Lecture 26: More Dictionaries & Hashing

CS 62

Spring 2018

Alexandra Papoutsaki & William Devanny

Naïve Version

- Warning: this code is simplified!

```
public class Map<K, V> {
  protected V[] entries;

  public V get(K key) {
    int index = key.hashCode() % entries.length;
    return entries[index];
  }
  public void put(K key, V value) {
    int index = key.hashCode() % entries.length;
    entries[index] = value;
  }
}
```

Hash Collisions

- `k1.hashCode() == k2.hashCode()` but `k1 != k2`
  - May also be caused by the modulus operation
- This is inevitable (e.g., the birthday paradox)
- A “good” hash function rarely collides

Two main strategies to avoid collisions

1) Open addressing (closed hashing):
   Each bucket can store at most one entry
   If hash falls in occupied bucket then search procedure for next empty bucket based on:
   - Linear probing
   - Quadratic probing
   - Double probing

2) Closed Addressing (open or external hashing/bucketing):
   Each bucket can store multiple entries
   - Separate chaining

Linear Probing

- If we collide, check next entry until one is empty. Wrap around when at the end of table
- Deletion is complicated
- Can only hold `entries.length` items
- Resizing the table requires rehashing everything
- Suffers from primary clustering

Linear Probing Example: $h(k) = k\%13$

Keys to insert: 17, 33, 18, 20, 44, 11, 19, 7 (ignore values)

0	1	2	3	4	5	6	7	8	9	10	11	12	
				17									
				17			33						
				17	18		33						
				17	18		33	20					Collision!
				17	18	44	33	20					Collision!
				17	18	44	33	20			11		
				17	18	44	33	20	19		11		Collision!
				17	18	44	33	20	19	7	11		Collision!

Linear Probing

- Keys with same hash will be clustered together
- The same thing can happen with unrelated keys forming primary clusters
- The more elements we add, the more collisions

Linear Probing Lookup

- Start at location returned by hashing function
  - If key was found → value
  - If key was not found search linearly until:
    - You find the key → value
    - You find an empty slot before you have found key → null
    - You wrapped around and ended up where you started → null
- Example: get(7) returns the value for 7

0	1	2	3	4	5	6	7	8	9	10	11	12
				17	18	44	33	20	19	7	11	

- Example: get(6) returns null

0	1	2	3	4	5	6	7	8	9	10	11	12
				17	18	44	33	20	19	7	11	

Quadratic Probing

- $h(k,i) = (h(k) + c_1 i + c_2 i^2)\ (mod\ n), c_2 \neq 0$
- If $c_2 = 0$ then degrades to linear probing
- E.g., $h(k,i) = (h(k) + i^2)\ (mod\ n)$, then for every probing
  $$h(k), h(k)+1, h(k)+4, \ldots$$
- Can result in cases where we don't try all slots
  - E.g., $n = 5$, and start with $h(k) = 1$.
  - Rehashings give $2, 0, 0, 2, 1, 2, 0, 0, \ldots$
  - The slots 3 and 4 will never be examined to see if they have room
- Secondary Clustering

Quadratic Probing: $h(k,i) = (k\%13) + i^2$

Keys to insert: 17, 33, 18, 20, 44, 11, 19, 7 (ignore values)

0	1	2	3	4	5	6	7	8	9	10	11	12	
				17									
				17			33						
				17	18		33						
				17	18		33	20					Collision!
				17	18	44	33	20					Collision!
				17	18	44	33	20			11		
				17	18	44	33	20		19	11		Collision!
			7	17	18	44	33	20		19	11		Collision!

Double Hashing

- Use second hash function on key to determine delta (interval) for next try
- $h(k,i) = (h_1(k) + i \cdot h_2(k))\ (mod\ n)$,
- E.g., $h_2(k) = (k\ mod\ (n\ - 2)) + 1$
- Helps with primary and secondary clustering

- Example:
  - Suppose $h_1(n) = n\ \%\ 5$
  - Then $h_1(1) = h_1(6) = h_1(11)$
  - However, $h_2(1) = 2, h_2(6) = 1, h_2(11) = 3$

Separate Chaining

- Turn each bucket into a linked list (or array, etc.)
- On collision add to the bucket
- Searching list is fast if lists are small
- Deletion is simple
- Can hold more than `entries.length` items easily

Load Factor

- Performance depends on *load factor*
- Load factor is $\alpha = \frac{n}{N}$ where n = items in table and N = size of table
- Higher load factor → more collisions → slow
- Can be > 1 for external chaining
- For open addressing usually want to ensure $\alpha < 0.75$
  - Generally $\alpha > 0.75$ means resize the table (& rehash everything)

Performance

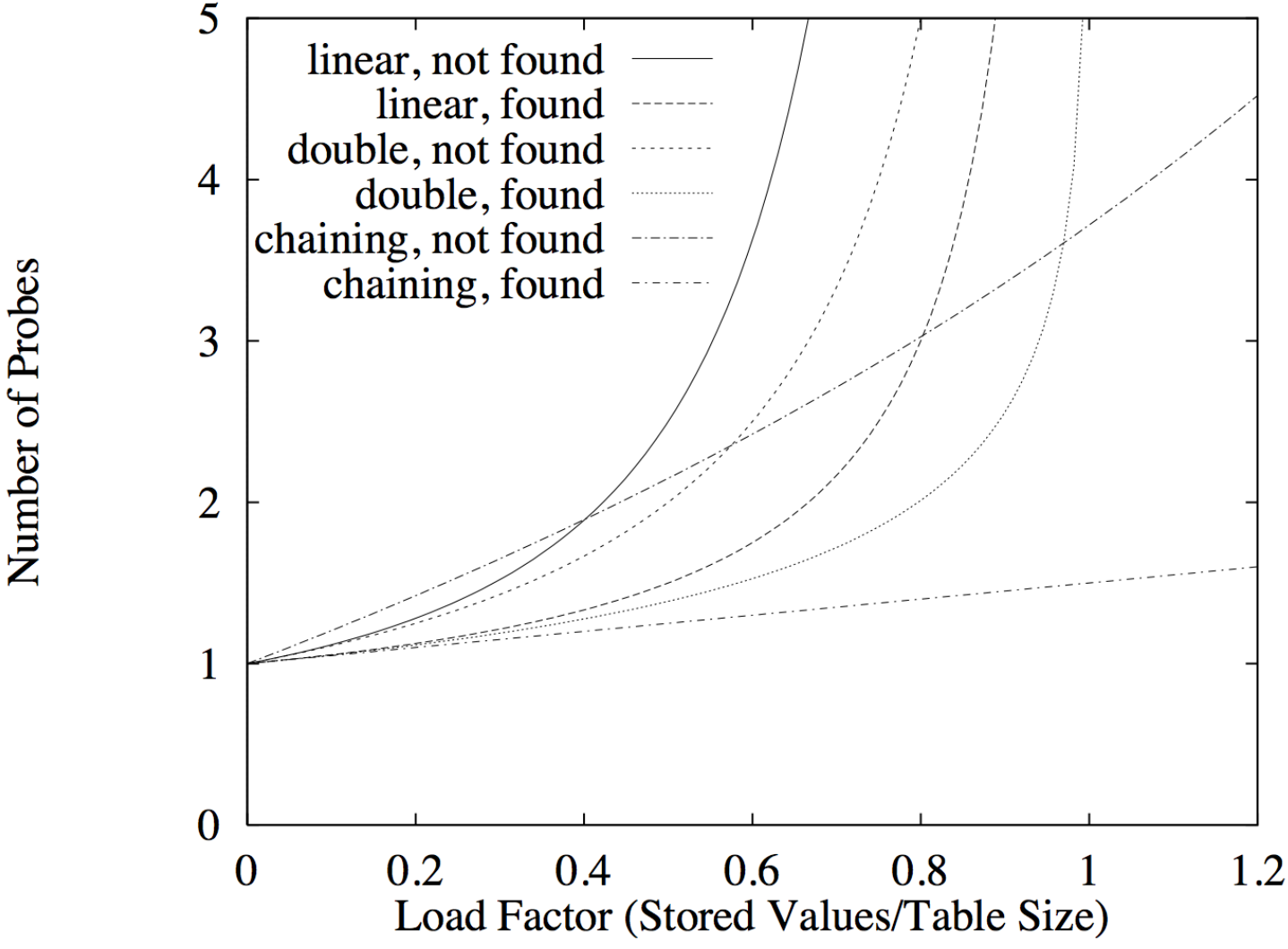
Strategy	Unsuccessful	Successful
Linear Probing	$1/2\ (1 + \ 1/(1-a)^2)$	$1/2\ (1 + \ 1/(1-a))$
Double Probing	$1/(1-a)$	$-(1/a)/log(1-a)$
External Chaining	$a + e^{-a}$	$1 + 1/2a$

Entries represent number of comparisons needed to find a specific element or demonstrate that it is not in the hash table

Performance for $a = .9$

Strategy	Unsuccessful	Successful
Linear Probing	55	5.5
Double Probing	10	~4
External Chaining	3	1.45

Entries represent number of comparisons needed to find a specific element or demonstrate that it is not in the hash table


linear, not found
linear, found
double, not found
double, found
chaining, not found
chaining, found
Number of Probes
Load Factor (Stored Values/Table Size)
0
1
2
3
4
5
0
0.2
0.4
0.6
0.8
1
1.2

Space requirements

- Open addressing: TableSize +n*objectsize
- External chaining: TableSize +(n*objectsize+1)

- Rule of thumb:
  - Small elements, small load factor: open addressing
  - Large elements, large load factor: external chaining