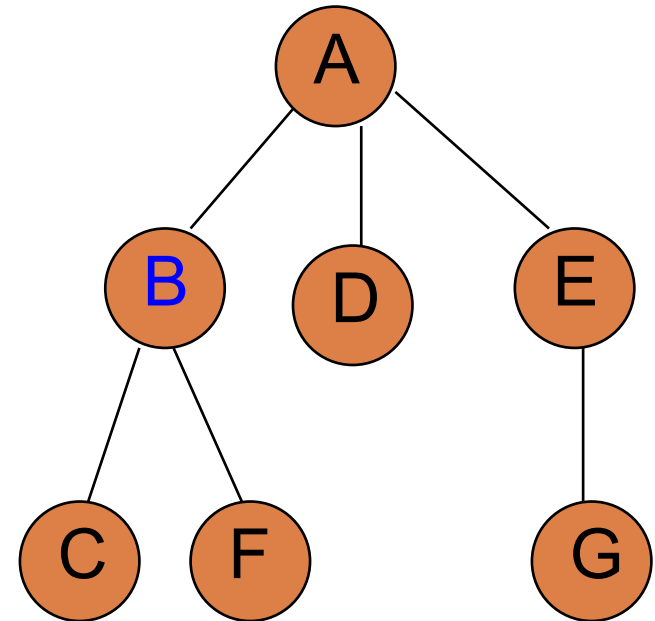


toVisit-queue: D E

printed: A **B**

Tree BFS

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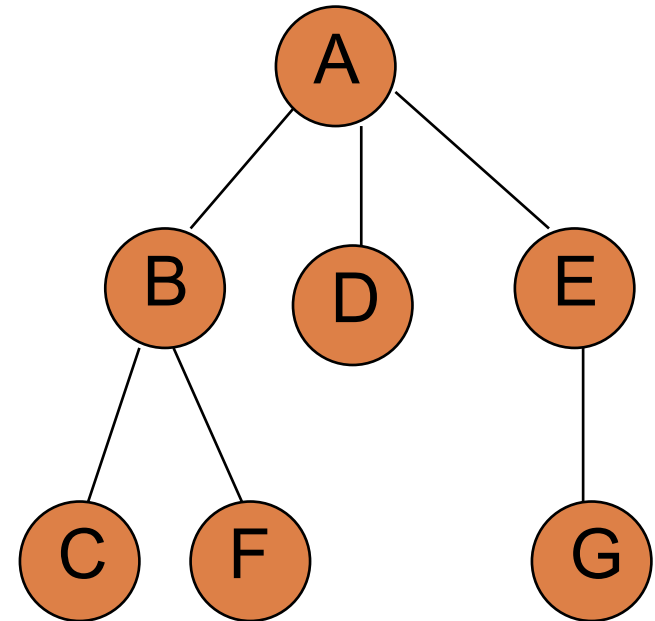


toVisit-queue: D E C F

printed: A B

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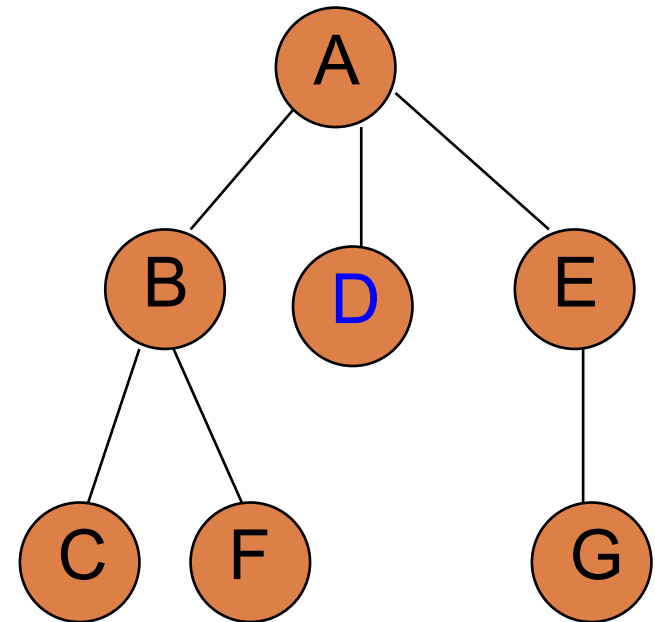


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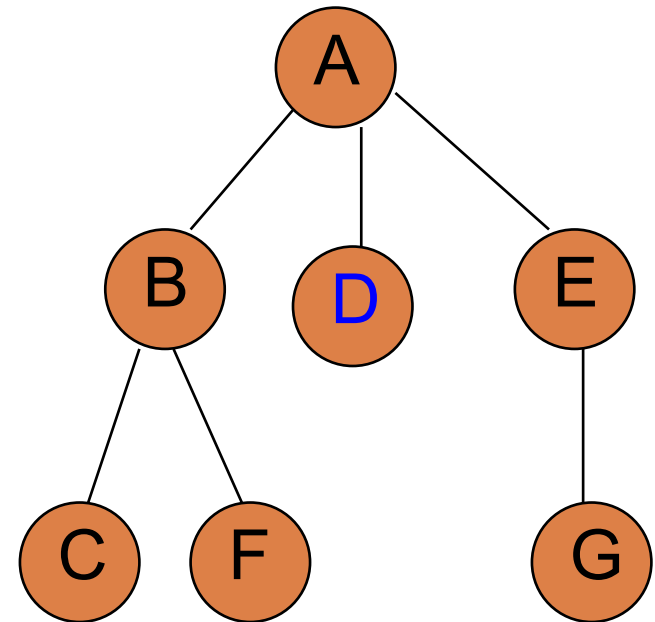


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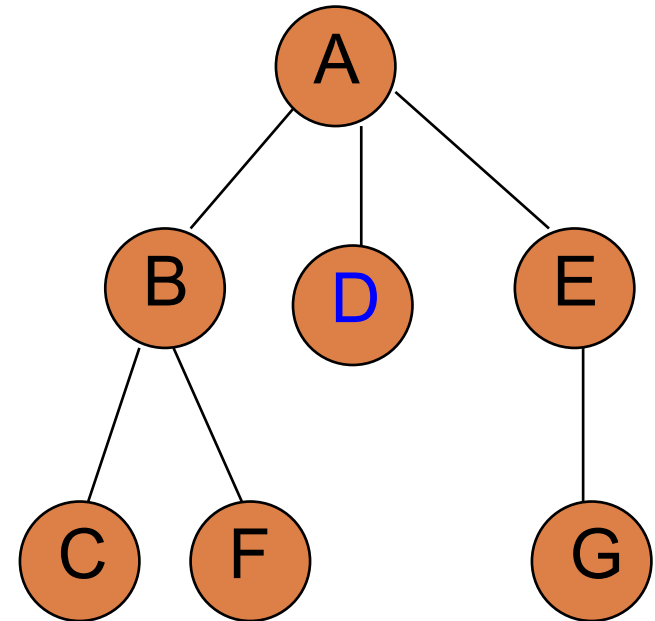


toVisit-queue: E C F

printed: A B **D**

Tree BFS

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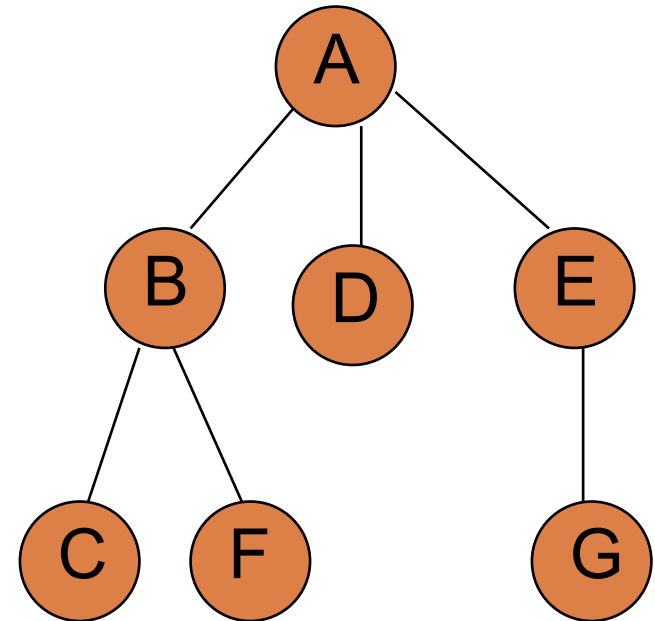


toVisit-queue: E C F

printed: A B D

Tree BFS

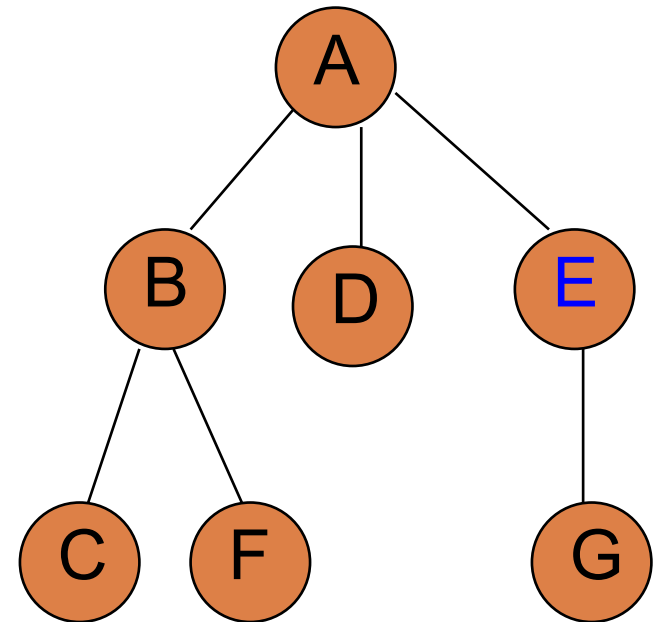
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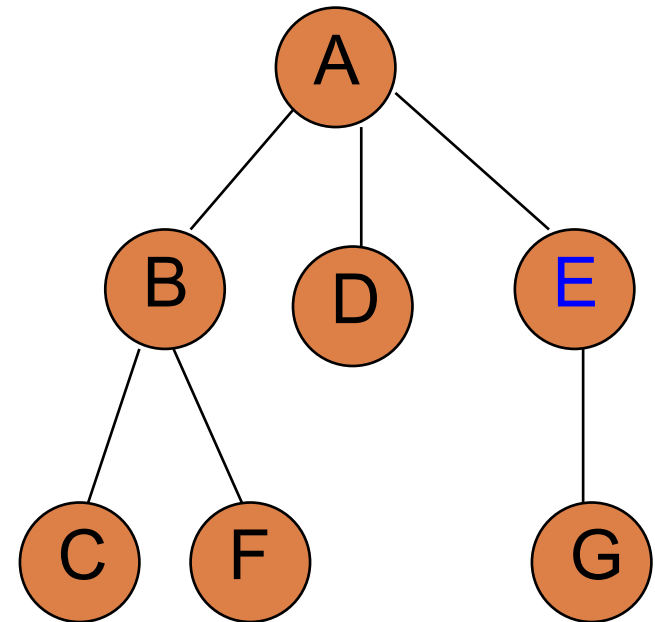


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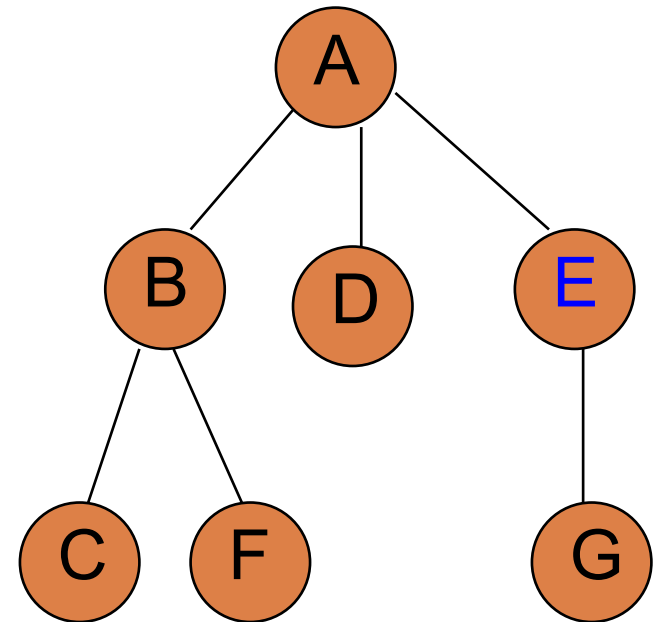


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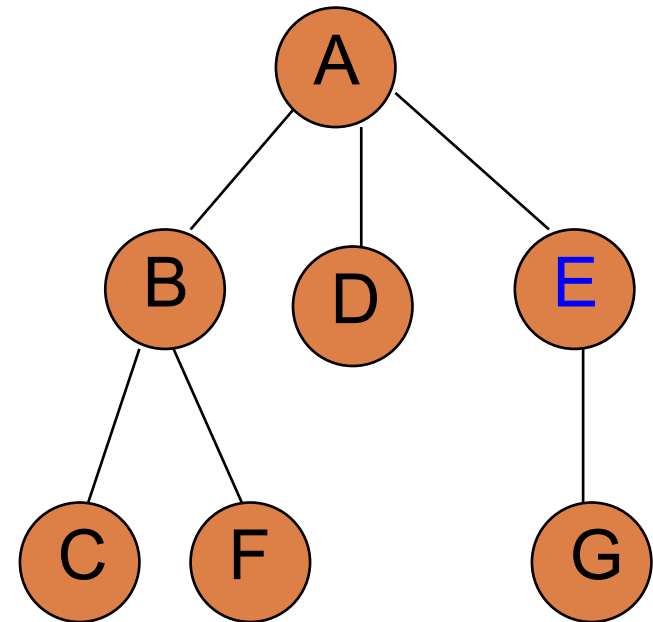


toVisit-queue: C F **G**

printed: A B D **E**

Tree BFS

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toVisit-queue: C F G

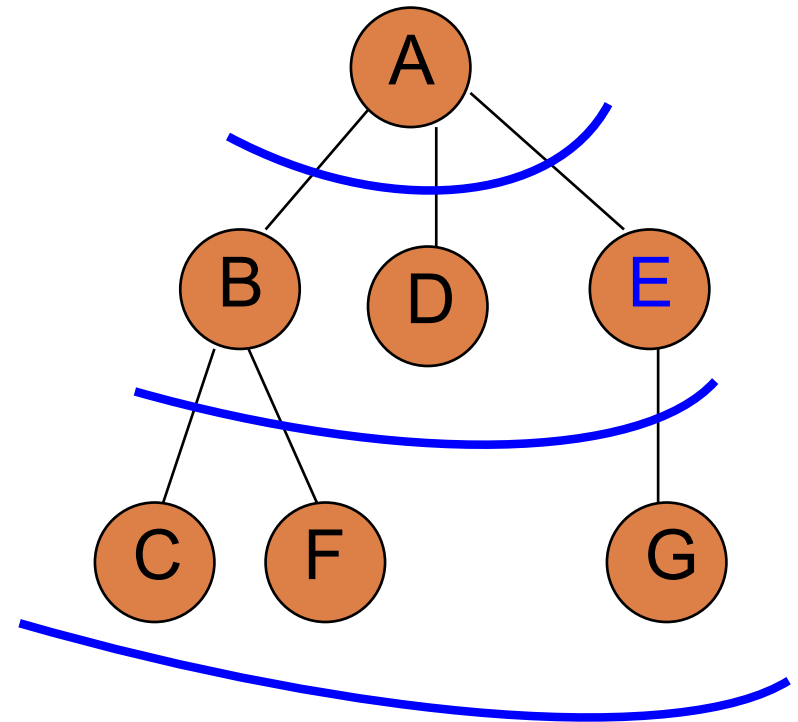
printed: A B D E

How are we exploring the vertices?

Tree BFS

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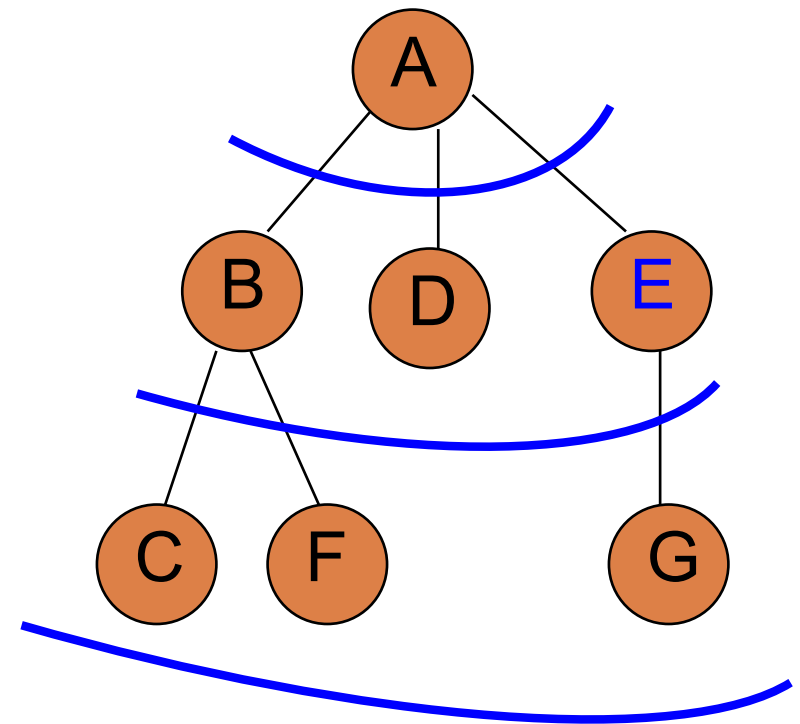
toVisit-queue: C F **G**
printed: A B D **E**



Frontier: all vertices a given number of edges from the start/root

Tree BFS = Tree breadth first search

```
treeSearch( toVisit )  
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```



toVisit-queue: C F **G**

printed: A B D **E**

Frontier: all vertices a given number of edges from the start/root

Tree BFS

```
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```

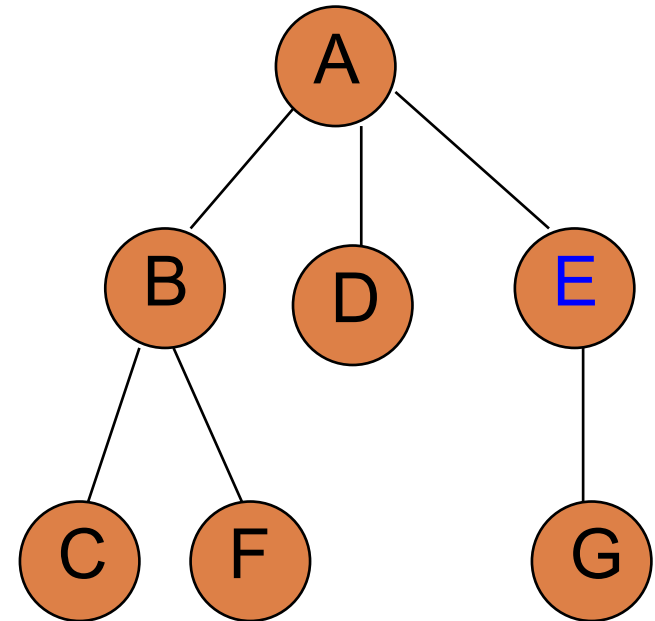
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toVisit-queue: C F G

printed: A B D E

Tree BFS

```
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```

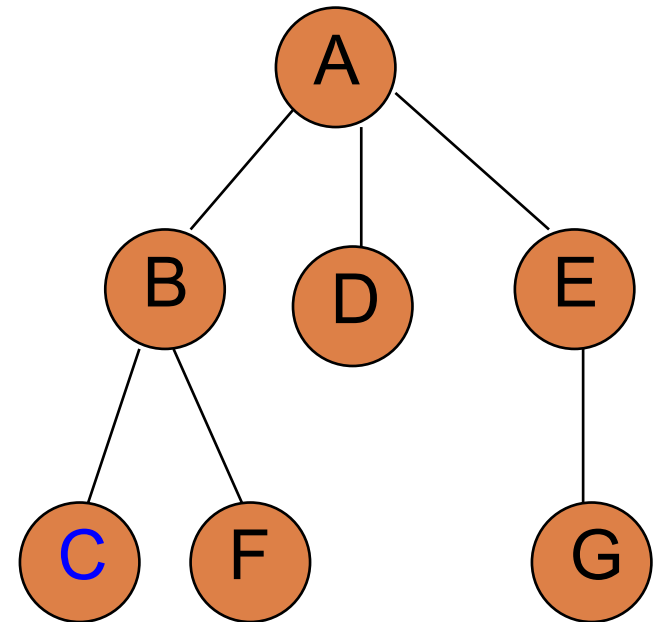
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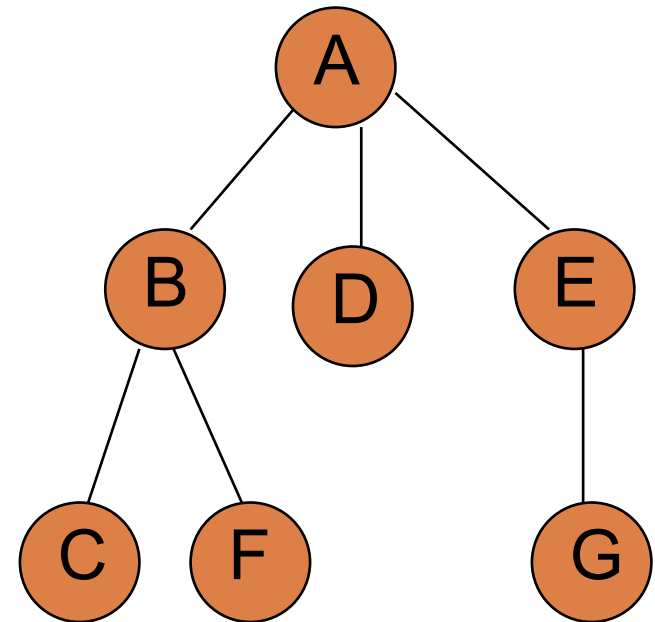


toVisit-queue: F G

printed: A B D E C

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toVisit-queue:

printed: A B D E C F G

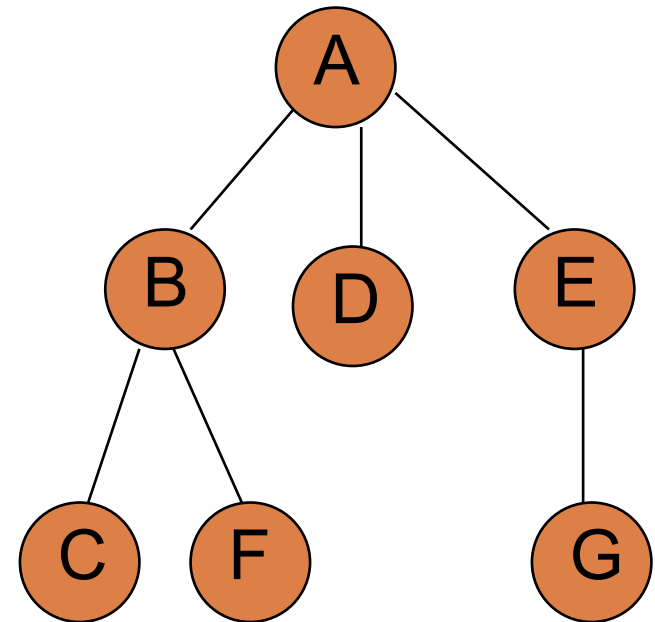
Tree DFS

```
treeDFS( start )
```

```
  s = new Stack()
```

```
  s.add(start)
```

```
  treeSearch(s)
```



S:

printed:

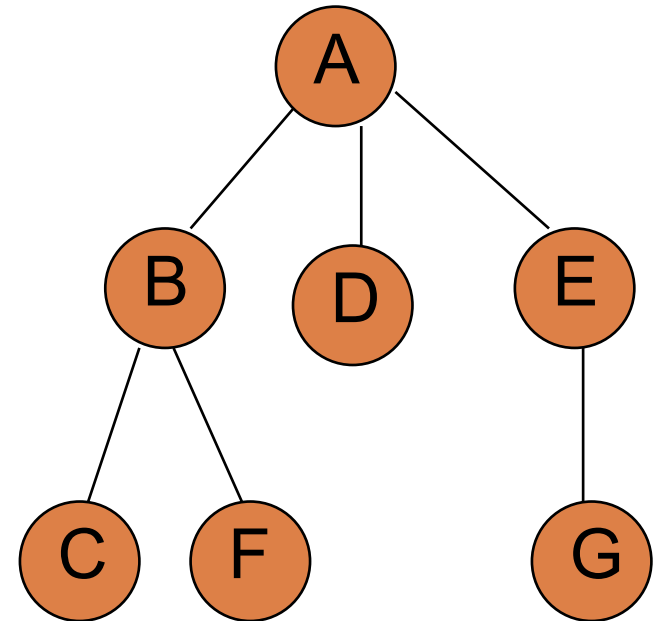
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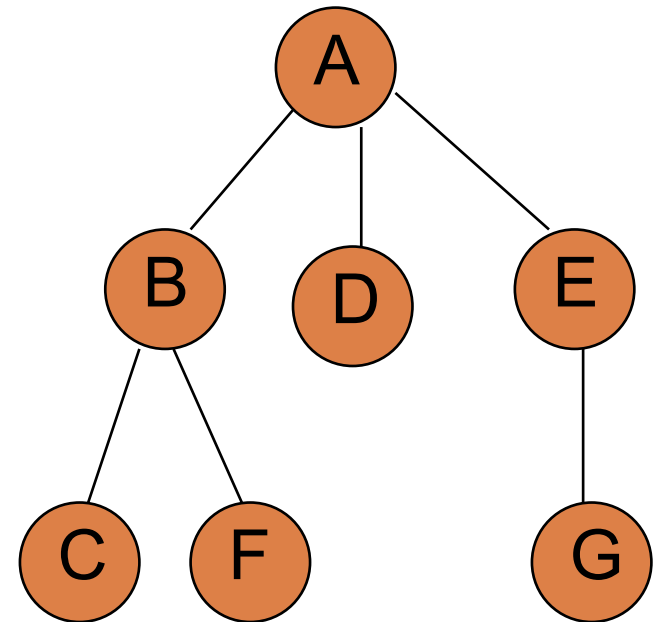


s: A

printed:

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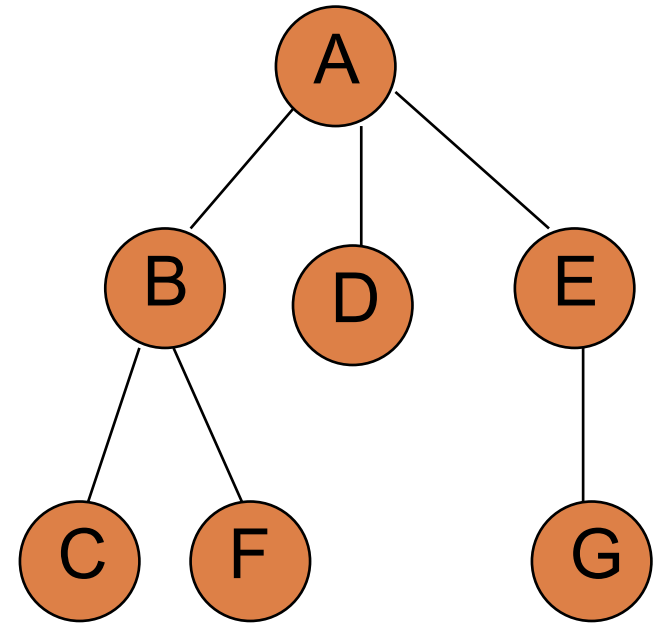


toVisit-stack: A
printed:

What order will the nodes get printed out?
Assume children are traversed left to right.

Tree DFS

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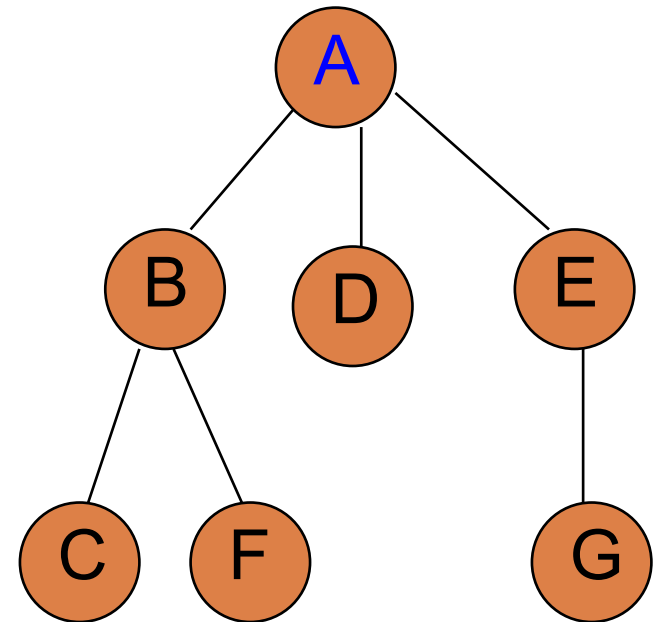


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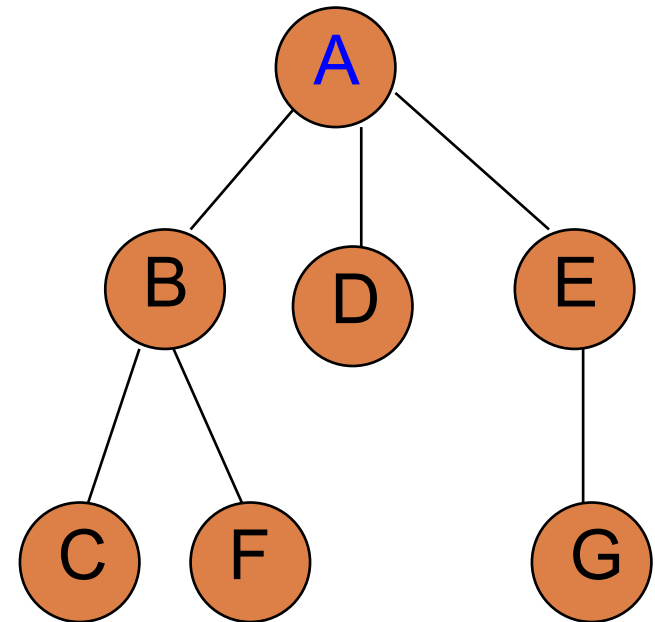


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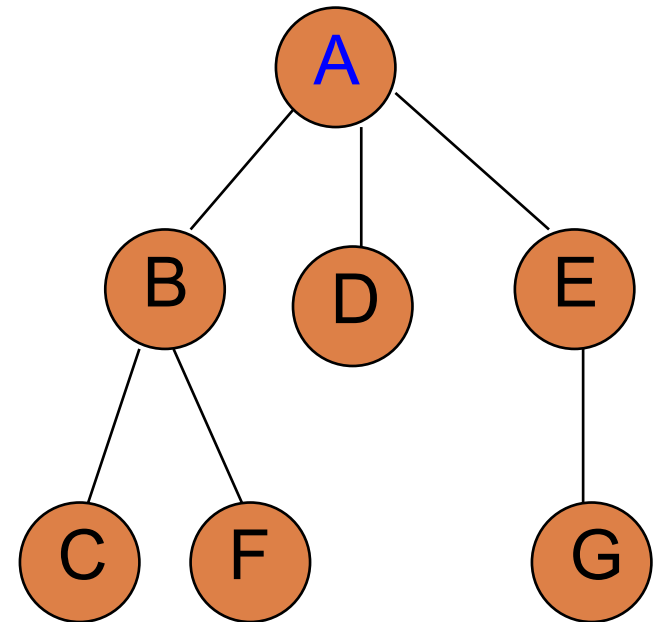


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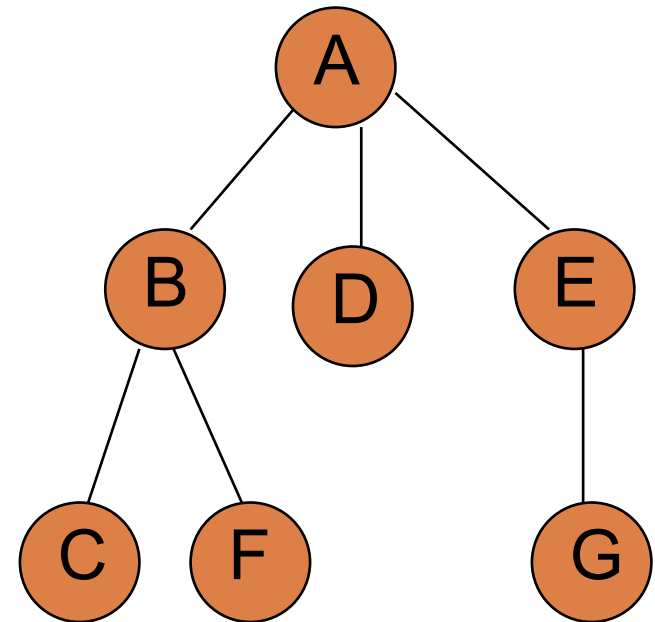


toVisit-stack: B D E

printed: A

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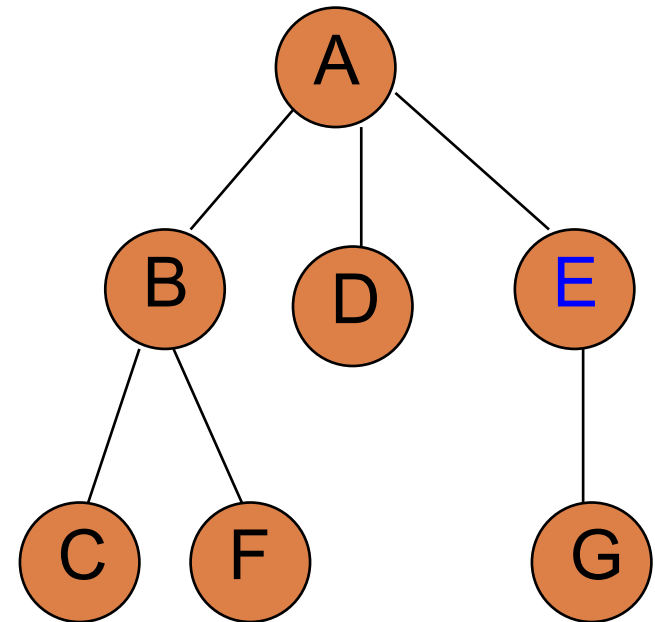


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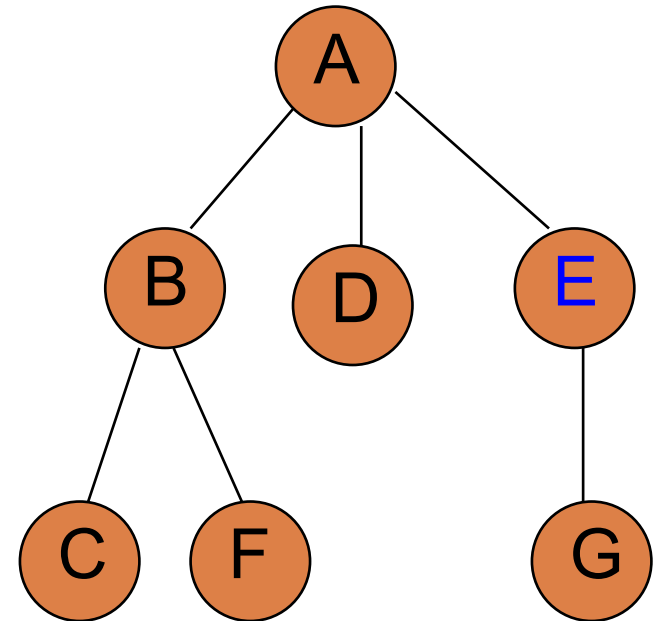


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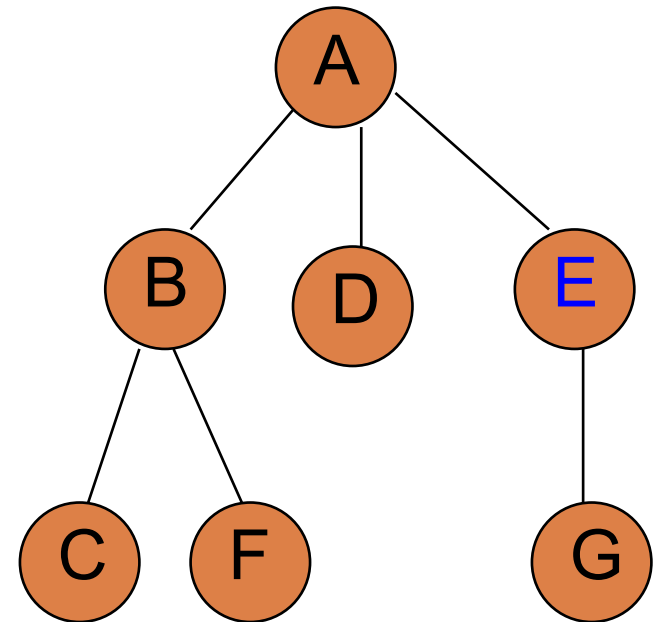


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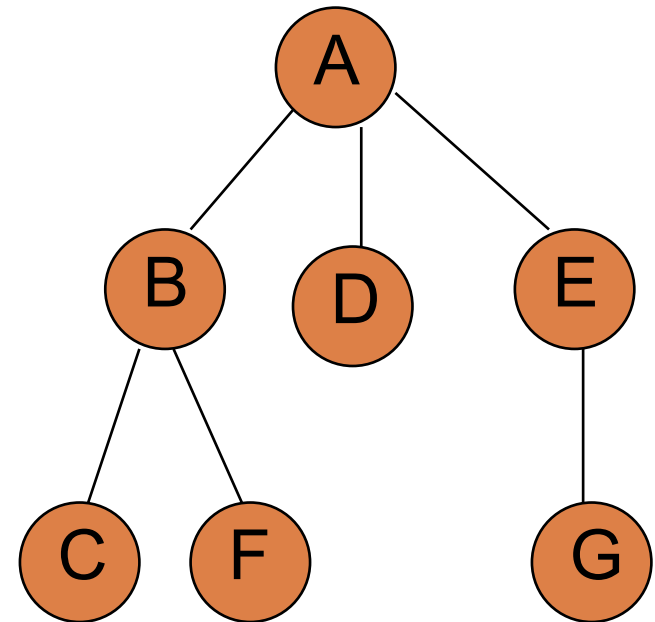


toVisit-stack: B D G

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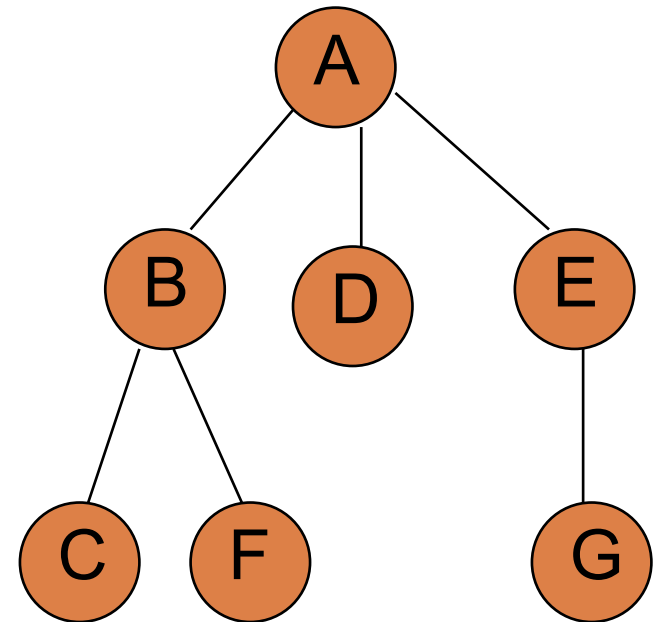


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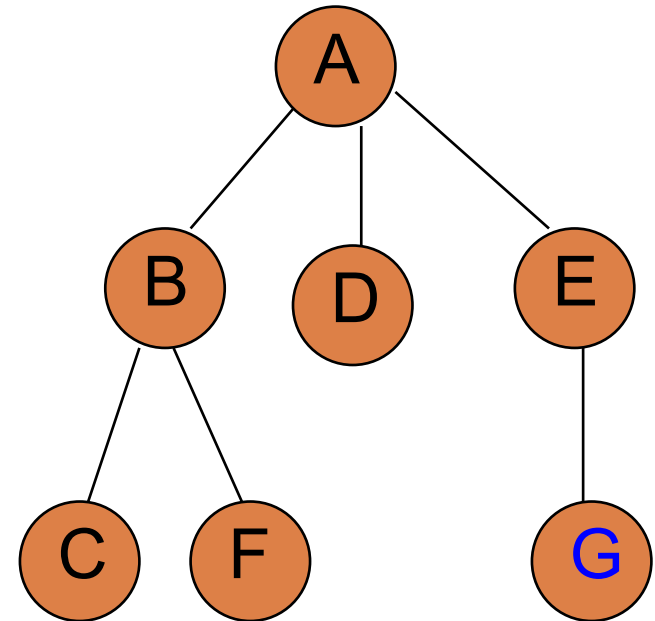


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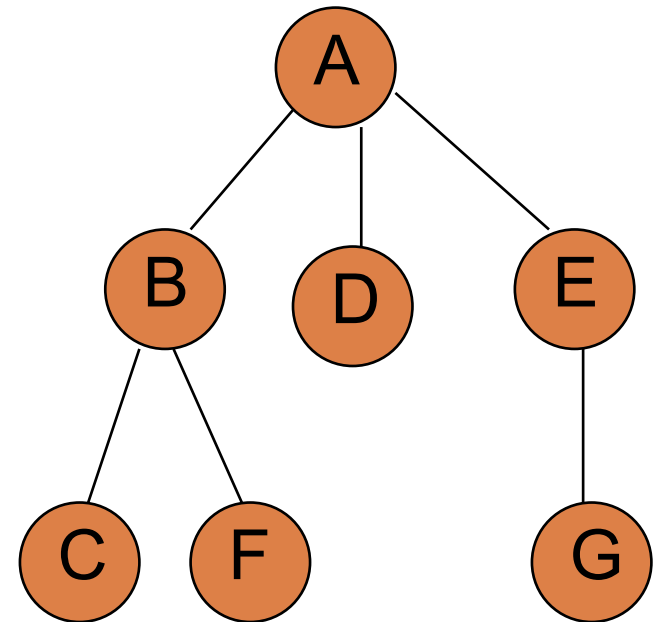


toVisit-stack: B D

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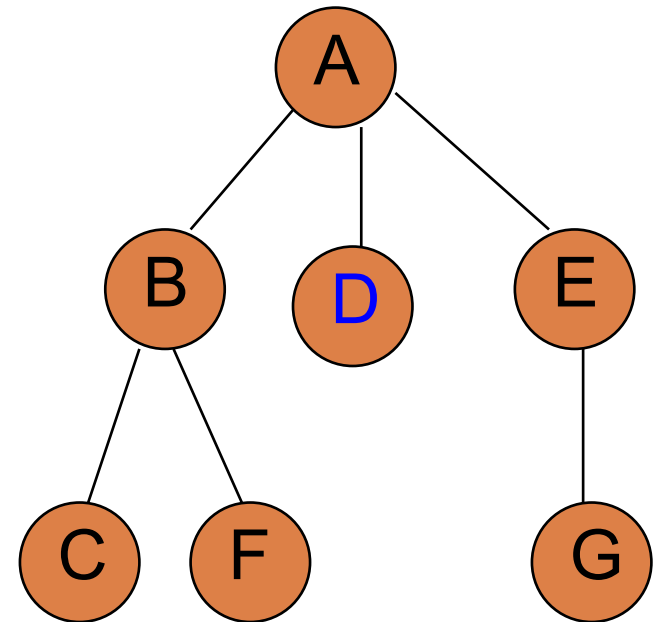


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        toVisit.add(c)
```

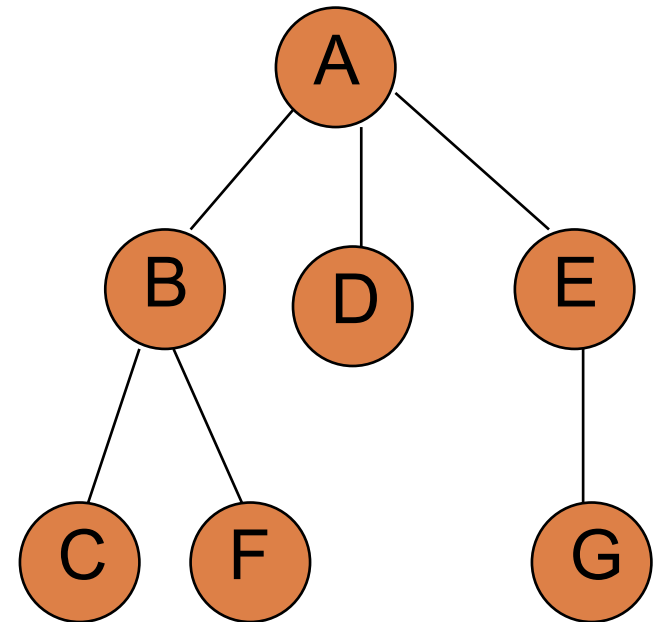


toVisit-stack: B

printed: A E G D

Tree DFS

```
treeSearch( toVisit )  
while !toVisit.empty()  
    v = toVisit.remove()  
    // visit v, e.g., print it out  
    for c in v.getChildren()  
        toVisit.add(c)
```



toVisit-stack: B

printed: A E G D

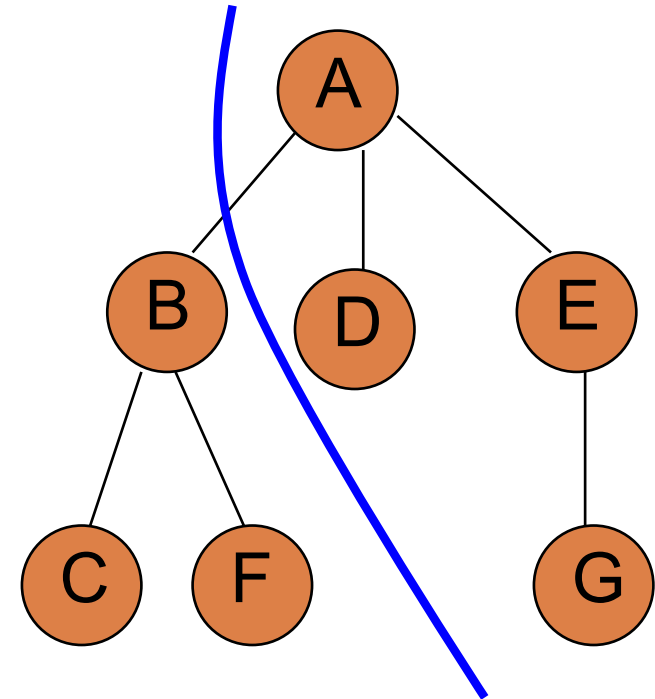
How are we exploring the vertices?

Tree DFS

```
treeSearch( toVisit )  
while !toVisit.empty()  
    v = toVisit.remove()  
    // visit v, e.g., print it out  
    for c in v.getChildren()  
        toVisit.add(c)
```

toVisit-stack: B

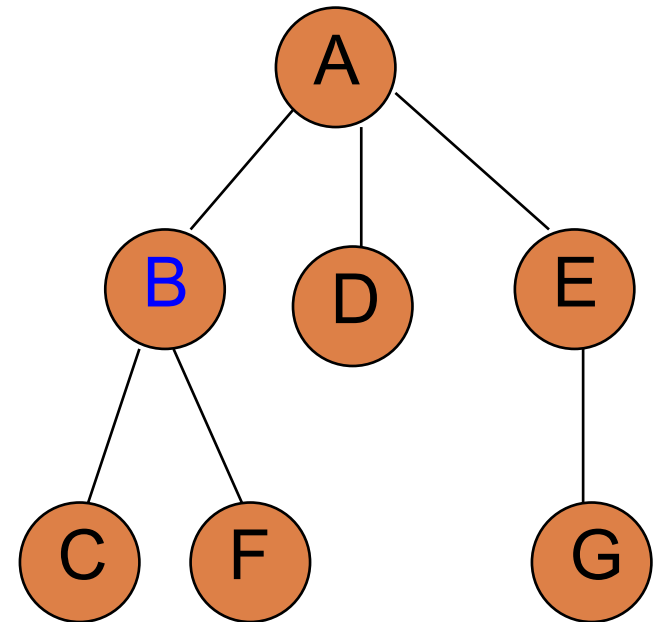
printed: A E G D



Frontier: go as far down one branch as possible, working right to left

Tree DFS

```
treeSearch( toVisit )  
while !toVisit.empty()  
    v = toVisit.remove()  
    // visit v, e.g., print it out  
    for c in v.getChildren()  
        toVisit.add(c)
```

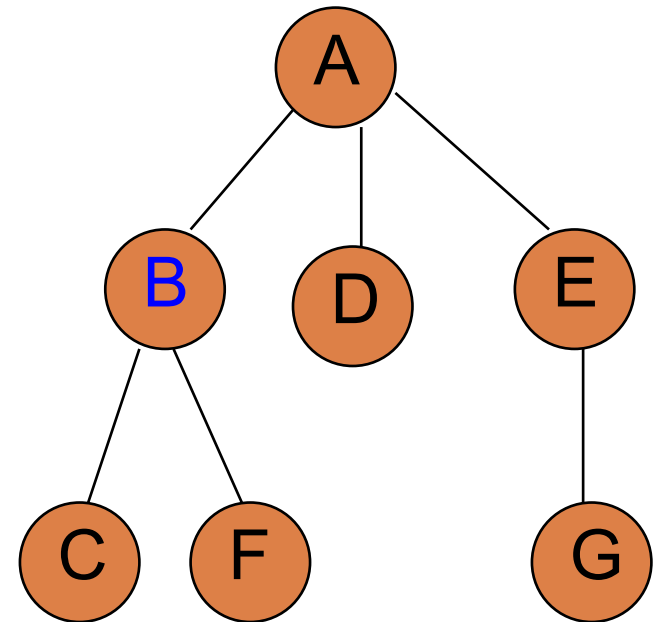


toVisit-stack:

printed: A E G D **B**

Tree DFS

```
treeSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    // visit v, e.g., print it out  
    for c in v.getChildren()  
      toVisit.add(c)
```

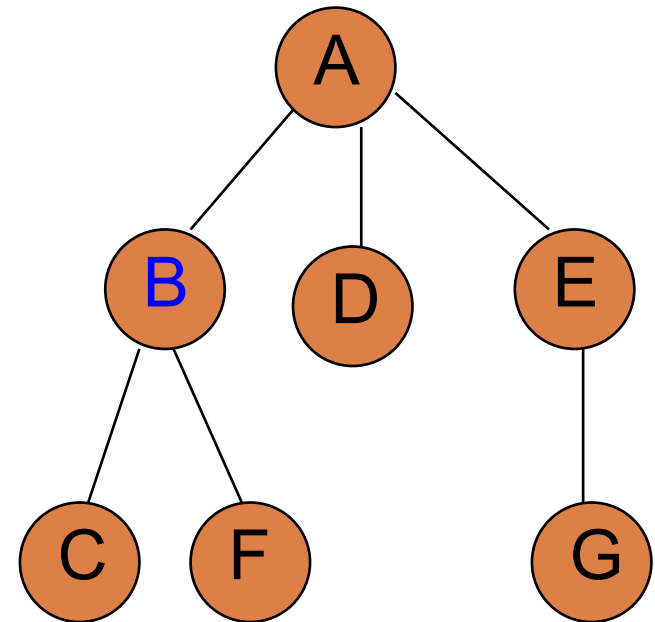


toVisit-stack:

printed: A E G D **B**

Tree DFS

```
treeSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    // visit v, e.g., print it out  
    for c in v.getChildren()  
      toVisit.add(c)
```



toVisit-stack: C F

printed: A E G D B

Tree DFS

```
treeSearch( toVisit )
```

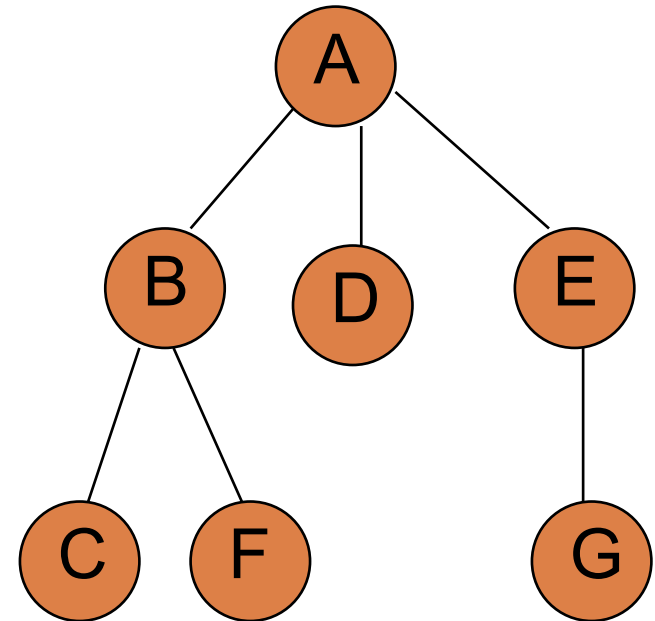
```
while !toVisit.empty()
```

```
    v = toVisit.remove()
```

```
    // visit v, e.g., print it out
```

```
    for c in v.getChildren()
```

```
        toVisit.add(c)
```



toVisit-stack: C F

printed: A E G D B

Tree DFS

```
treeSearch( toVisit )
```

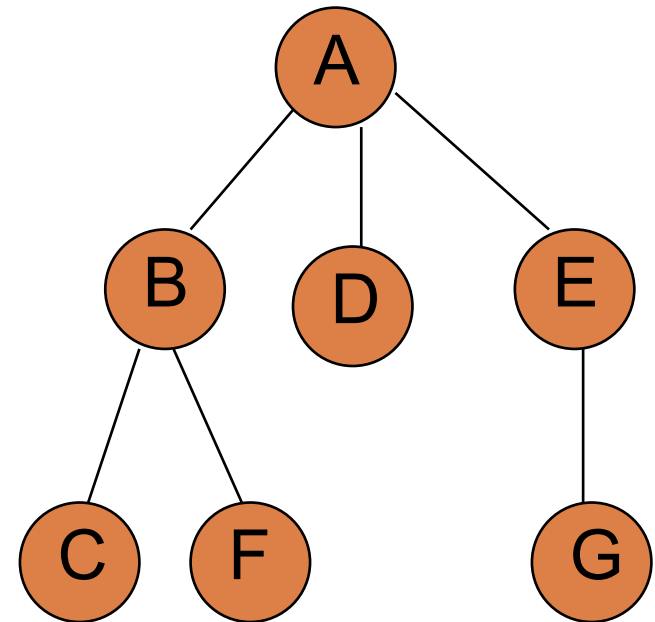
```
while !toVisit.empty()
```

```
    v = toVisit.remove()
```

```
    // visit v, e.g., print it out
```

```
    for c in v.getChildren()
```

```
        toVisit.add(c)
```



toVisit-stack:

printed: A E G D B F C

Run-time of graph algorithms

A graph is a set of vertices V and a set of edges $(u,v) \in E$ where $u,v \in V$

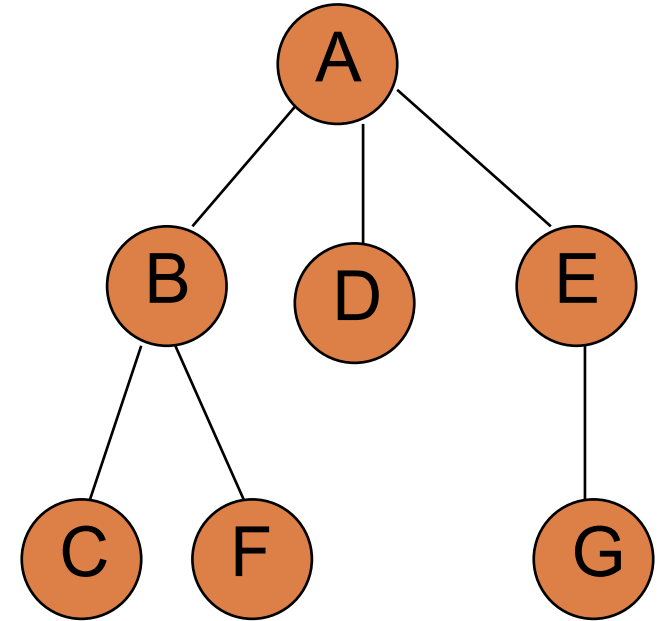
When we analyze graph algorithms, the run-time often includes both the number of vertices **AND** the number of edges:

- ▣ $|V|$ = number of vertices
- ▣ $|E|$ = number of edges

Sometimes, in big- O notation, we'll use just V and E to represent these to simplify notation

treeSearch run-time

```
treeSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    // visit v, e.g., print it out  
    for c in v.getChildren()  
      toVisit.add(c)
```



What is the big-O run-time of treeSearch?

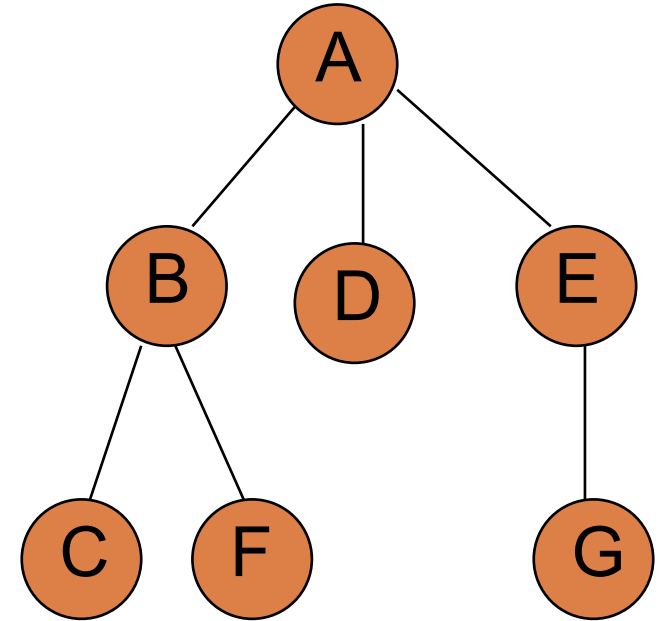
Assume all of the stack/queue operations are constant.

How many times do we visit each vertex?

How many times do we traverse each edge (vis the for loop)?

treeSearch run-time

```
treeSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    // visit v, e.g., print it out  
    for c in v.getChildren()  
      toVisit.add(c)
```



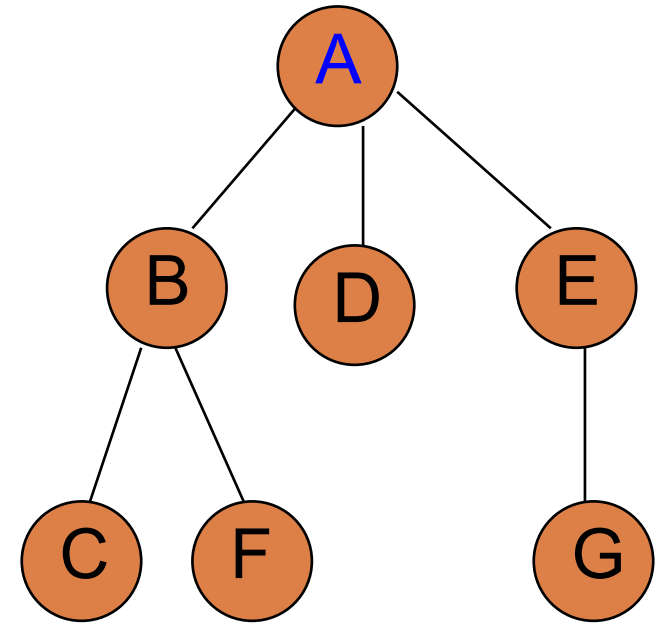
How many times do we visit each vertex? Exactly once

How many times do we traverse each edge (vis the for loop)? Exactly once

What is the big-O run-time of treeSearch? $O(|V| + |E|)$. Linear algorithm.

What algorithm is this?

```
search( v )  
  // visit v, e.g., print it out  
  for c in v.getChildren()  
    search(c)
```



What order will the vertices get printed out?
Assume children are traversed left to right.

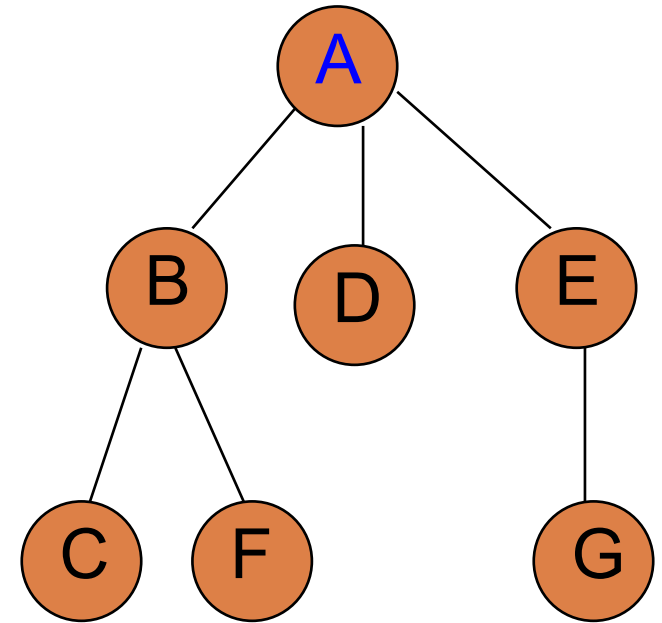
What algorithm is this?

```
search( v )
```

```
  // visit v, e.g., print it out
```

```
  for c in v.getChildren()
```

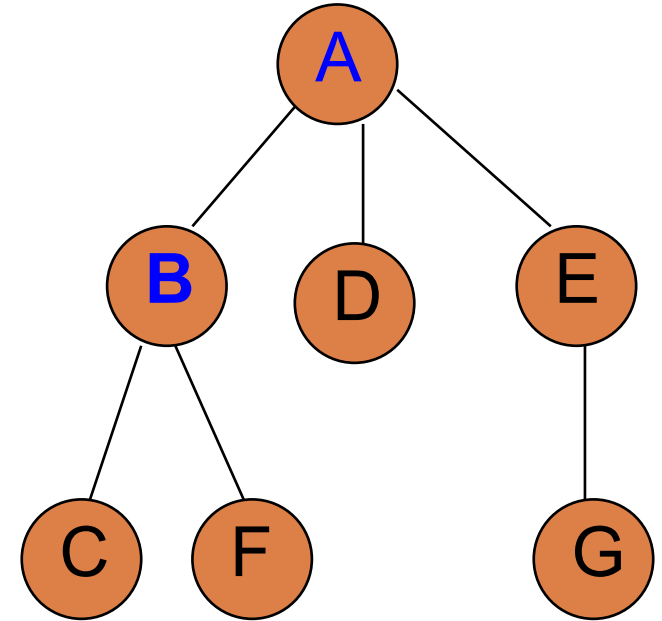
```
    search(c)
```



Visited: A

What algorithm is this?

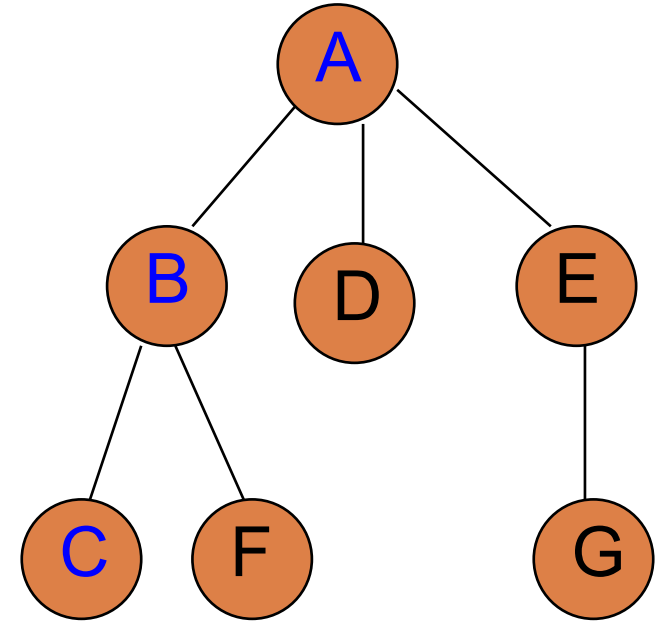
```
search( v )  
  // visit v, e.g., print it out  
  for c in v.getChildren()  
    search(c)
```



Visited: A B

What algorithm is this?

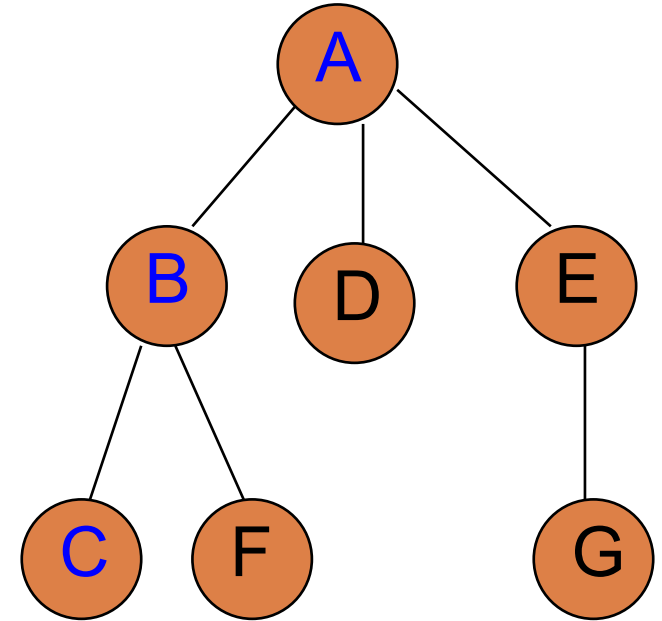
```
search( v )  
  // visit v, e.g., print it out  
  for c in v.getChildren()  
    search(c)
```



Visited: A B C

What algorithm is this?

```
search( v )  
  // visit v, e.g., print it out  
  for c in v.getChildren()  
    search(c)
```

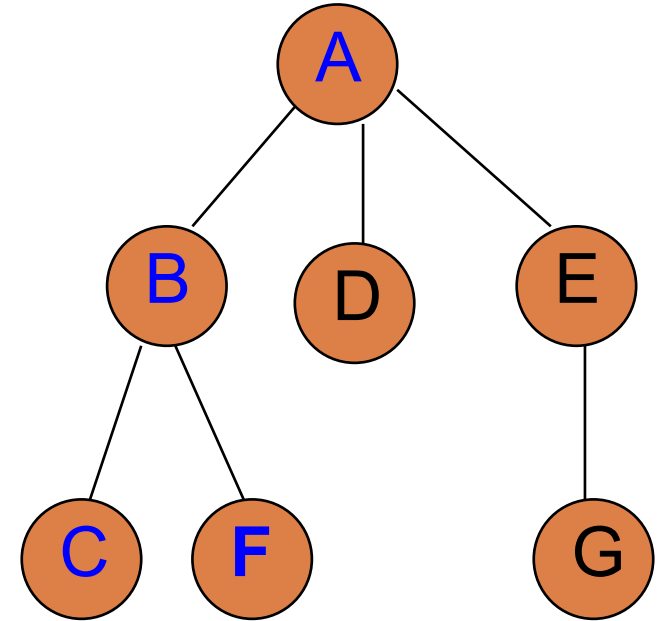


Visited: A B C

Now where?

What algorithm is this?

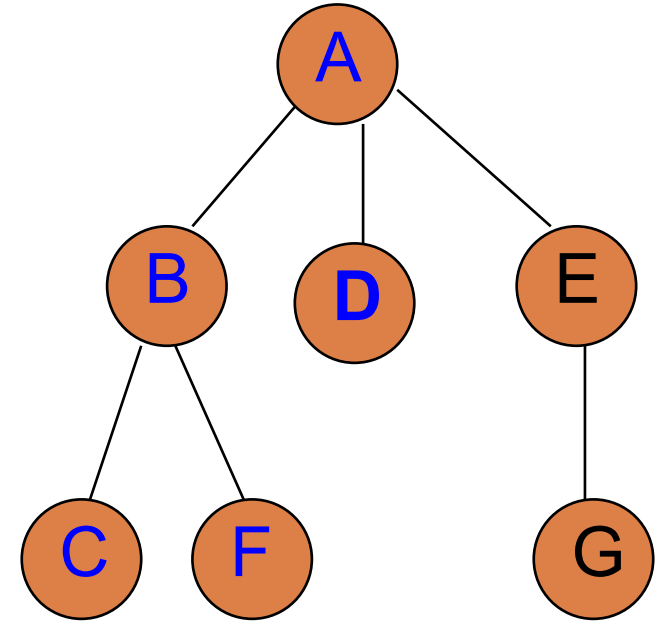
```
search( v )  
  // visit v, e.g., print it out  
  for c in v.getChildren()  
    search(c)
```



Visited: A B C F

What algorithm is this?

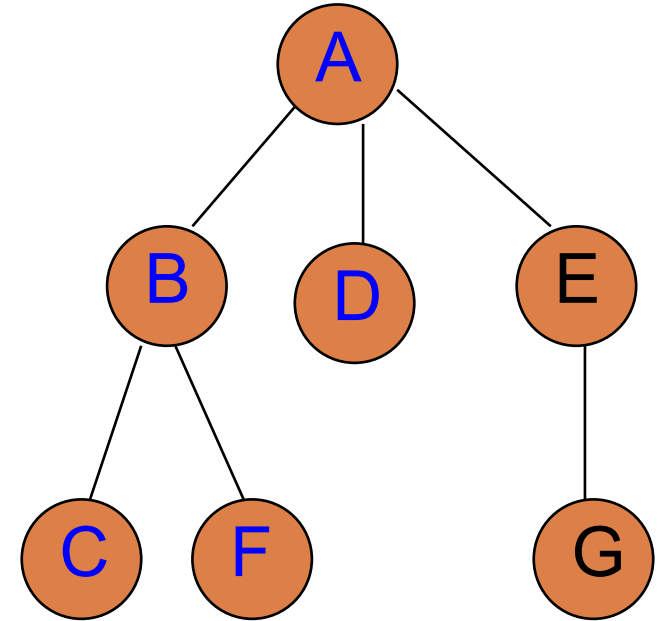
```
search( v )  
  // visit v, e.g., print it out  
  for c in v.getChildren()  
    search(c)
```



Visited: A B C F D

What algorithm is this?

```
search( v )  
  // visit v, e.g., print it out  
  for c in v.getChildren()  
    search(c)
```

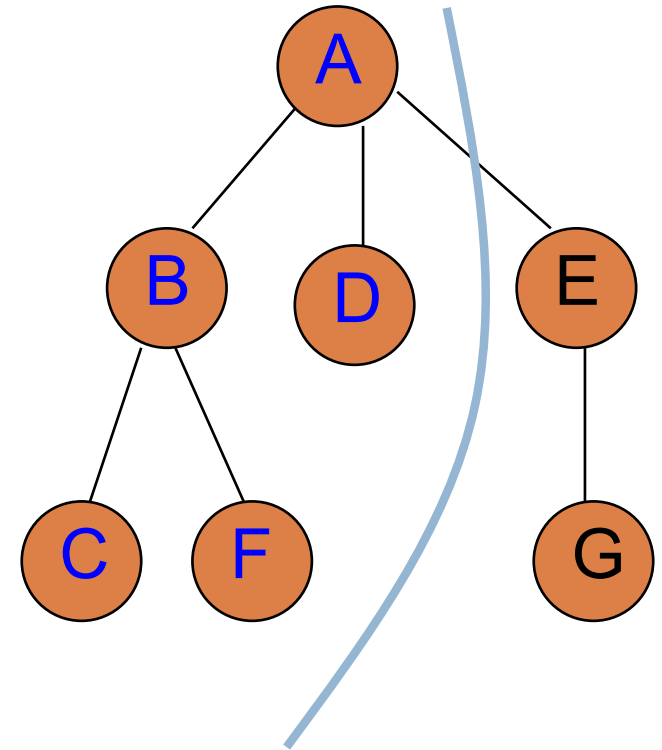


Visited: A B C F D

What algorithm is this?

What algorithm is this?

```
search( v )  
  // visit v, e.g., print it out  
  for c in v.getChildren()  
    search(c)
```

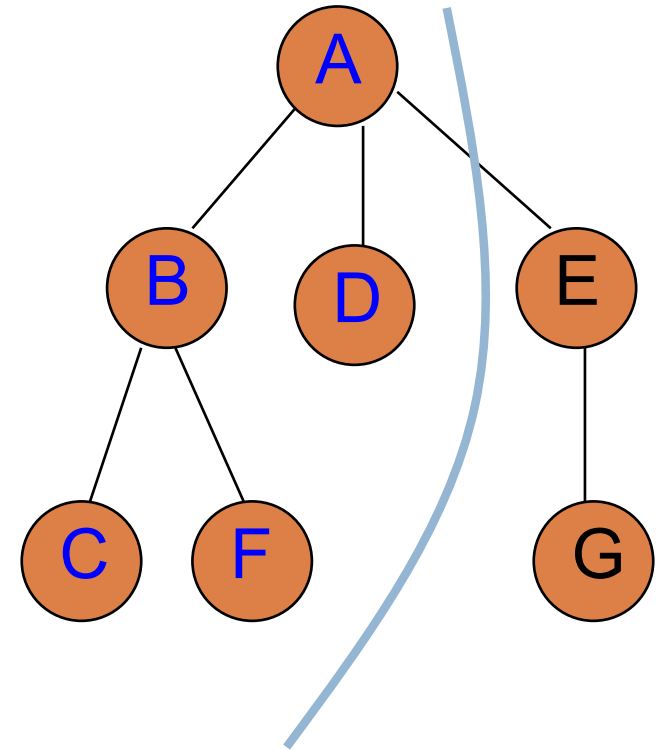


Depth first search!

Visited: A B C F D

What algorithm is this?

```
search( v )  
  // visit v, e.g., print it out  
  for c in v.getChildren()  
    search(c)
```

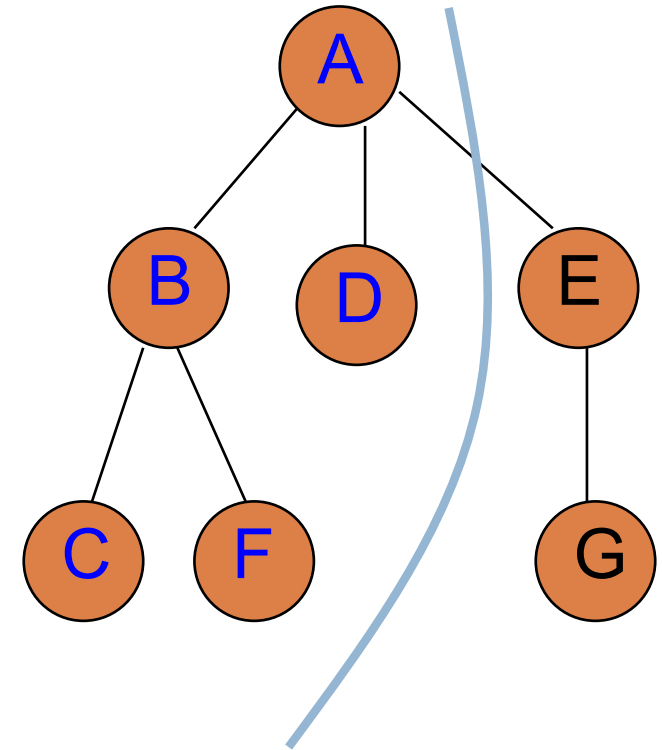


Any difference between this version and the stack version?

Visited: A B C F D

What algorithm is this?

```
search( v )  
  // visit v, e.g., print it out  
  for c in v.getChildren()  
    search(c)
```



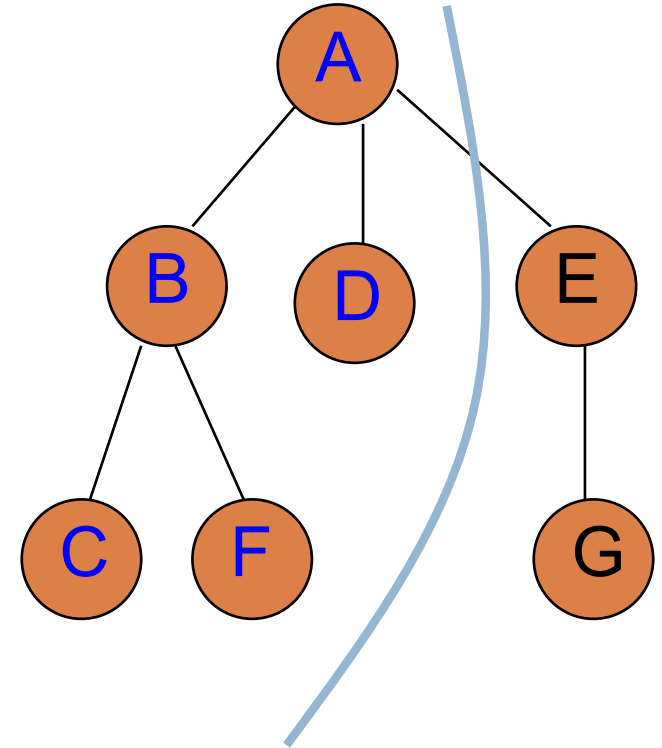
Any difference between this version and the stack version?

Visited: A B C F D

Traverses in the other direction (left to right in this case).

What algorithm is this?

```
search( v )  
  // visit v, e.g., print it out  
  for c in v.getChildren()  
    search(c)
```

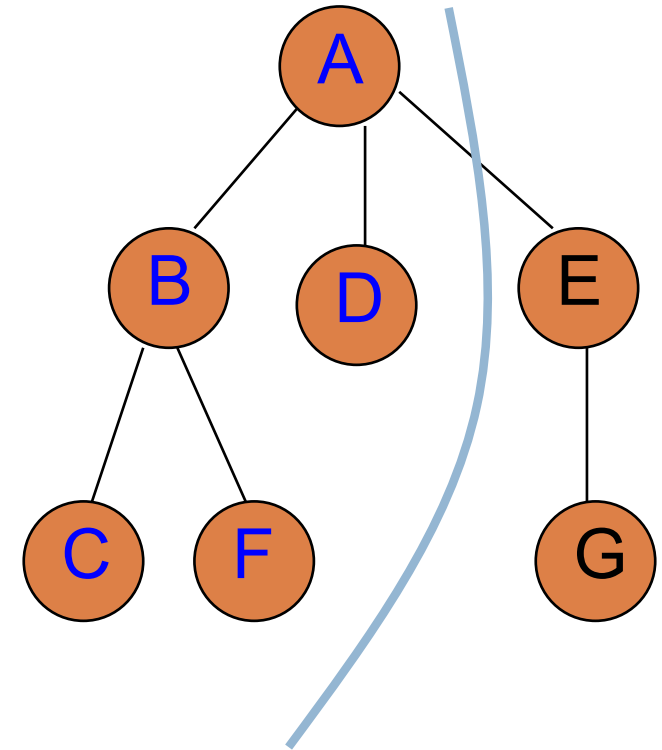


treeDFS used a stack.
Is there a stack here?

Visited: A B C F D

What algorithm is this?

```
search( v )  
  // visit v, e.g., print it out  
  for c in v.getChildren()  
    search(c)
```



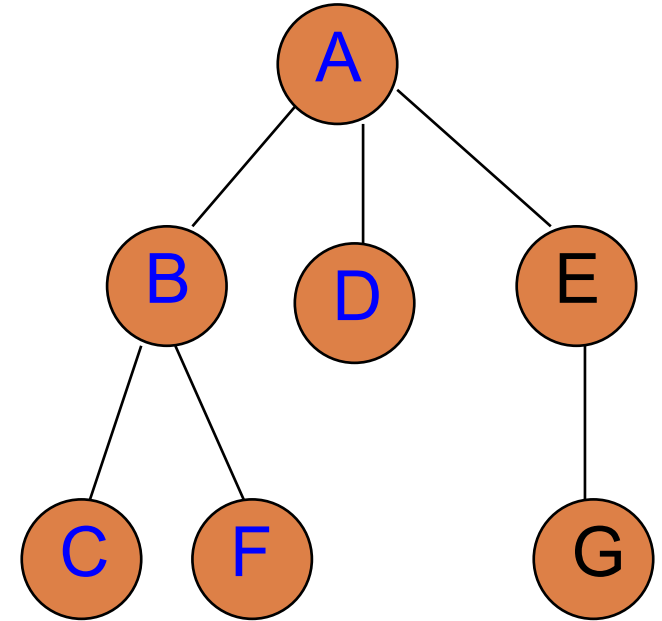
treeDFS used a stack.
Is there a stack here?

Visited: A B C F D

The run-time stack keeping track of recursive calls!

What algorithm is this?

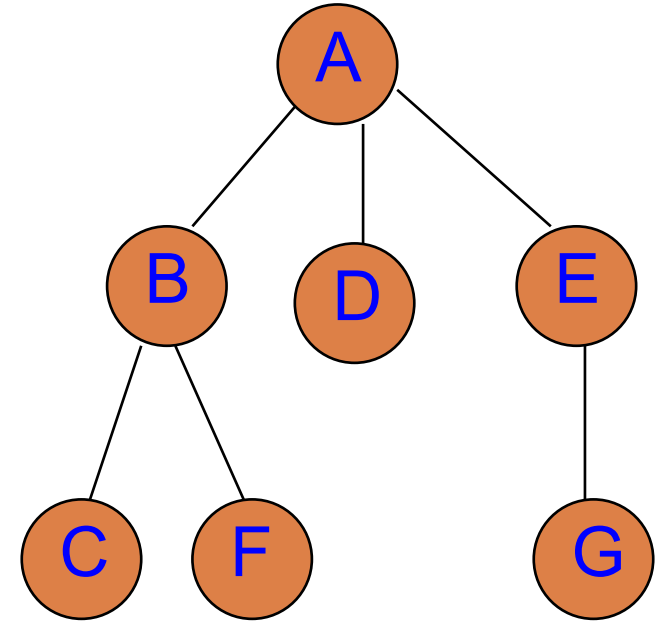
```
search( v )  
  // visit v, e.g., print it out  
  for c in v.getChildren()  
    search(c)
```



Visited: A B C F D

What algorithm is this?

```
search( v )  
  // visit v, e.g., print it out  
  for c in v.getChildren()  
    search(c)
```



Visited: A B C F D E G

DFS versions

```
treeDFS( start )
```

```
    s = new Stack()
```

```
    s.add(start)
```

```
    treeSearch(s)
```

```
treeSearch( toVisit )
```

```
    while !toVisit.empty()
```

```
        v = toVisit.remove()
```

```
        // visit v, e.g., print it out
```

```
        for c in v.getChildren()
```

```
            toVisit.add(c)
```

```
treeRecursiveDFS( v )
```

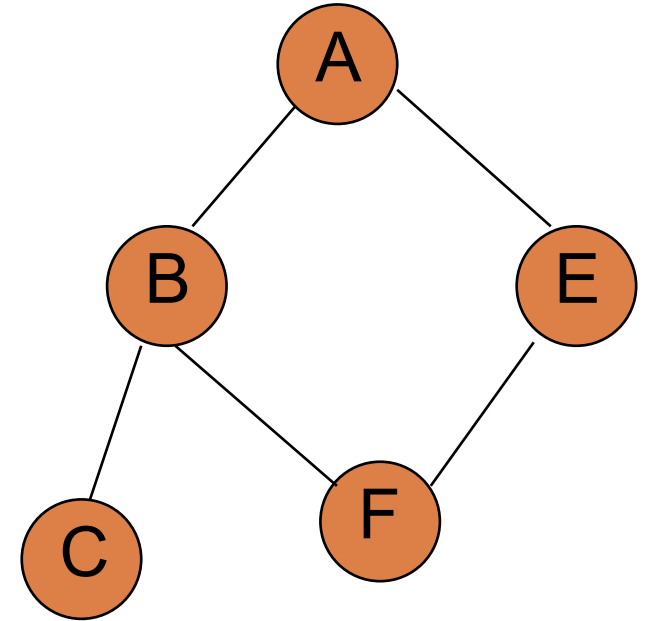
```
    // visit v, e.g., print it out
```

```
    for c in v.getChildren()
```

```
        treeRecursiveDFS(c)
```

treeSearch on graphs

```
treeSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    // visit v, e.g., print it out  
    for c in v.getChildren()  
      toVisit.add(c)
```

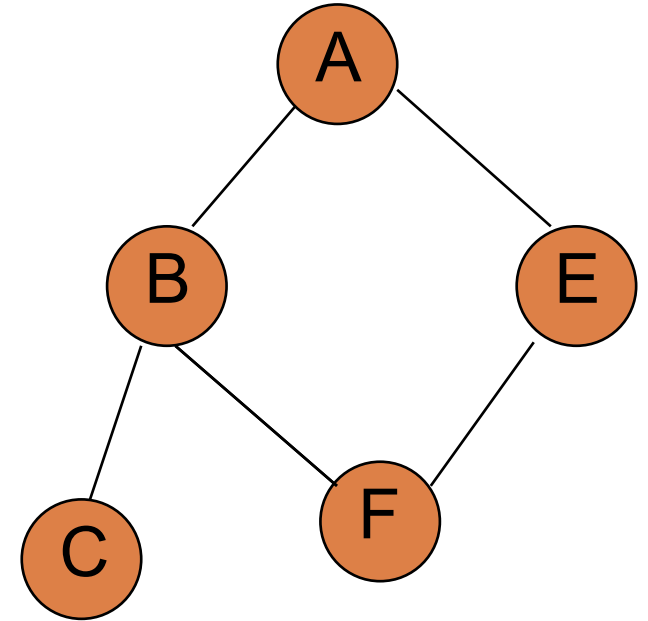


What would happen if we ran treeSearch on this graph?

Won't ever end!

treeSearch on graphs

```
treeSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    // visit v, e.g., print it out  
    for c in v.getChildren()  
      toVisit.add(c)
```



How can we fix this?

Keep track of the vertices that we've visited

Searching on graphs

```
treeSearch( toVisit )
  while !toVisit.empty()
    v = toVisit.remove()
    // visit v, e.g., print it out
    for c in v.getChildren()
      toVisit.add(c)
```

```
graphSearch( toVisit )
  while !toVisit.empty()
    v = toVisit.remove()
    if !visited[v]
      visited[v] = true
      for c in v.getAdjacent()
        if !visited[c]
          toVisit.add(c)
```

Searching on graphs

```
graphBFS( start )
```

```
  q = new Queue()
```

```
  q.add(start)
```

```
  treeSearch(q)
```

```
graphDFS( start )
```

```
  s = new Stack()
```

```
  s.add(start)
```

```
  treeSearch(s)
```

```
graphSearch( toVisit )
```

```
  while !toVisit.empty()
```

```
    v = toVisit.remove()
```

```
    if !visited[v]
```

```
      visited[v] = true
```

```
      for c in v.getAdjacent()
```

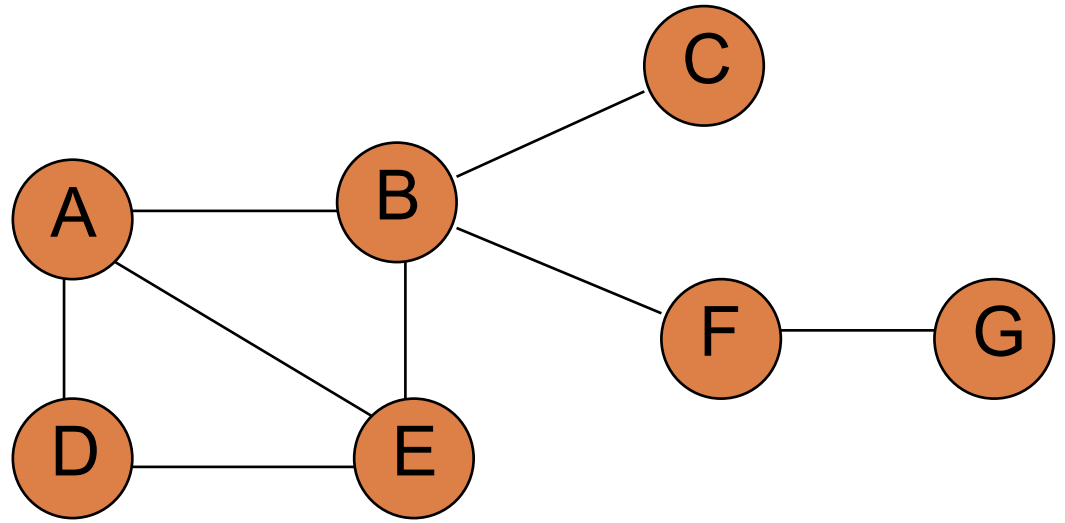
```
        if !visited[c]
```

```
          toVisit.add(c)
```

BFS

```
graphBFS( start )
```

```
q = new Queue()  
q.add(start)  
treeSearch(q)
```

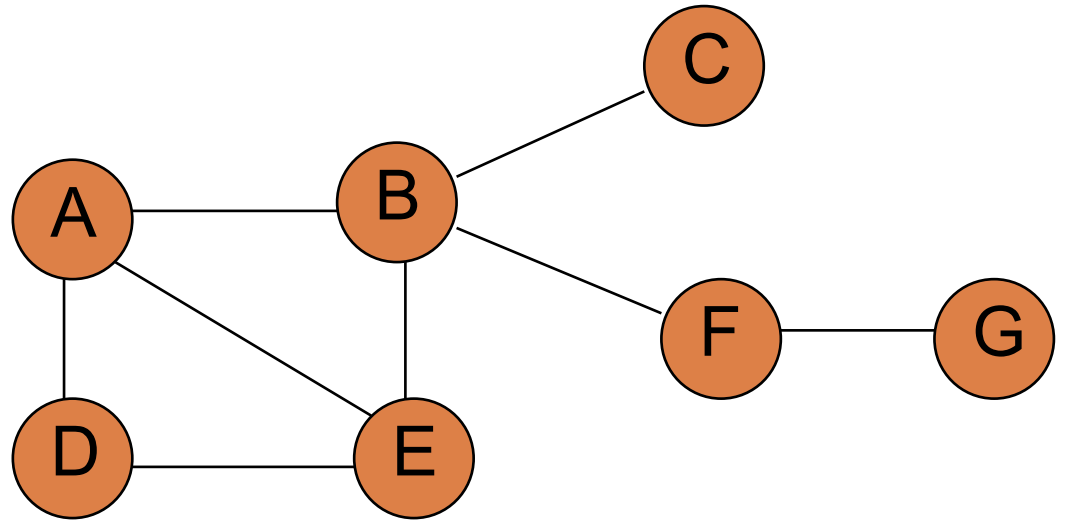


toVisit-queue: **A**

visited:

BFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```

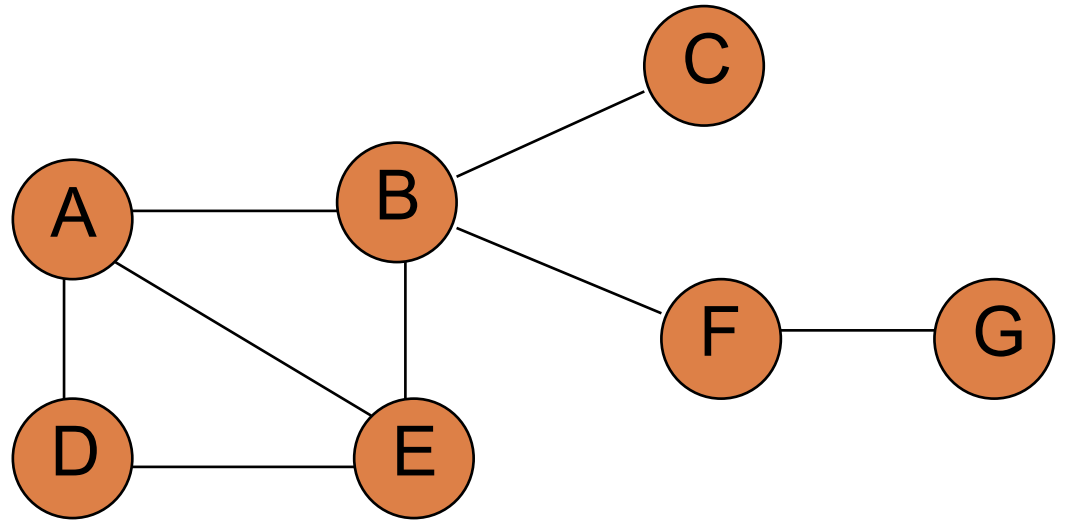


toVisit-queue: A
visited:

What order will the nodes get printed out?
Assume edges are traversed alphabetically.

BFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```

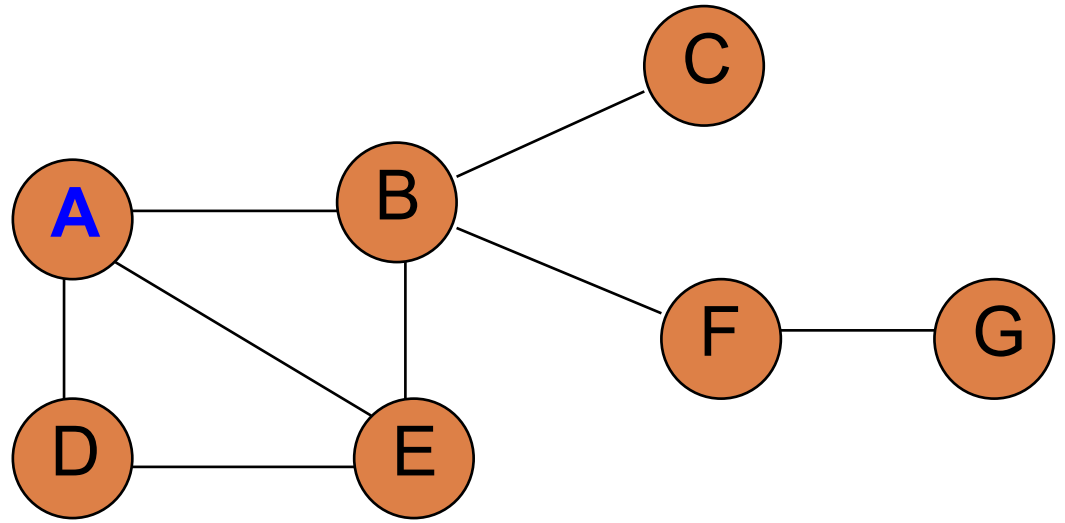


toVisit-queue: A

visited:

BFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```

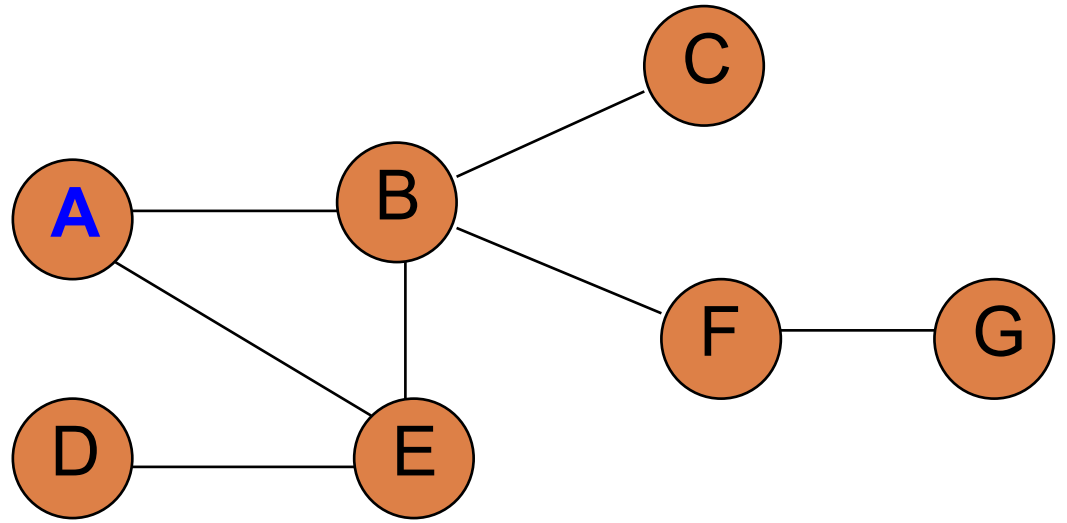


toVisit-queue:

visited: A

BFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```

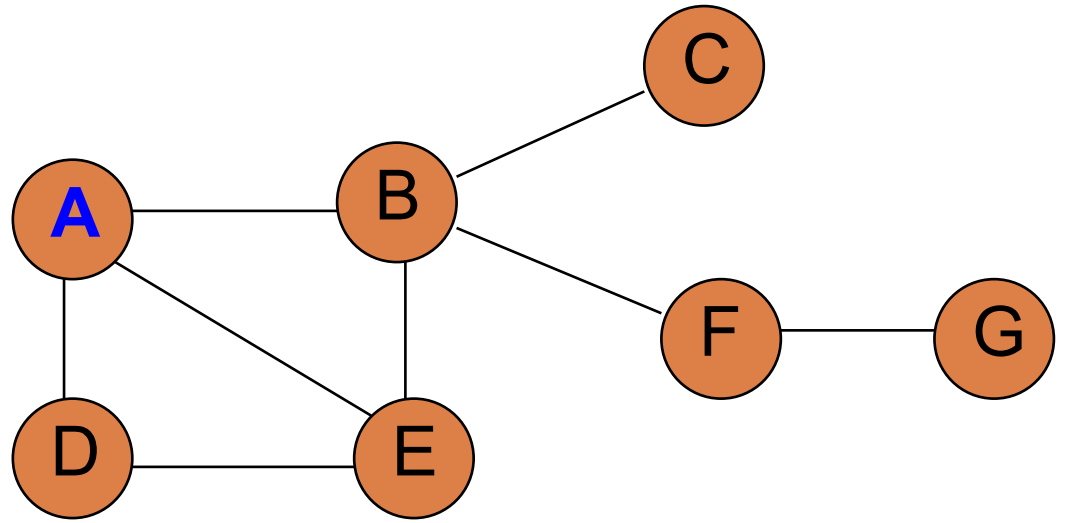


toVisit-queue:

visited: A

BFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```

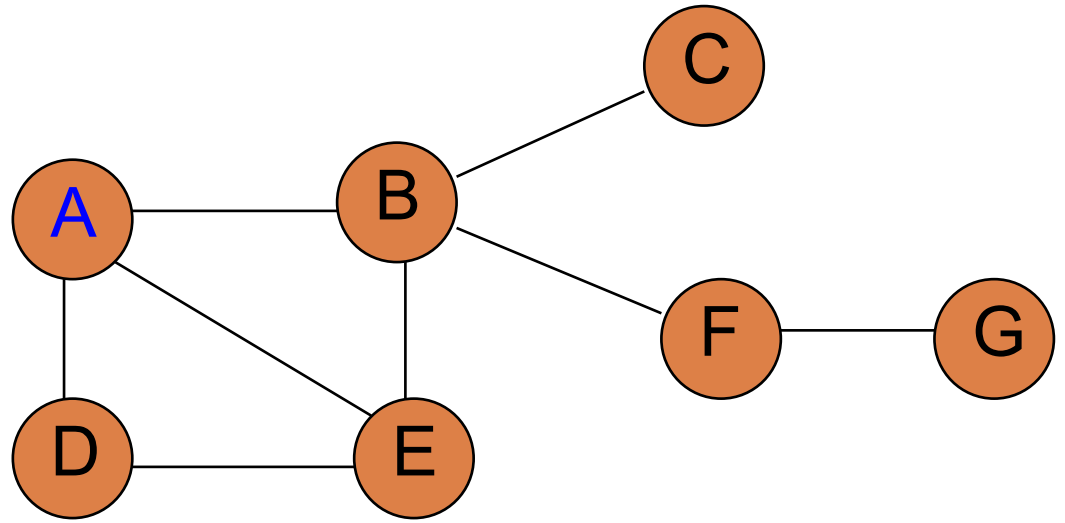


toVisit-queue: B D E

visited: A

BFS

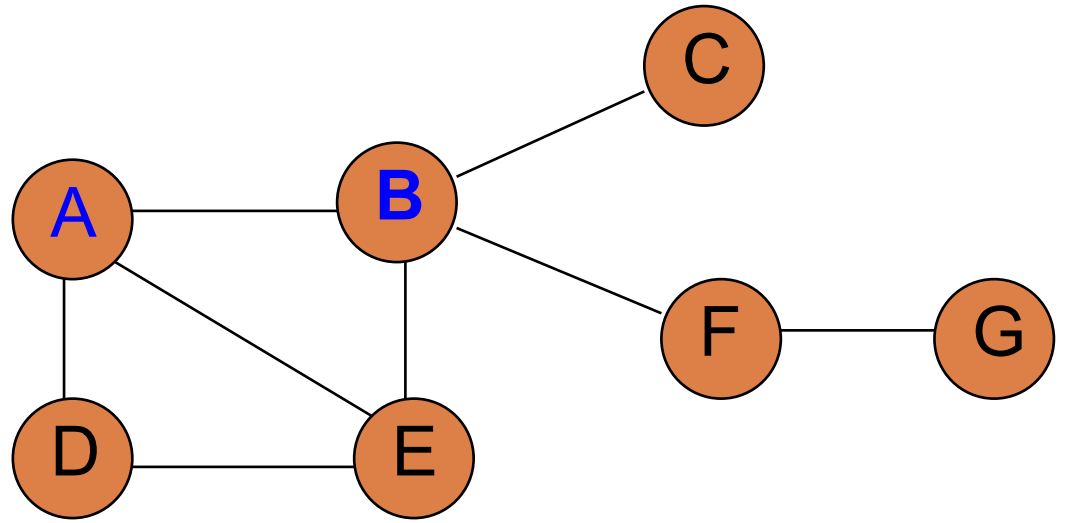
```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```



toVisit-queue: B D E
visited: A

BFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```

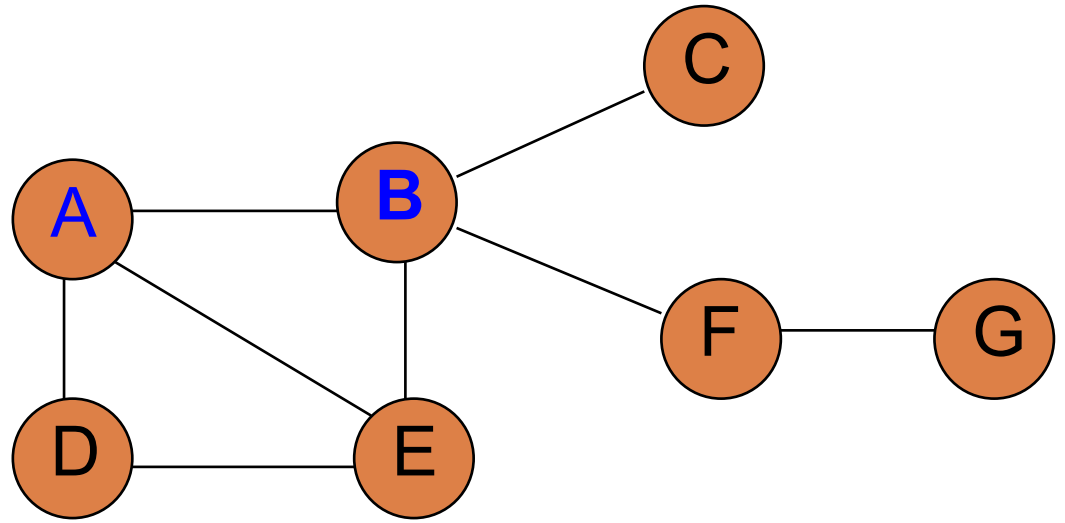


toVisit-queue: D E

visited: A B

BFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```

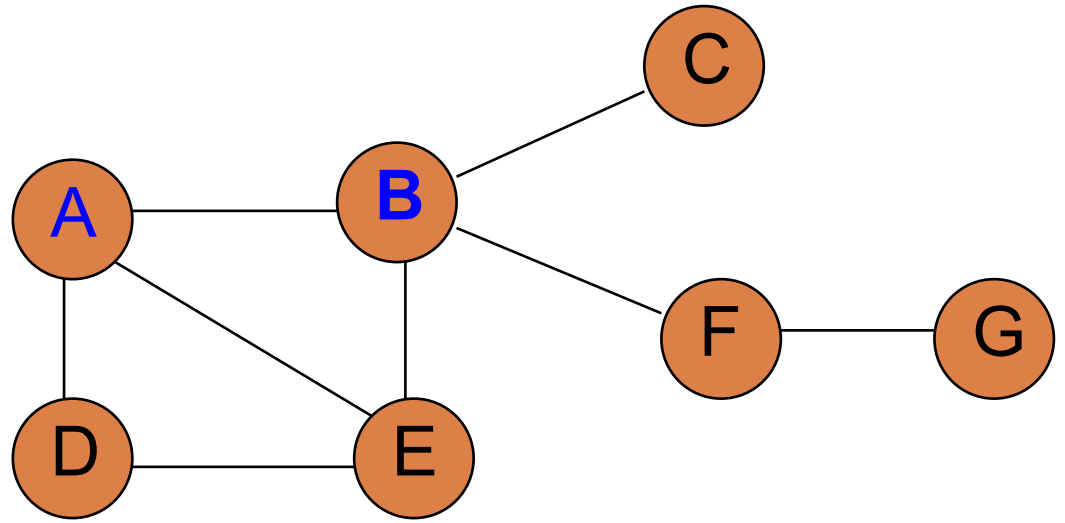


toVisit-queue: D E

visited: A B

BFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```

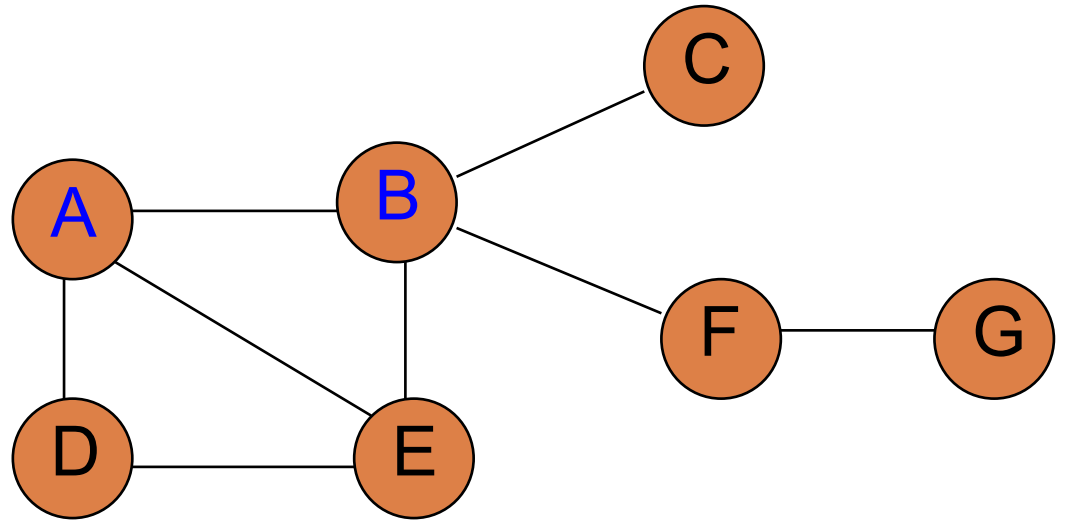


toVisit-queue: D E C E F
visited: A B

Notice that we do add E again to toVisit since we haven't visited it yet

BFS

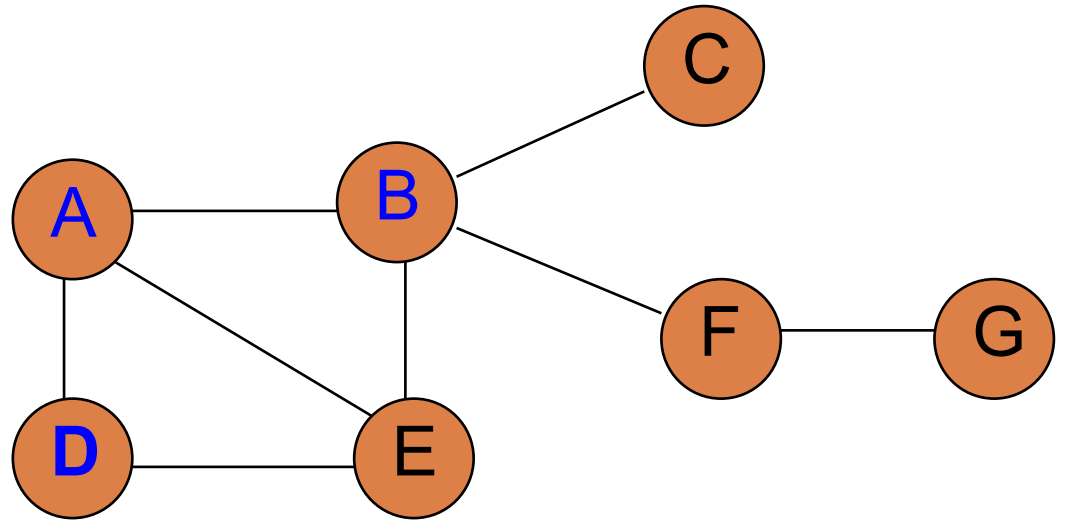
```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```



toVisit-queue: D E C E F
visited: A B

BFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```

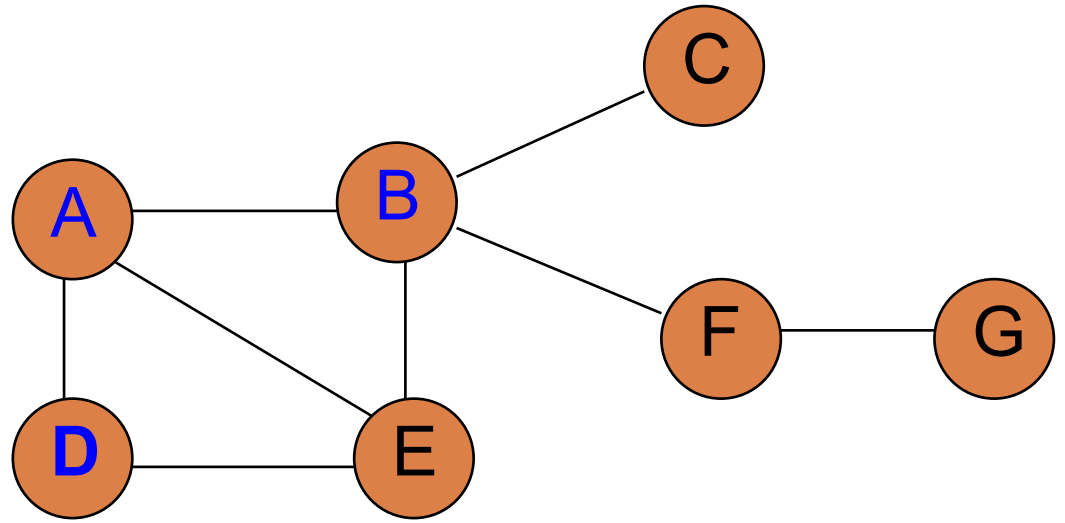


toVisit-queue: E C E F

visited: A B **D**

BFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```

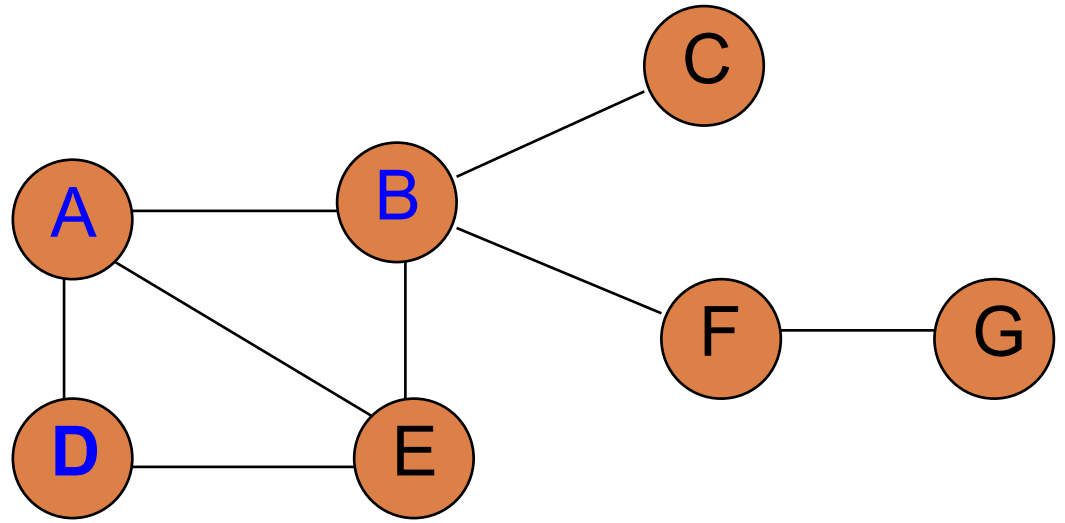


toVisit-queue: E C E F

visited: A B D

BFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```

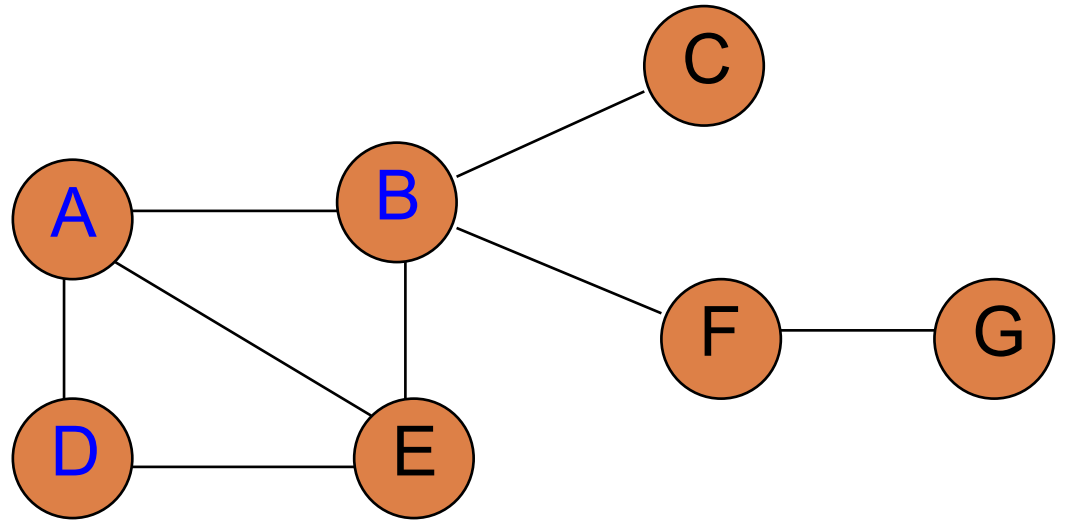


toVisit-queue: E C E F **E**
visited: A B D

We add E again to toVisit since we still haven't visited it yet

BFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```

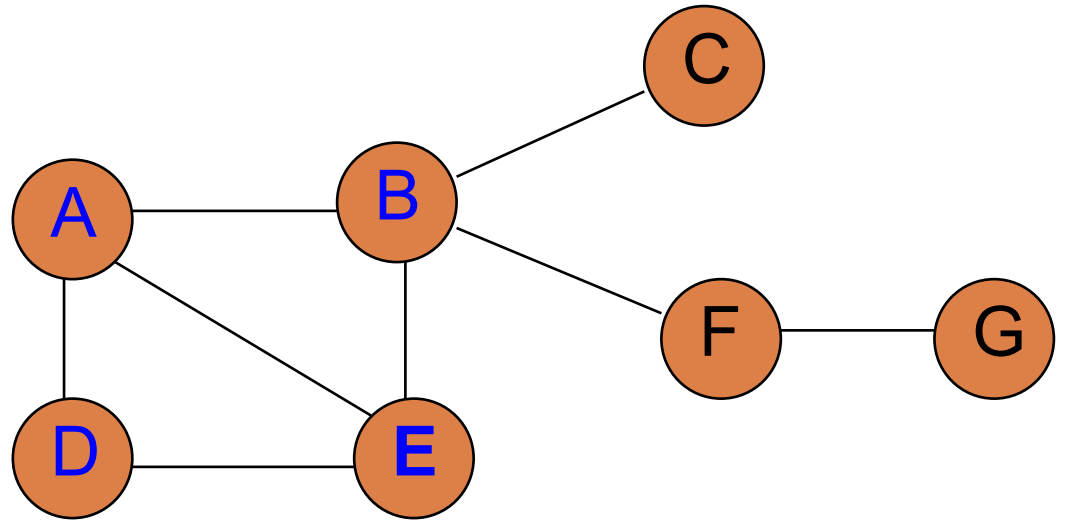


toVisit-queue: E C E F E

visited: A B D

BFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```

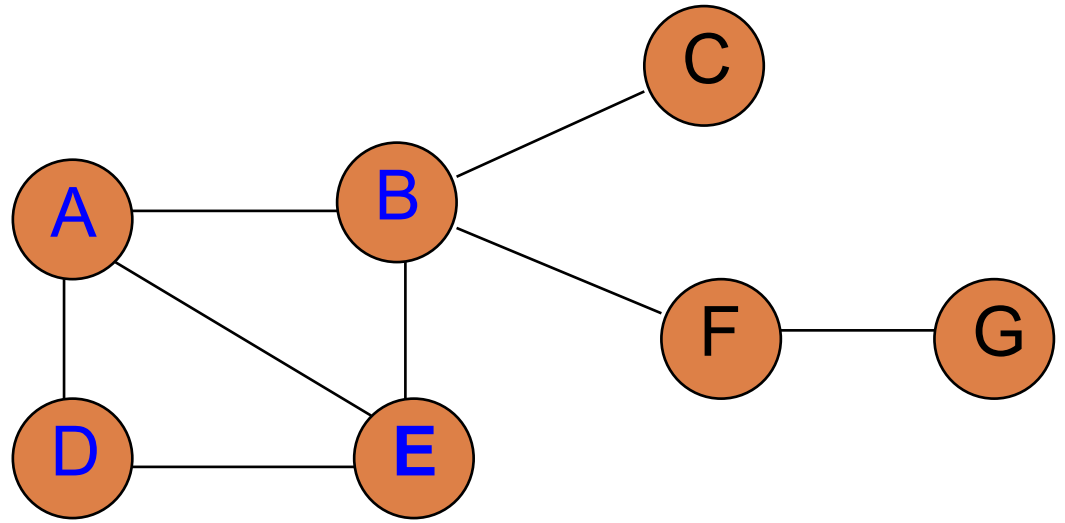


toVisit-queue: E C E F E

visited: A B D E

BFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```

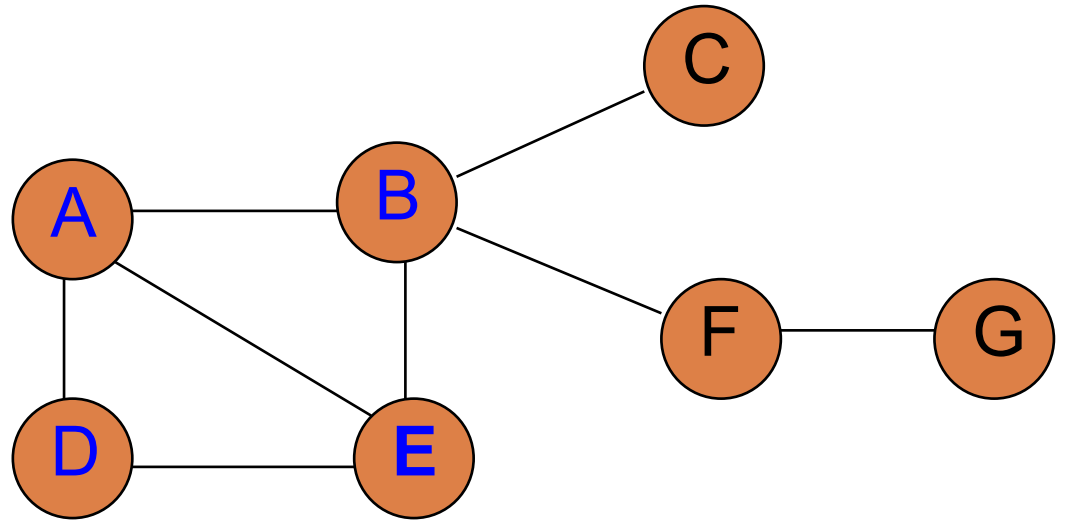


toVisit-queue: E C E F E

visited: A B D E

BFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```

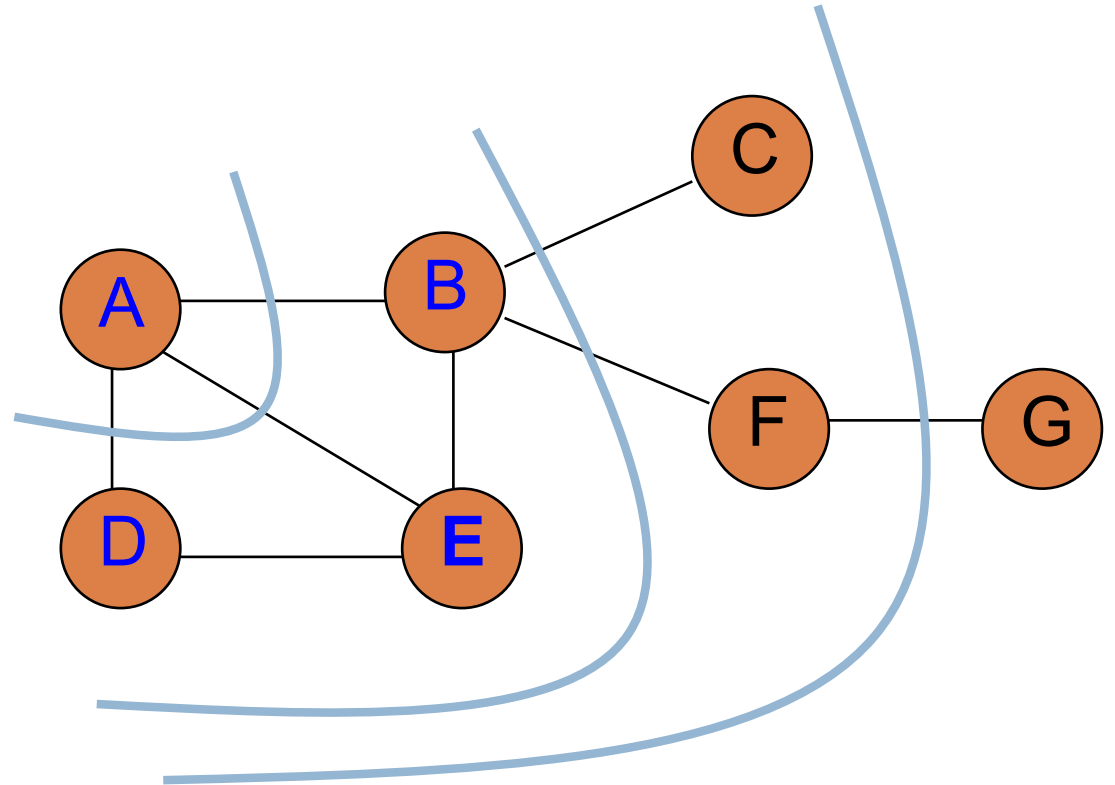


toVisit-queue: E C E F E
visited: A B D E

No adjacent vertices that
haven't been visited

BFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```

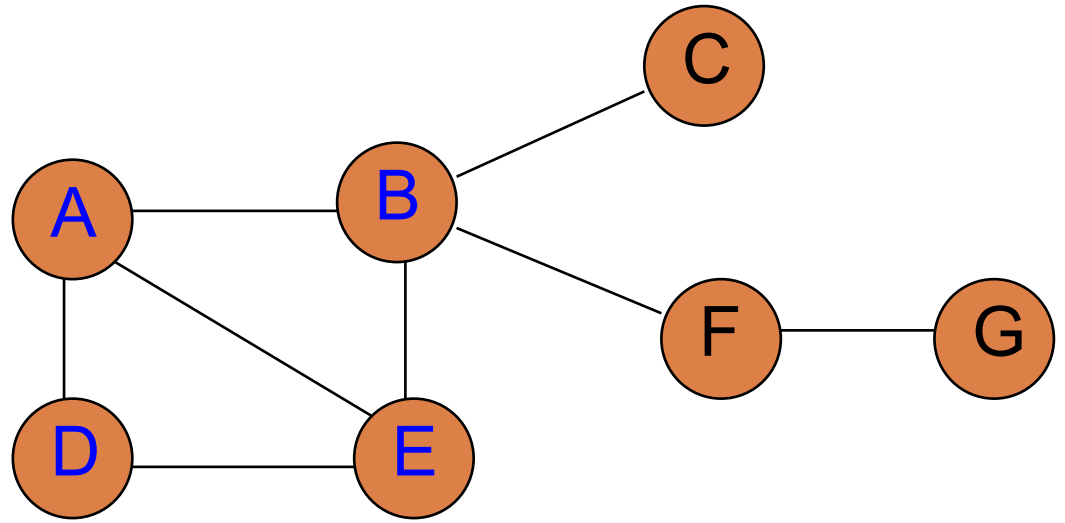


toVisit-queue: E C E F E
visited: A B D E

Frontier: all vertices a given number of edges from the start/root

BFS

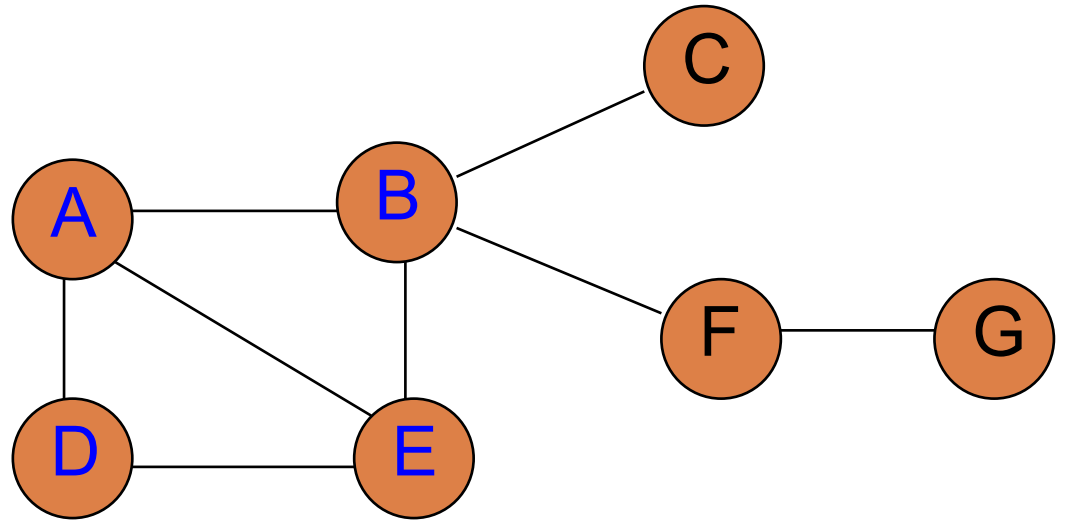
```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```



toVisit-queue: E C E F E
visited: A B D E

BFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```

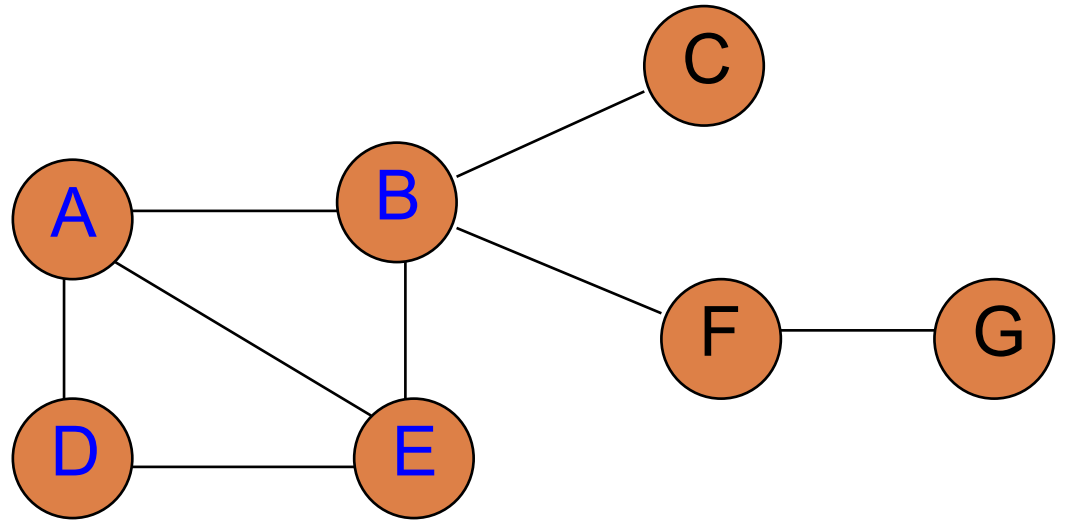


toVisit-queue: C E F E
visited: A B D E

E has already been visited

BFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```

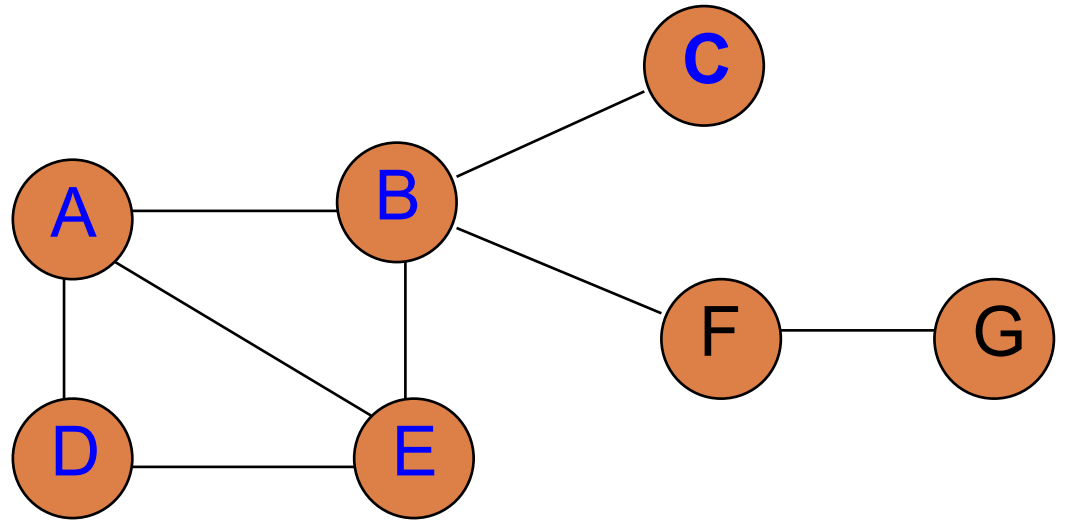


toVisit-queue: C E F E

visited: A B D E

BFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```

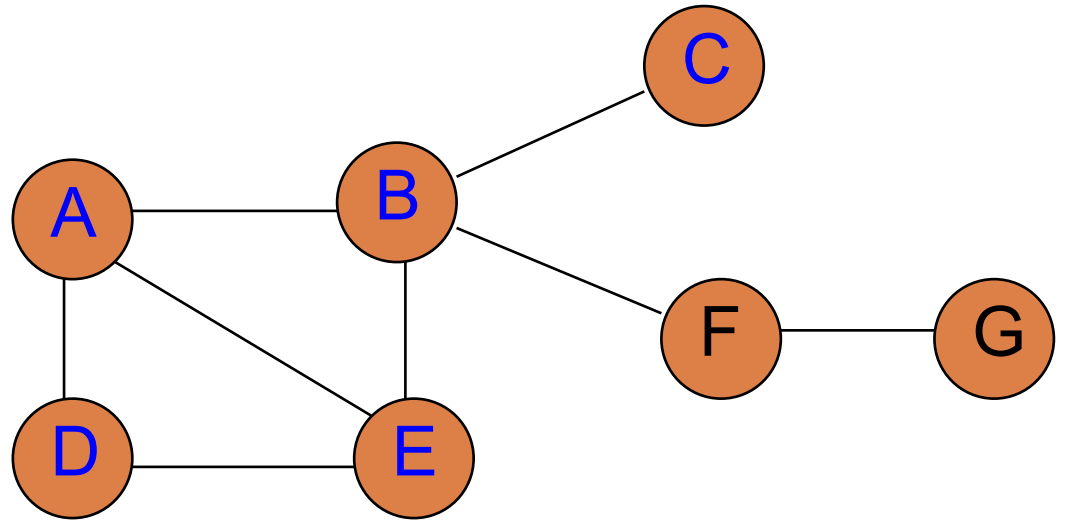


toVisit-queue: E F E

visited: A B D E C

BFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```



toVisit-queue: E F E

visited: A B D E C

BFS

```
graphSearch( toVisit )
```

```
while !toVisit.empty()
```

```
  v = toVisit.remove()
```

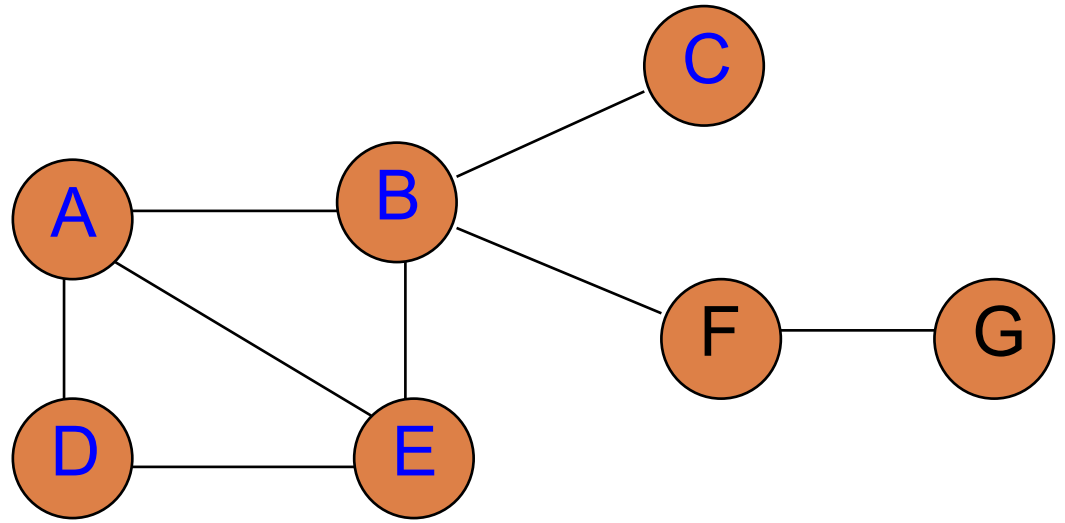
```
  if !visited[v]
```

```
    visited[v] = true
```

```
    for c in v.getAdjacent()
```

```
      if !visited[c]
```

```
        toVisit.add(c)
```



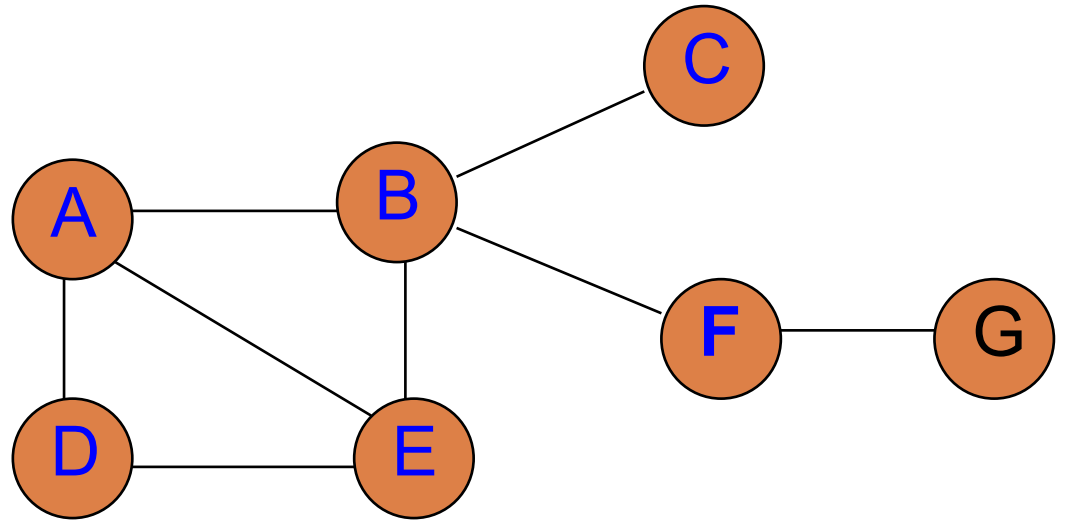
toVisit-queue: F E

visited: A B D E C

E has already been visited

BFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```

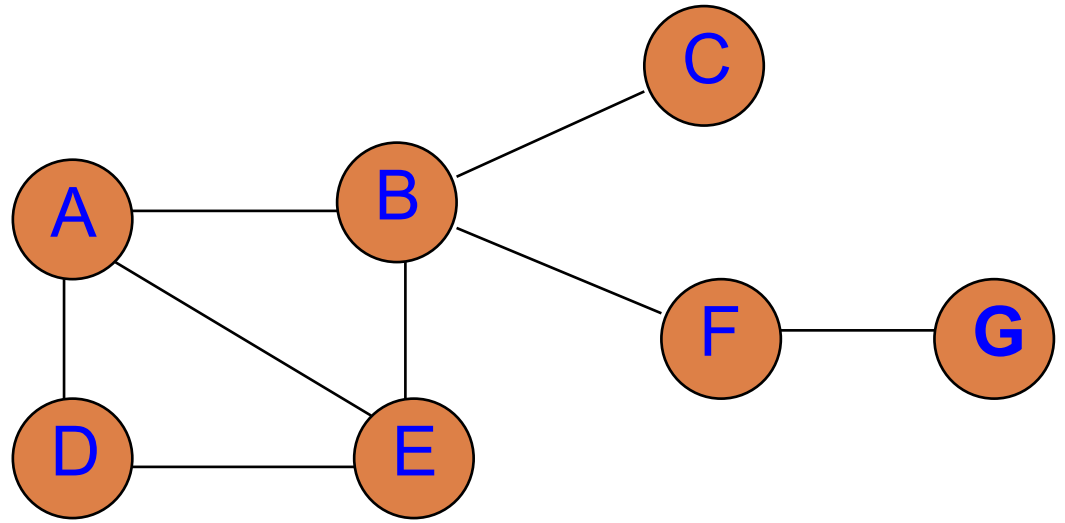


toVisit-queue: E G

visited: A B D E C F

BFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```



toVisit-queue:

visited: A B D E C F G

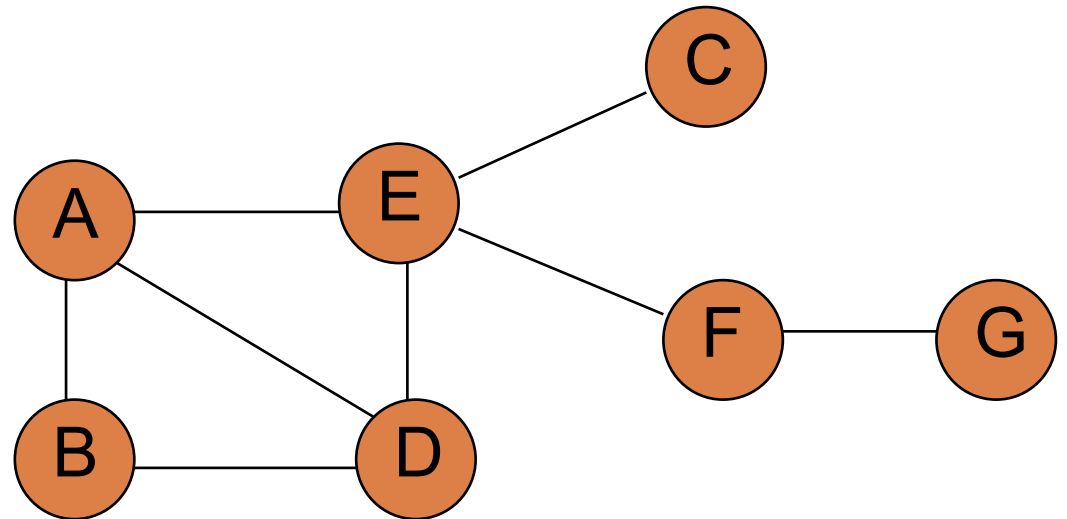
DFS

```
graphDFS( start )
```

```
  s = new Stack()
```

```
  s.add(start)
```

```
  treeSearch(s)
```

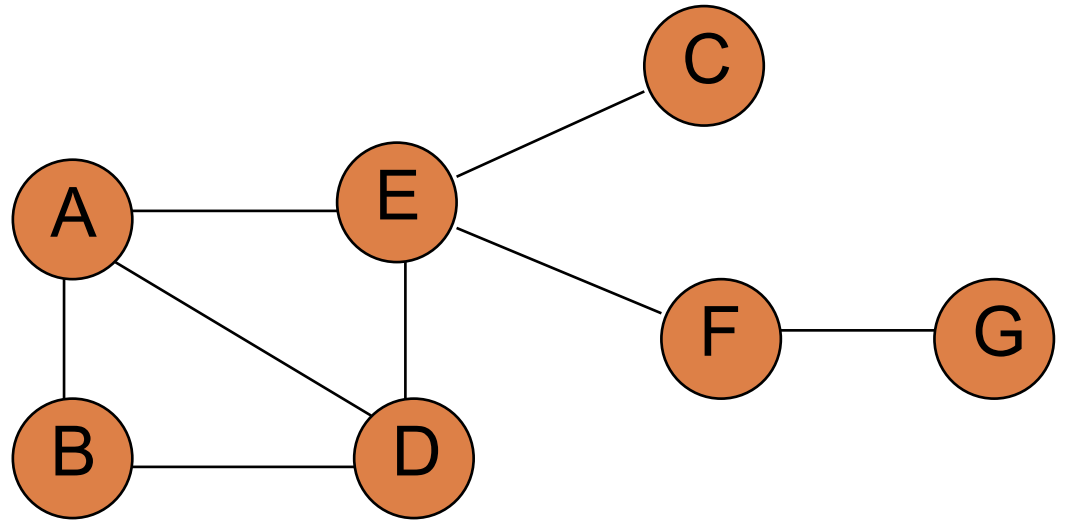


toVisit-stack: A

visited:

DFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```



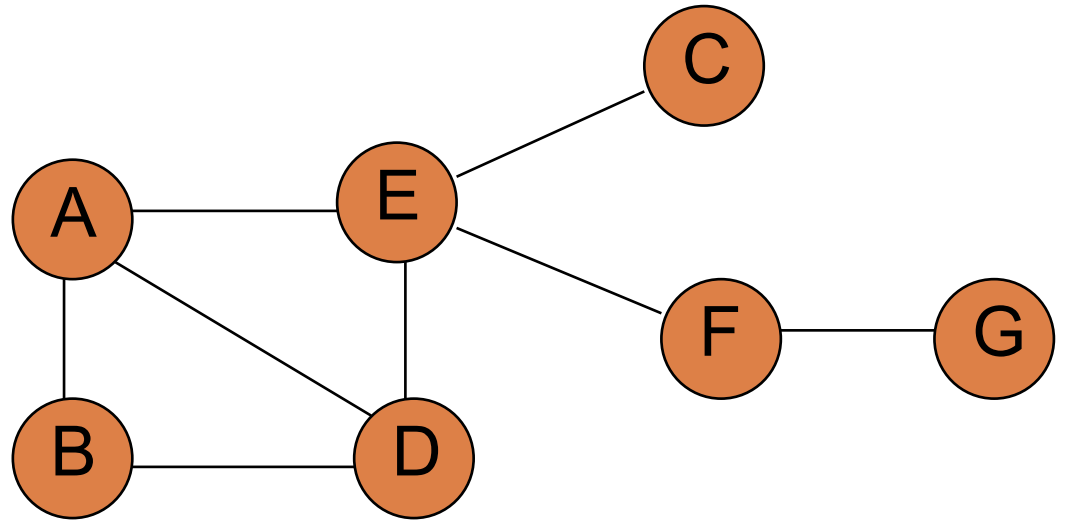
toVisit-stack: A

visited:

What order will the nodes get printed out?
Assume edges are traversed alphabetically.

DFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```

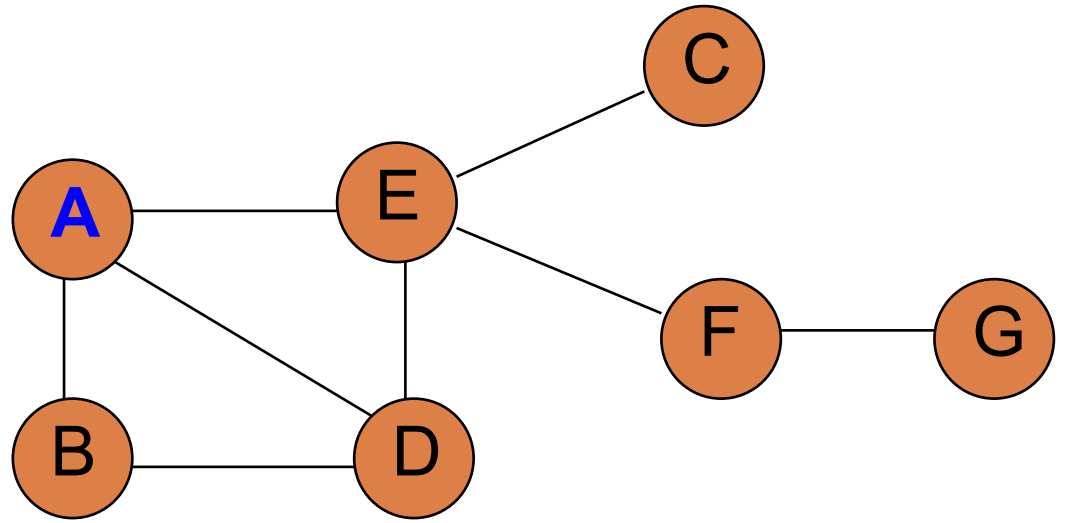


toVisit-stack: A

visited:

DFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```

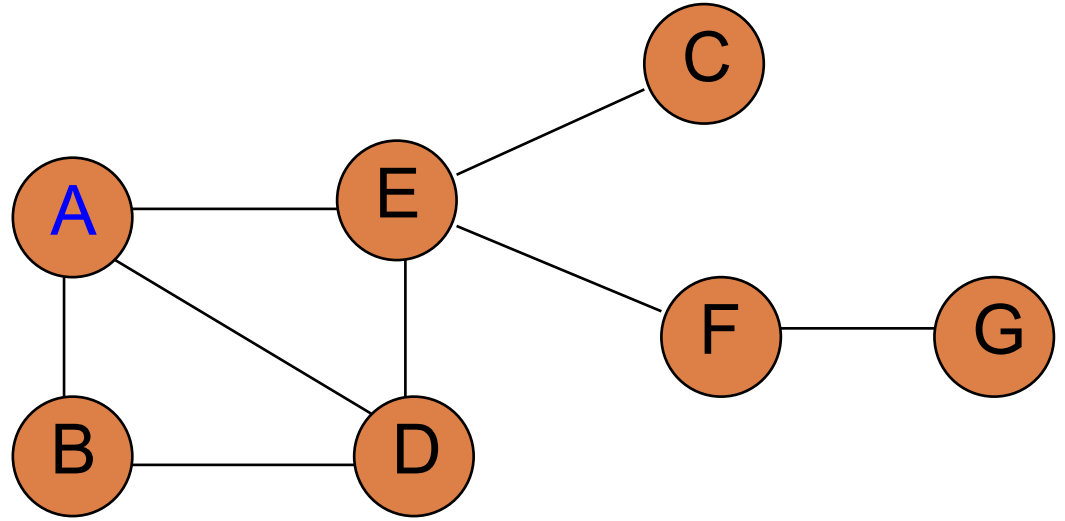


toVisit-stack: B D E

visited: A

DFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```

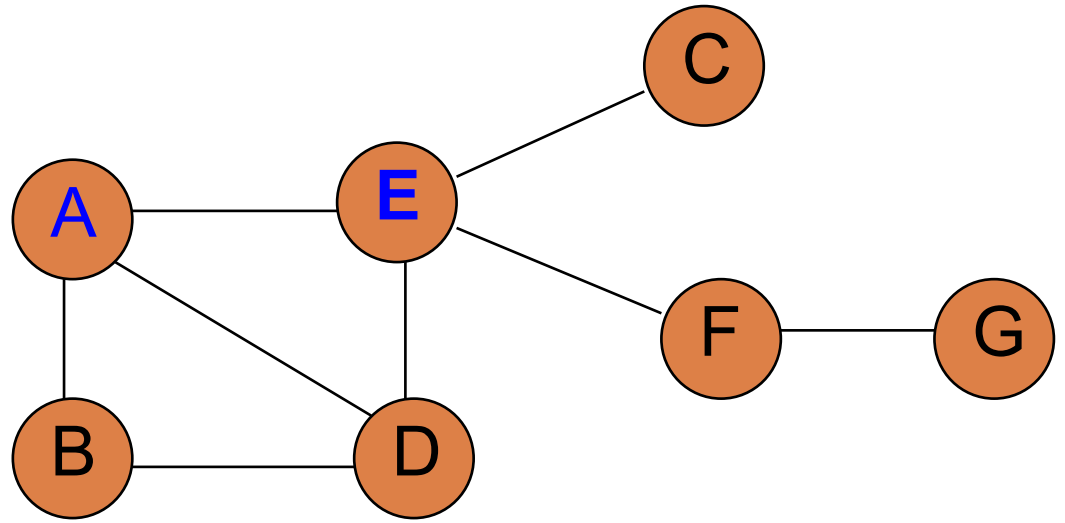


toVisit-stack: B D E

visited: A

DFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```



toVisit-stack: B D C D F

visited: A E

DFS

```
graphSearch( toVisit )
```

```
while !toVisit.empty()
```

```
  v = toVisit.remove()
```

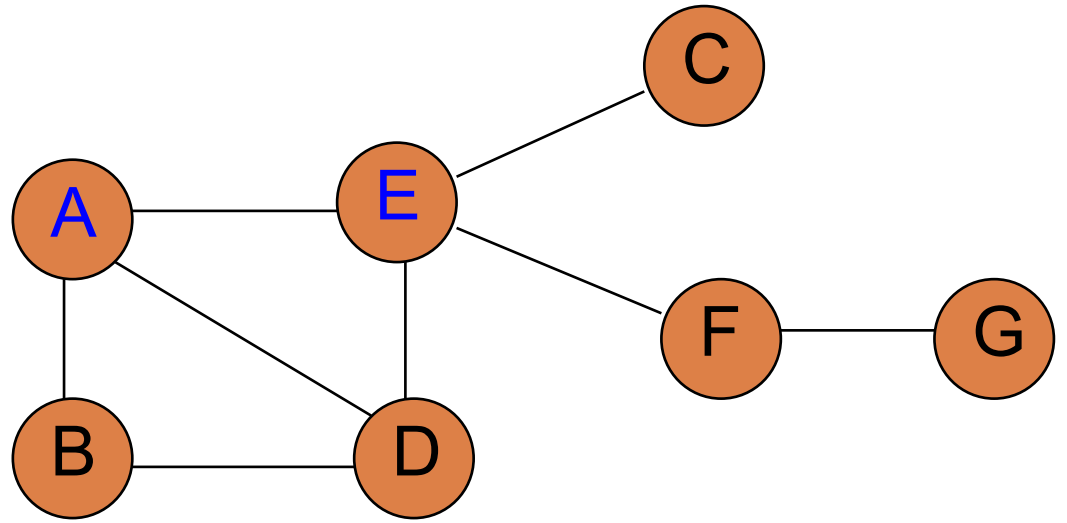
```
  if !visited[v]
```

```
    visited[v] = true
```

```
    for c in v.getAdjacent()
```

```
      if !visited[c]
```

```
        toVisit.add(c)
```

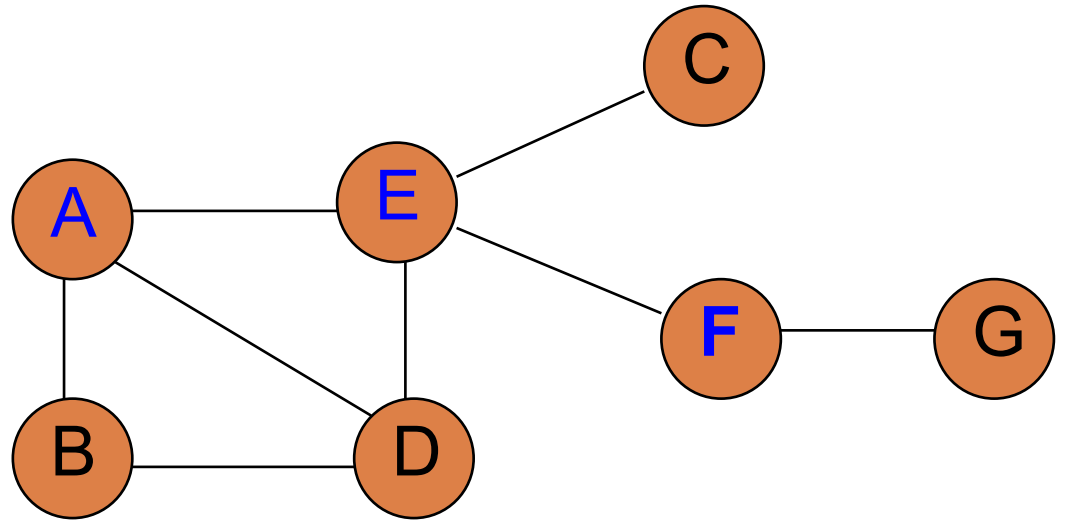


toVisit-stack: B D C D F

visited: A E

DFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```

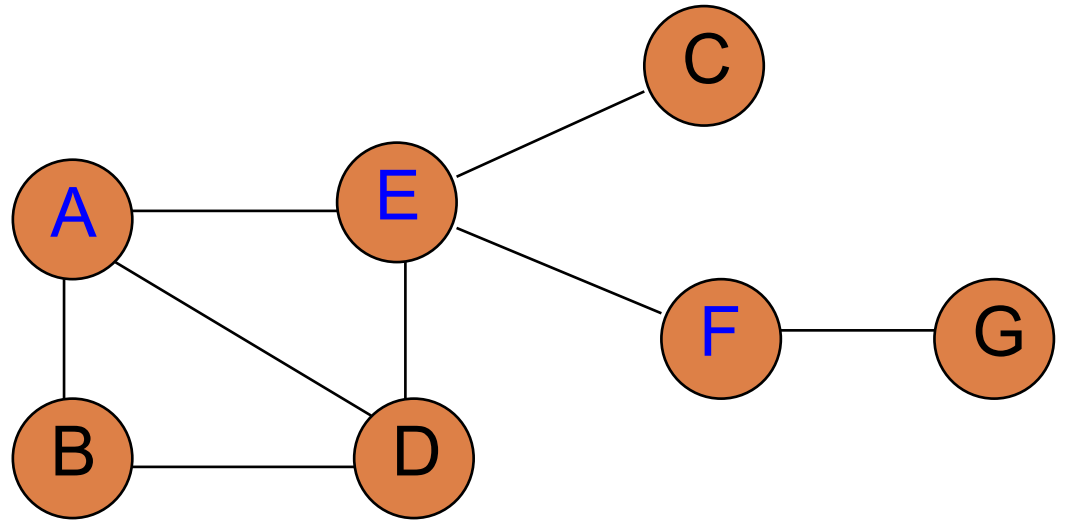


toVisit-stack: B D C D G

visited: A E F

DFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```

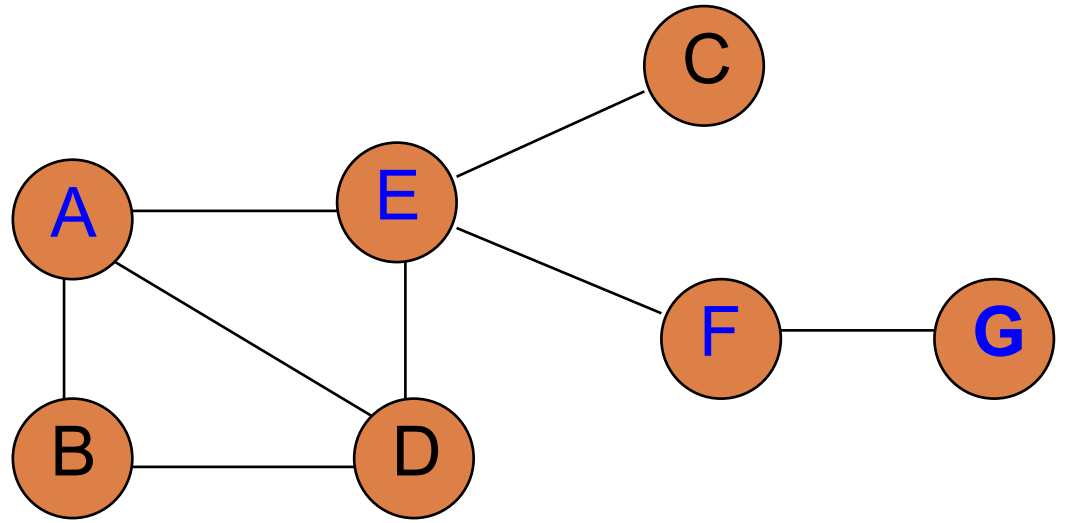


toVisit-stack: B D C D G

visited: A E F

DFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```

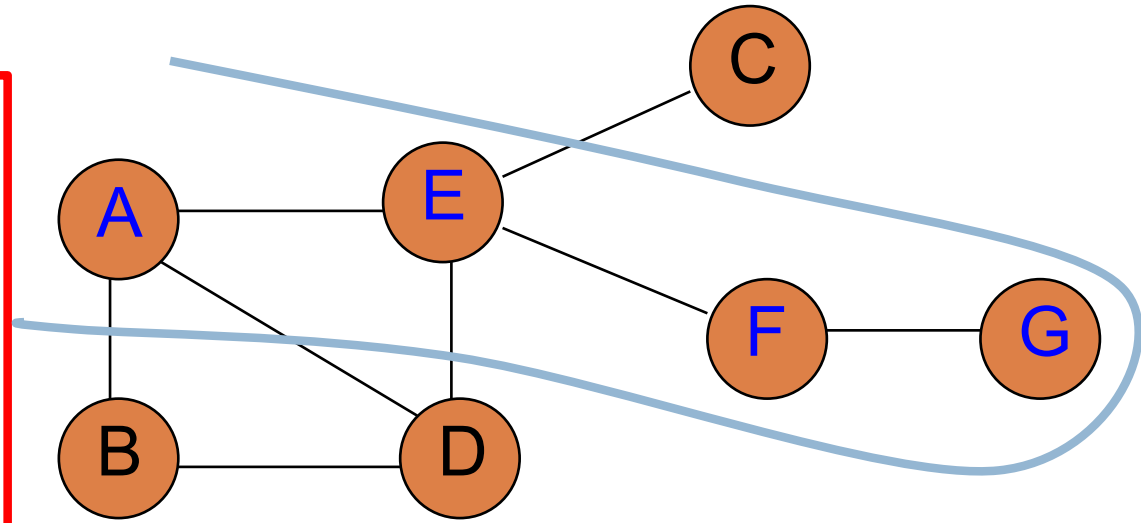


toVisit-stack: B D C D

visited: A E F **G**

DFS

```
graphSearch( toVisit )  
while !toVisit.empty()  
  v = toVisit.remove()  
  if !visited[v]  
    visited[v] = true  
    for c in v.getAdjacent()  
      if !visited[c]  
        toVisit.add(c)
```

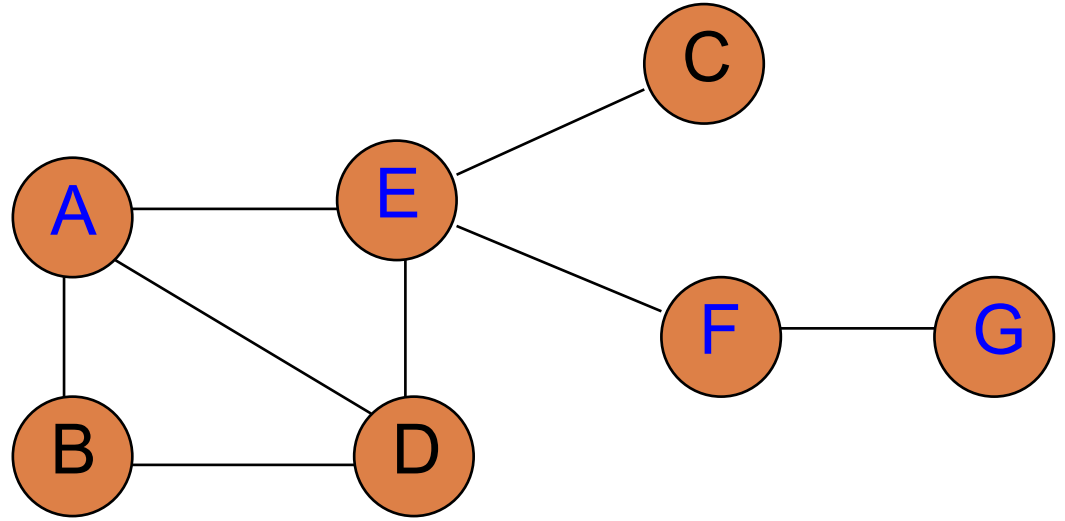


toVisit-stack: B D C D
visited: A E F G

Frontier: Go as far down one path as possible

DFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```



toVisit-stack: B D C D

visited: A E F G

DFS

```
graphSearch( toVisit )
```

```
while !toVisit.empty()
```

```
  v = toVisit.remove()
```

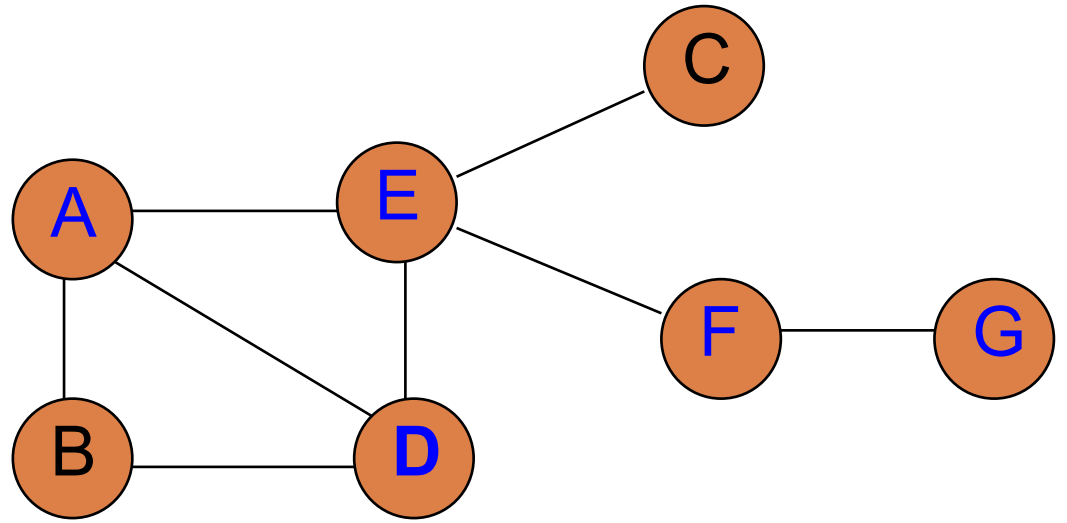
```
  if !visited[v]
```

```
    visited[v] = true
```

```
    for c in v.getAdjacent()
```

```
      if !visited[c]
```

```
        toVisit.add(c)
```



toVisit-stack: B D C B

visited: A E F G D

DFS

```
graphSearch( toVisit )
```

```
while !toVisit.empty()
```

```
  v = toVisit.remove()
```

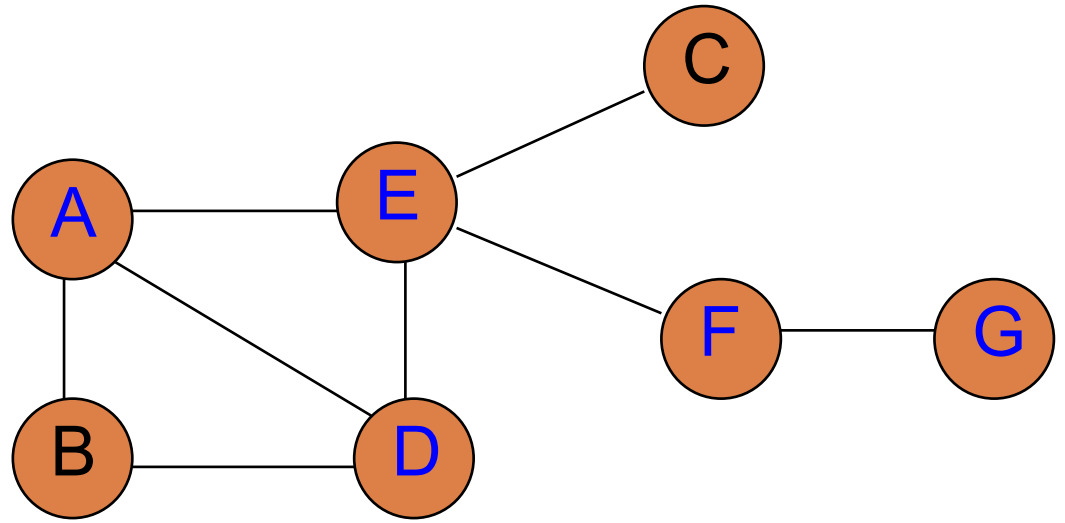
```
  if !visited[v]
```

```
    visited[v] = true
```

```
    for c in v.getAdjacent()
```

```
      if !visited[c]
```

```
        toVisit.add(c)
```



toVisit-stack: B D C B

visited: A E F G D

DFS

```
graphSearch( toVisit )
```

```
while !toVisit.empty()
```

```
  v = toVisit.remove()
```

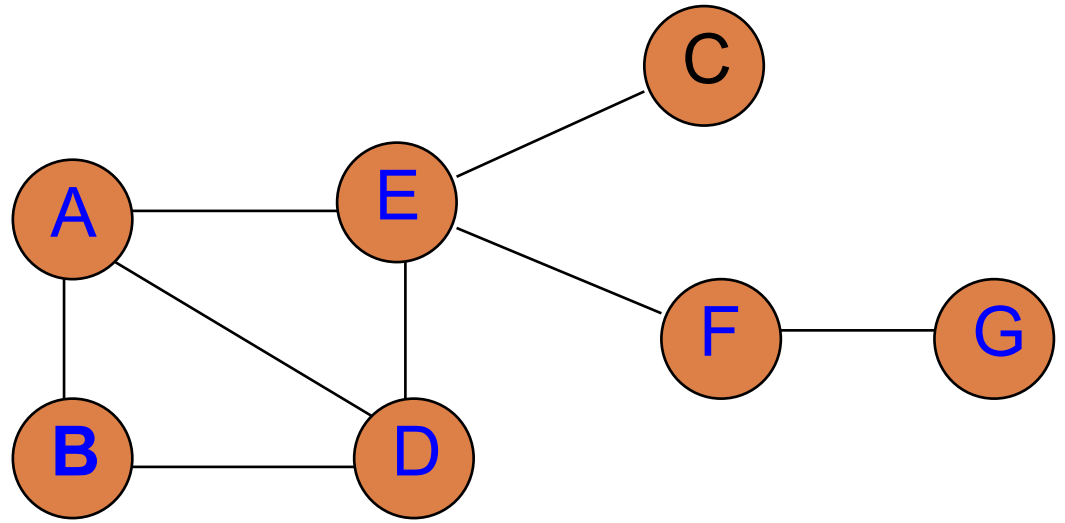
```
  if !visited[v]
```

```
    visited[v] = true
```

```
    for c in v.getAdjacent()
```

```
      if !visited[c]
```

```
        toVisit.add(c)
```



toVisit-stack: B D C

visited: A E F G D **B**

DFS

```
graphSearch( toVisit )
```

```
while !toVisit.empty()
```

```
  v = toVisit.remove()
```

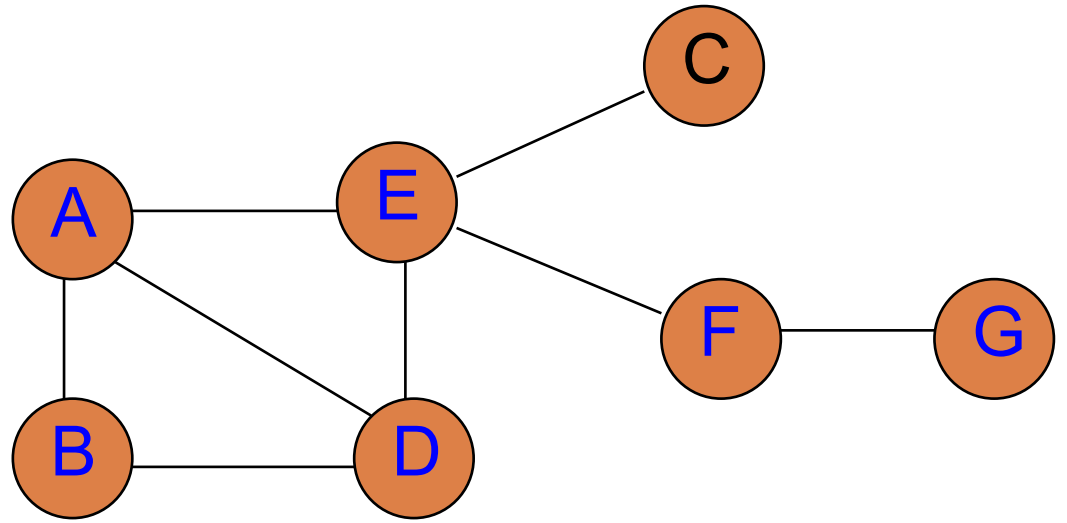
```
  if !visited[v]
```

```
    visited[v] = true
```

```
    for c in v.getAdjacent()
```

```
      if !visited[c]
```

```
        toVisit.add(c)
```

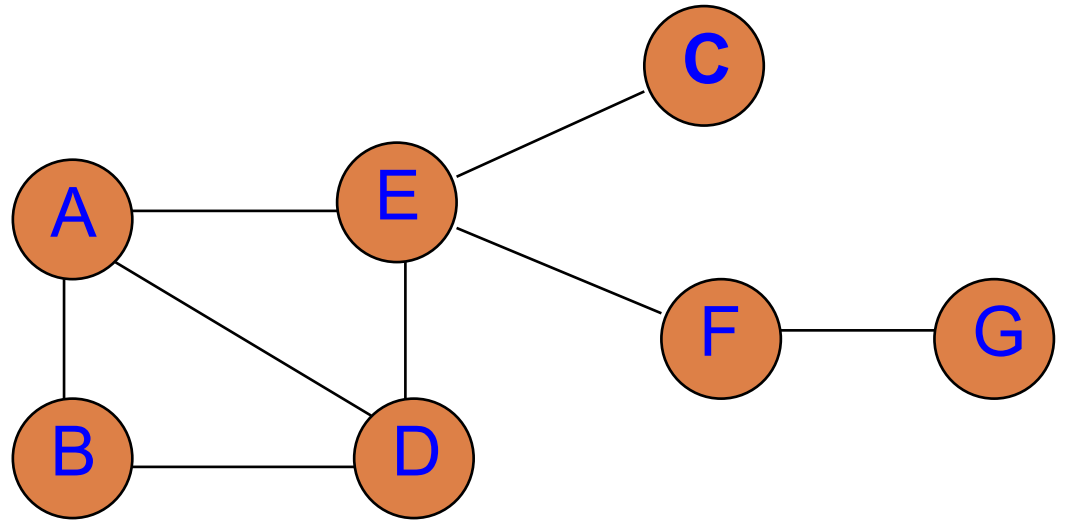


toVisit-stack: B D C

visited: A E F G D B

DFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```

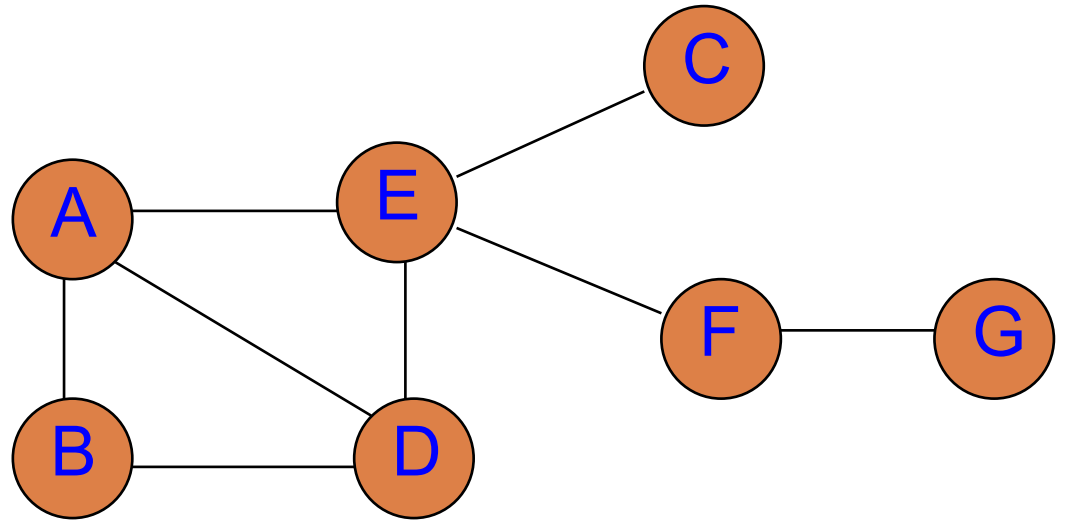


toVisit-stack: B D

visited: A E F G D B C

DFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```

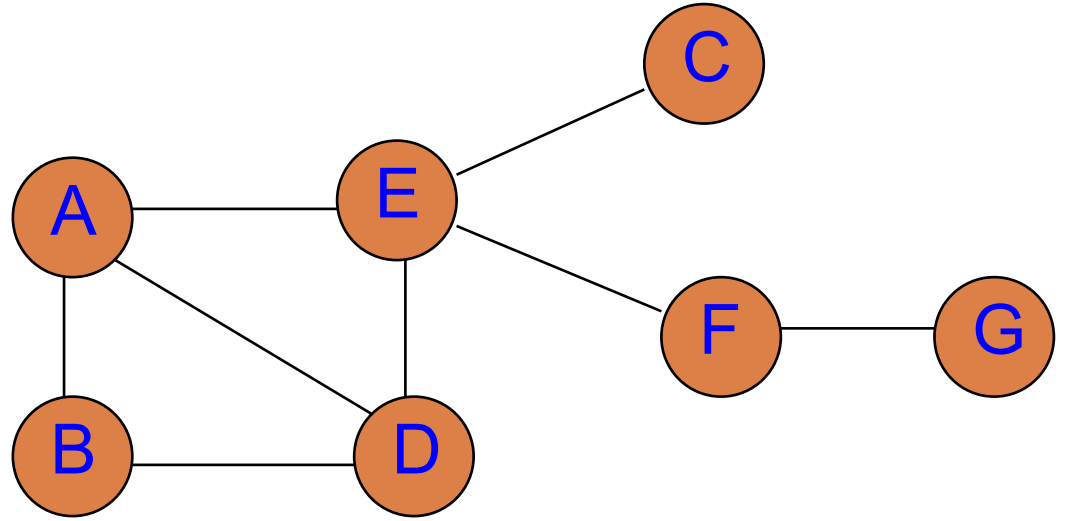


toVisit-stack: B D

visited: A E F G D B C

DFS

```
graphSearch( toVisit )  
  while !toVisit.empty()  
    v = toVisit.remove()  
    if !visited[v]  
      visited[v] = true  
      for c in v.getAdjacent()  
        if !visited[c]  
          toVisit.add(c)
```



toVisit-stack:

visited: A E F G D B C

graphSearch run-time

```
graphSearch( toVisit )
  while !toVisit.empty()
    v = toVisit.remove()
    if !visited[v]
      visited[v] = true
      for c in v.getAdjacent()
        if !visited[c]
          toVisit.add(c)
```

What is the big-O run-time of graphSearch?

Assume all of the stack/queue operations are constant.

How many times do we visit each vertex?

How many times do we traverse each edge (vis the for loop)?

graphSearch run-time

```
graphSearch( toVisit )
  while !toVisit.empty()
    v = toVisit.remove()
    if !visited[v]
      visited[v] = true
      for c in v.getAdjacent()
        if !visited[c]
          toVisit.add(c)
```

How many times do we visit each vertex? Exactly once

How many times do we traverse each edge (vis the for loop)? Exactly once

What is the big-O run-time of treeSearch? $O(|V| + |E|)$. Linear algorithm.

Nothing changes from treeSearch!