CS062

DATA STRUCTURES AND ADVANCED PROGRAMMING

Midterm II Review

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she/her/hers

Midterm Review

- Sorting
- Heaps/Priority Queues
- Dictionaries
- Misc
- Practice Problems
- Answers

Sorting

- Selection sort
- Insertion sort
- Merge sort
- Quick sort
- Heap sort

Sorting

- Given an array of n items, sort them in non-descending order based on a comparable key.
- Cost model counts comparisons and exchanges (or array accesses).
- Not in place: If linear extra memory is required.
- Stable: If duplicate elements stay in the same order that they appear in the input.
- Practice: https://visualgo.net/en/sorting (minus quick sort).

Selection sort - Algorithm

```
public static <E extends Comparable<E>> void selectionSort(E[] a) {
     int n = a.length;

     for (int i = 0; i < n; i++) {
          int min = i;
          for (int j = i+1; j < n; j++) {
              if (a[j].compareTo(a[min])<0){
                   min = j;
              }
          }
          E temp = a[i];
          a[i]=a[min];
          a[min]=temp;
     }
}
```

Selection sort - Key characteristics

- At the end of each iteration `i`:
    - `a[0…i]` is sorted.
    - no smaller item exists in `a[i+1…n-1]`.
- In-place.
- Not stable.
- $O(n^2)$ comparisons for best/average/worst case.
    - $O(n)$ exchanges.
- Slowest. Realistically, never used in practice.

Selection sort - Example

- Sort: 1,4,9,3,8,2.

i iteration	Result
0	1,4,9,3,8,2
1	1,2,9,3,8,4
2	1,2,3,9,8,4
3	1,2,3,4,8,9
4	1,2,3,4,8,9
5	1,2,3,4,8,9

Sorting

- Selection sort
- Insertion sort
- Merge sort
- Quick sort
- Heap sort

Insertion sort - Algorithm

```
public static <E extends Comparable<E>> void insertionSort(E[] a) {
     int n = a.length;

     for (int i = 0; i < n; i++) {
         for (int j = i; j > 0; j--) {
              if (a[j].compareTo(a[j-1])<0){
                 E temp = a[j];
                 a[j]=a[j-1];
                 a[j-1]=temp;
              }
              else{
                 break;
              }
          }
      }
}
```

Insertion sort - Key characteristics

- At the end of each iteration `i`:
    - `a[0…i]` is partially sorted.
- In-place.
- Stable.
- $O(n^2)$ comparisons/exchanges for average/worst case.
- $O(n)$ comparisons and 0 exchanges for best case (already sorted array).
- Slow but in practice such little overhead that can be even faster than quick sort for small arrays. Often used below certain thresholds for merge sort and quick sort.

Insertion sort - Example

- Sort: 1,4,9,3,8,2.

i iteration	Result
0	1,4,9,3,8,2
1	1,4,9,3,8,2
2	1,4,9,3,8,2
3	1,3,4,9,8,2
4	1,3,4,8,9,2
5	1,2,3,4,8,9

Sorting

- Selection sort
- Insertion sort
- Merge sort
- Quick sort
- Heap sort

Merge sort - Algorithm

```java
private static <E extends Comparable<E>> void merge(E[] a, E[] aux, int lo, int mid, int hi) {
    for (int k = lo; k <= hi; k++){
        aux[k] = a[k];
    }
    int i = lo, j = mid + 1;
    for (int k = lo; k <= hi; k++) {
        if (i > mid) { // ran out of elements in the left subarray
            a[k] = aux[j++];
        } else if (j > hi) { // ran out of elements in the right subarray
            a[k] = aux[i++];
        } else if (aux[j].compareTo(aux[i]) < 0) {
            a[k] = aux[j++];
        } else {
            a[k] = aux[i++];
        }
    }
}
```

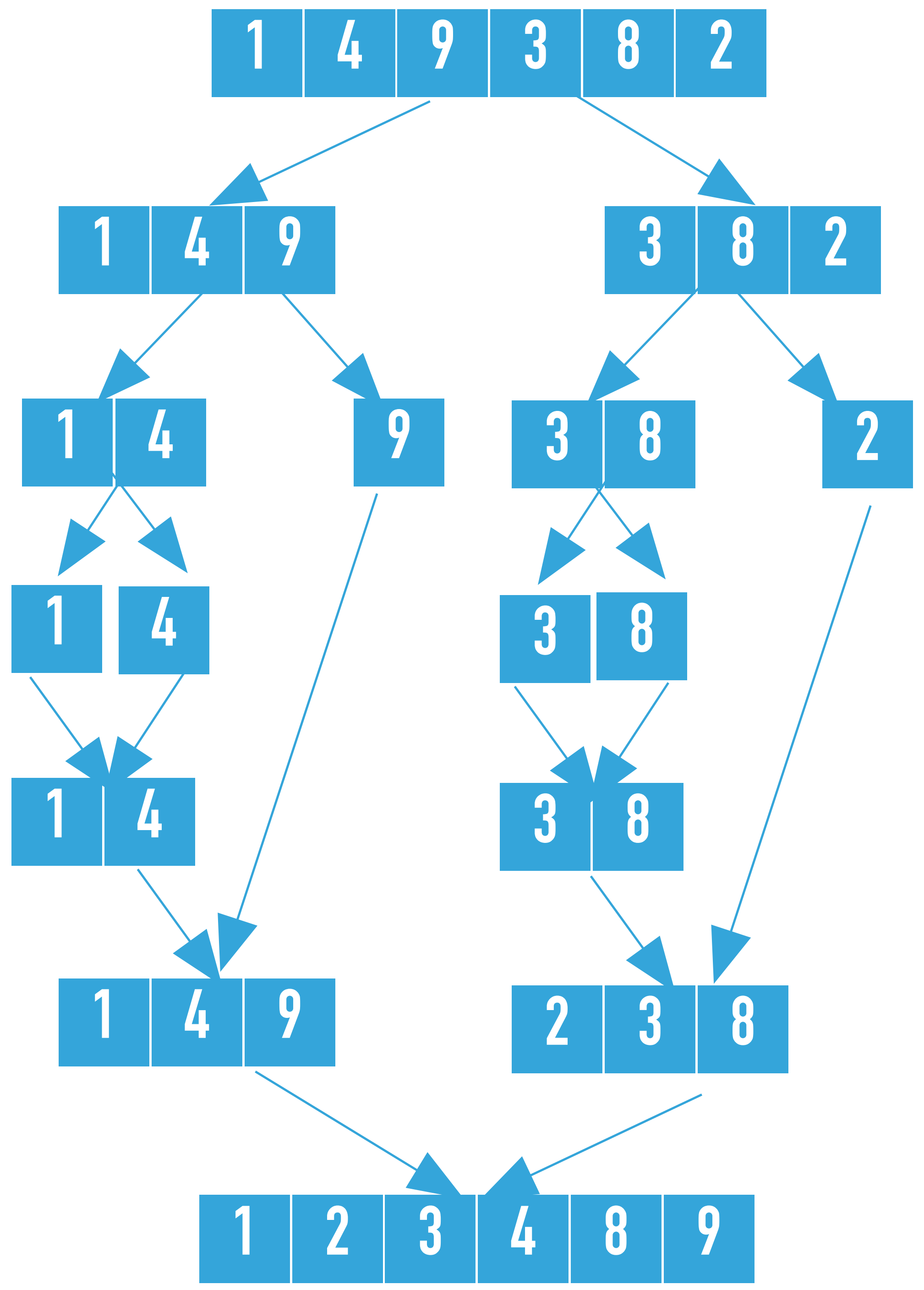
```java
public static <E extends Comparable<E>> void mergeSort(E[] a) {
    E[] aux = (E[]) new Comparable[a.length];
    mergeSort(a, aux, 0, a.length - 1);
}

private static <E extends Comparable<E>> void mergeSort(E[] a, E[] aux, int lo, int hi) {
    if (hi <= lo){
        return;
    }
    int mid = lo + (hi - lo) / 2;
    mergeSort(a, aux, lo, mid);
    mergeSort(a, aux, mid+1, hi);
    merge(a, aux, lo, mid, hi);
}
```

Merge sort - Key characteristics

- Divide till you reach an array of a single element and conquer by merging two already-sorted subarrays into a sorted larger one.
- Not in-place, requires linear extra memory. On-disk sort assignment showed how to use the disk if memory is not enough.
- Stable.
- $O(n \log n)$ comparisons/array accesses for best/average/worst case.
- Stable performance, preferred for arrays of objects due to stability. Slower than quick sort on average. Not in-place so not good when memory is in short supply (e.g., embedded systems).

Merge sort - Example

- Sort: 1,4,9,3,8,2.



Sorting

- Selection sort
- Insertion sort
- Merge sort
- Quick sort
- Heap sort

Quick sort - Algorithm

```
private static <E extends Comparable<E>> int partition(E[] a, int lo, int hi) {
        int i = lo - 1;
        E pivot = a[hi]

        for (int j = lo; j < hi; j++) {
            if (a[j].compareTo(pivot) <= 0) {
                i++;
                E temp = a[i];
                a[i] = a[j];
                a[j] = temp;
            }
        }

        i++;
        E temp = a[i];
        a[i] = a[hi];
        a[hi] = temp;

        return i;
    }
```

```
private static <E extends Comparable<E>> void quickSort(E[] a)
{
   quickSort(a, 0, a.length-1);
}
private static <E extends Comparable<E>> void quickSort(E[] a,
int lo, int hi) {
    if (lo < hi){
       int pivot = partition(a, lo, hi);
       quickSort(a, lo, pivot-1);
       quickSort(a, pivot+1, hi);
    }
}
```

Quick sort - Key characteristics

- Swap smaller elements than pivot to go to left, and larger elements to go to right subarray.
- In-place.
- Not stable.
- $O(n \log n)$ comparisons/exchanges for best/average case.
- $O(n^2)$ comparisons/exchanges for worst case (already (reversely) sorted array, where pivot is always the smallest/largest element).
- Preferred for arrays of primitives since stability does not matter. Fastest on average but if unlucky quadratic. In-place so good choice for memory efficient applications with tolerance for occasional slowdowns.

Quick sort - Example

- Sort: 1,4,9,3,8,2

lo	hi	Index of pivot returned	Result
0	5	1	1,2,9,3,8,4
2	5	3	1,2,3,4,8,9
4	5	5	1,2,3,4,8,9
	Final result:		1,2,3,4,8,9

Sorting

- Selection sort
- Insertion sort
- Merge sort
- Quick sort
- Heap sort

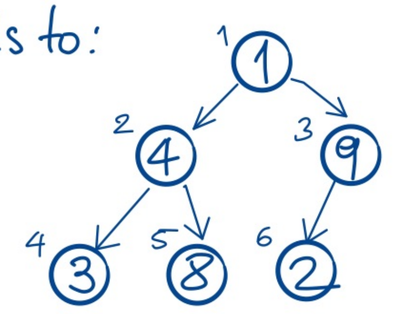
Heap sort - Key characteristics

- Heap construction in $O(n)$: heapify subtrees rooted in internal nodes.
  - There is also a slower $O(n\log n)$ version with n insertions. Avoid it.
  - Sortdown in $O(n\log n)$: Repeat: exchange root with last element and sink.
- In-place.
- Not stable.
- $O(n\log n)$ comparisons/exchanges for best/average/worst case.
- Slower than merge sort (and quick sort) but does not require extra memory. Good choice for memory efficient applications that need stable performance.

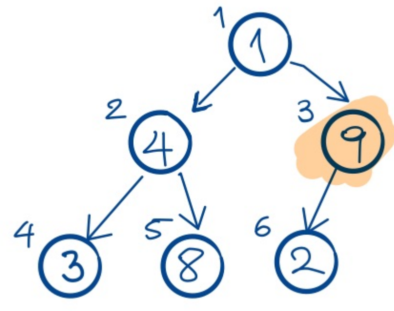
Heap sort - Example

- Sort: 1,4,9,3,8,2,

Corresponds to:

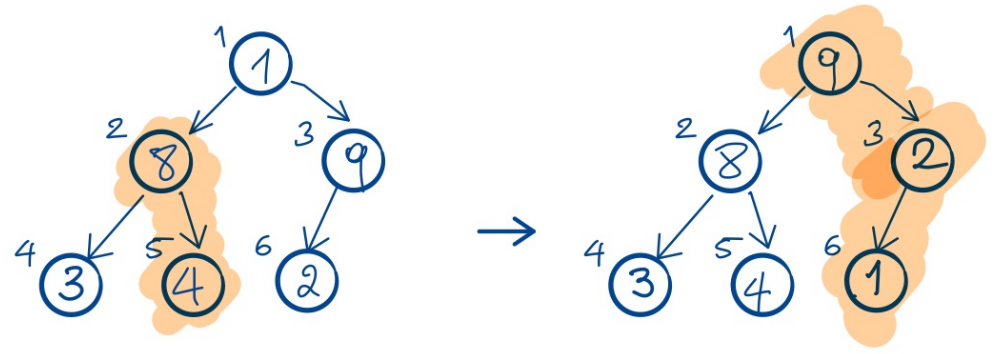


Heap construction: Start at first internal node, k=6/2=3
sink(k). k-- till you reach root.



Do not sink, does not violate heap-order.

$k=3$



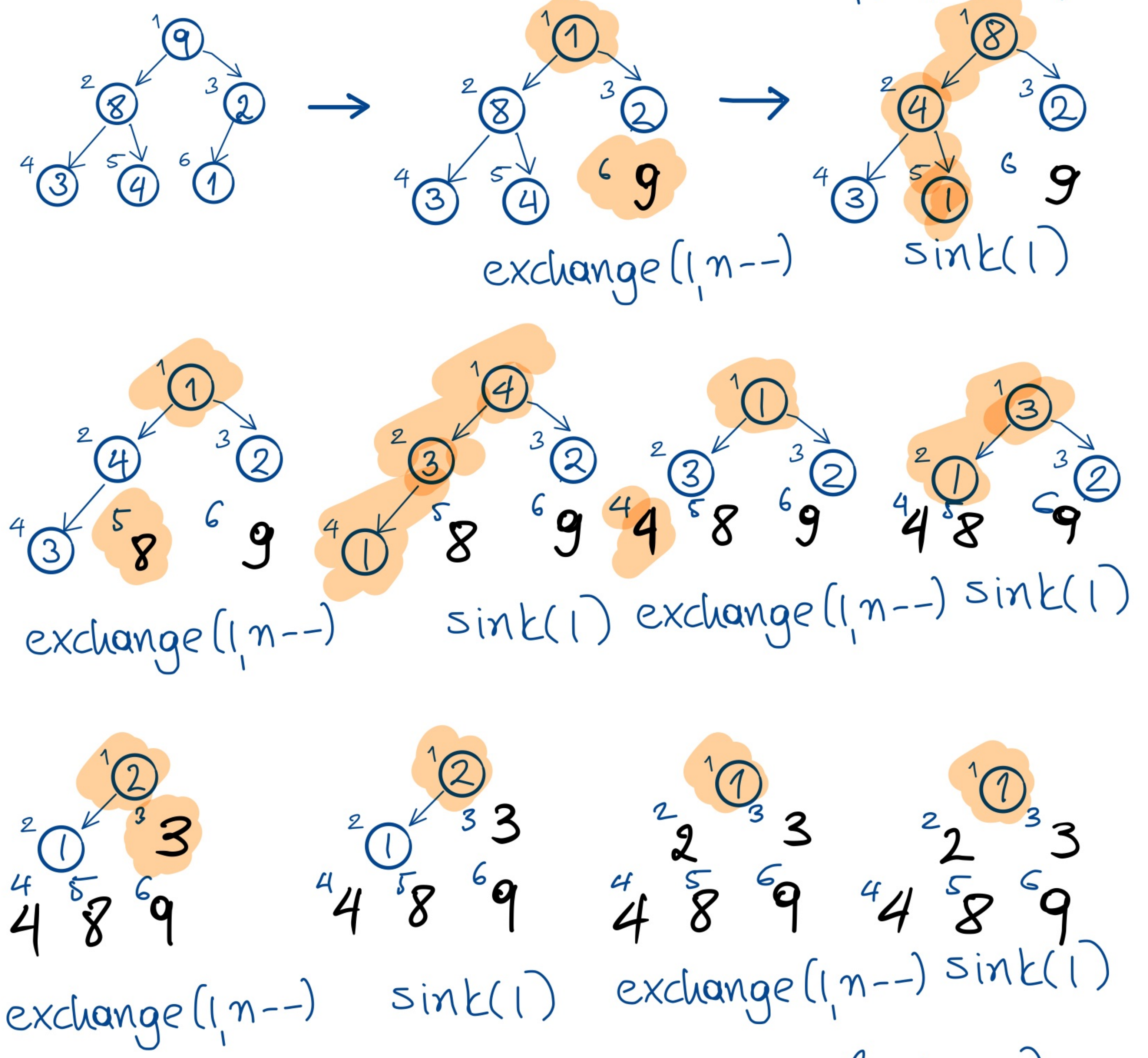
$k=2$ $k=1$

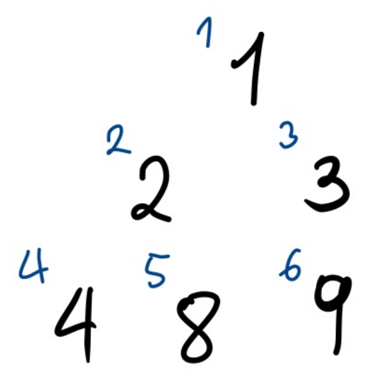
Total cost: $O(n)$

Heap sort - Example

- Sort: 1,4,9,3,8,2,

Sortdown: Given binary heap, repeatedly exchange last node with root and sink new root to its appropriate place





Total cost: $O(n \log n)$

Heapsort $= O(n) + O(n \log n) = O(n \log n)$

Heaps

- Insertion
- Deletion

Heaps

- Array representation of binary trees (at most 2 children for each node) which are complete ($log n$ minimal height and nodes in last level as left as possible) and heap-ordered (every node is larger/equal to both of its children - if any).
- For node k, left child can be found at 2k, right child at 2k+1, and parent at k/2. Elements start at index 1.
- Heaps and priority queues are often considered synonyms.
- Practice: https://visualgo.net/en/heap (including heap sort).

Heaps

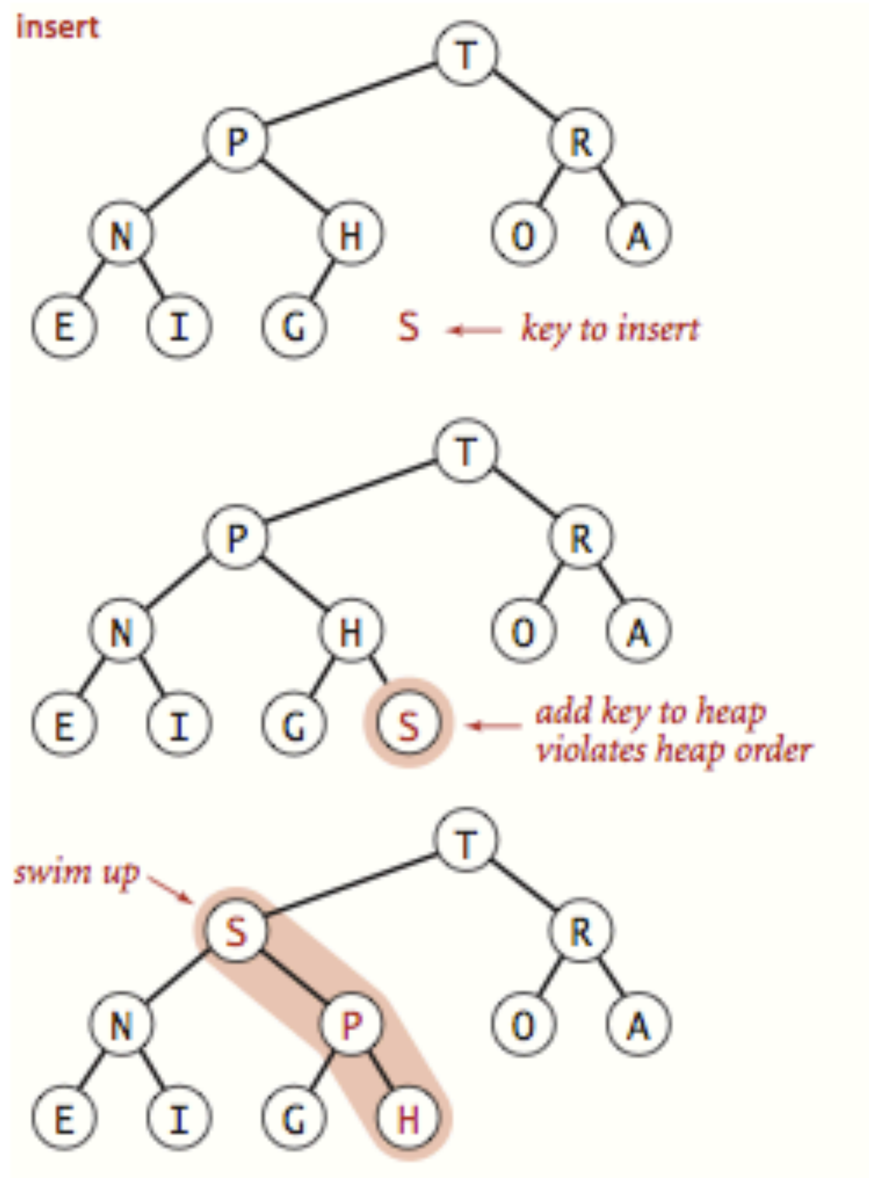
- Insertion
- Deletion

Heaps - Insertion

- Insert node at last level, as left as possible (or create a new level if last level is full). Swim newly-added node to its proper place so that heap-ordered property is satisfied. At most $O(\log n)$ comparisons.

```
public void insert(Key x) {
   pq[++n] = x;
   swim(n);
}
```

```
private void swim(int k) {
   while (k > 1 && less(k/2, k)) {
      exch(k, k/2);
      k = k/2;
   }
}
```



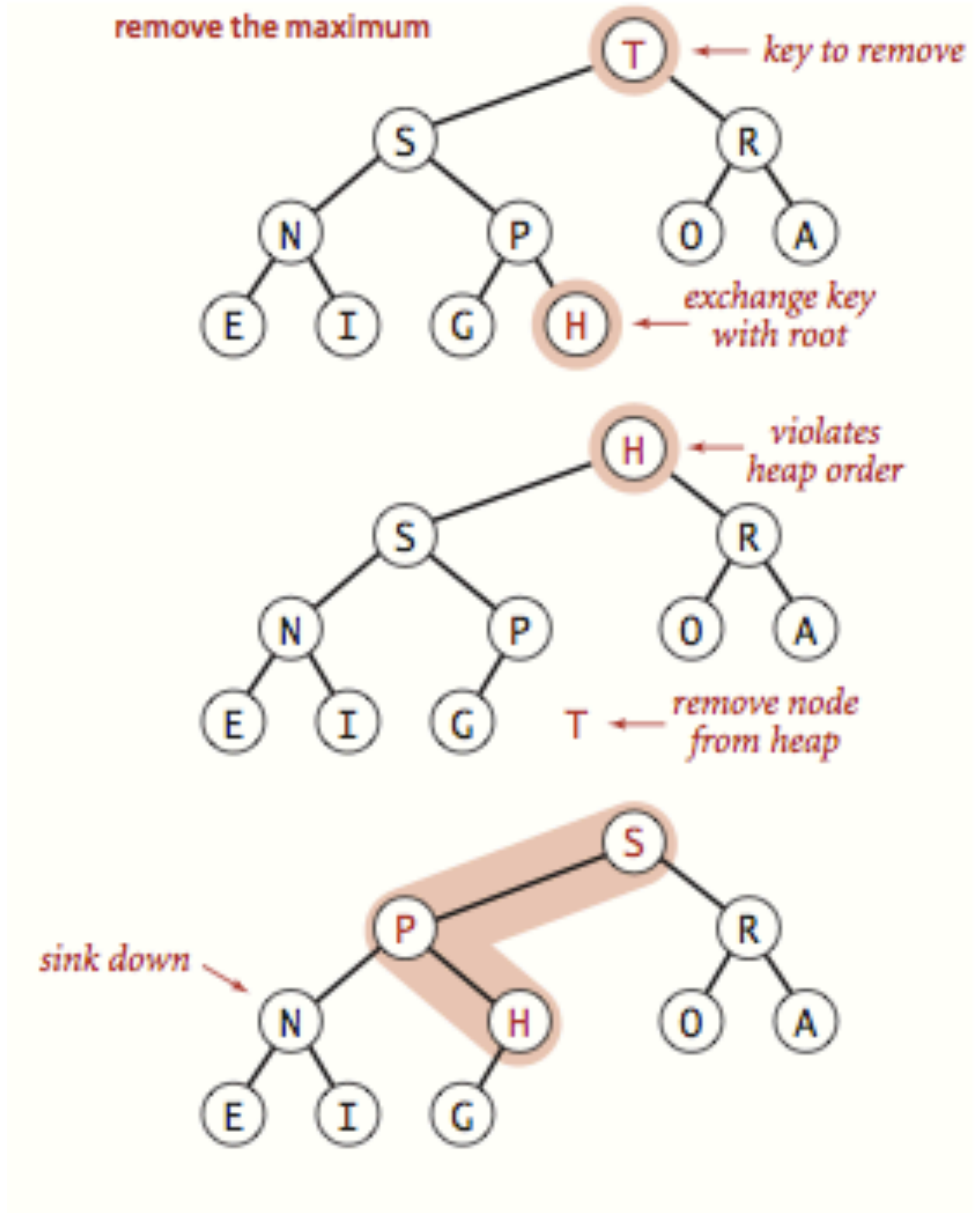
Heaps

- Insertion
- Deletion

Heaps - Deletion

- Exchange root with last element. Sink down the new root to its proper place so that heap-ordered property is satisfied. Nullify index of deleted element and return it. At most $O(\log n)$ comparisons.

```
public Key delMax() {
    Key max = pq[1];
    exch(1, n--);
    sink(1);
    pq[n+1] = null;
    return max;
}

private void sink(int k) {
    while (2*k <= n) {
        int j = 2*k;
        if (j < n && less(j, j+1))
            j++;
        if (!less(k, j))
            break;
        exch(k, j);
        k = j;
    }
}
```



Dictionaries

- Binary search trees

Dictionaries

- (Possibly ordered by key) collections of key-value pairs. Keys are comparable and unique. Values cannot be null.
- Ultimate goal is to achieve fast search based on key.
- Support insertion, deletion, and possibly ordered operations.

Binary search trees

- Binary trees with symmetric order (every node contains key larger than all keys in left subtree and smaller than all keys in right subtree).
- Height can vary from $O(\log n)$ (compact like complete trees) all the way to $O(n)$ (sticks/twigs).
- Practice: https://visualgo.net/en/bst

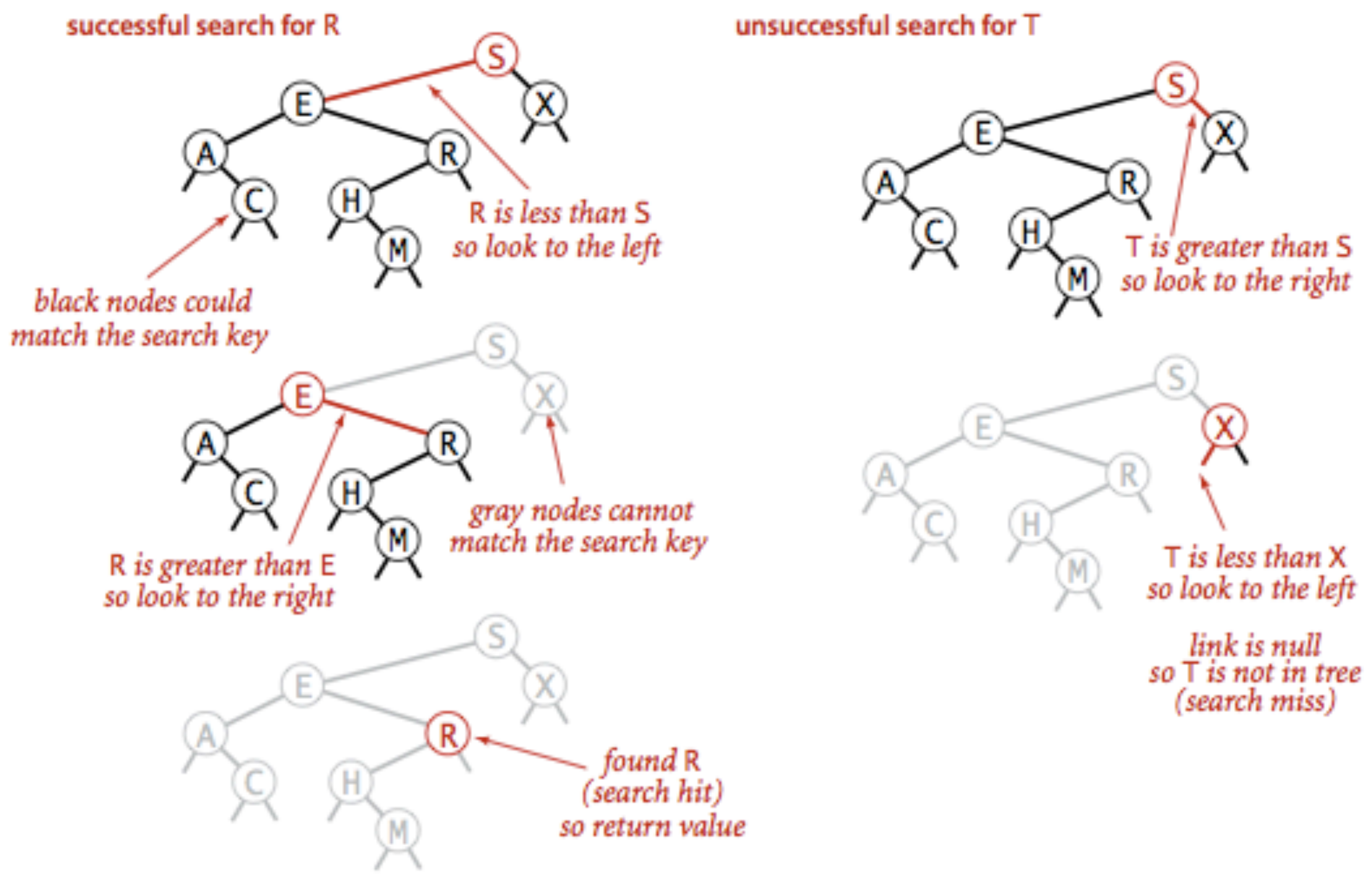
```
public class BST<Key extends Comparable<Key>, Value> {
   private Node root;             // root of BST

   private class Node {
        private Key key;          // sorted by key
        private Value val;        // associated value
        private Node left, right;  // roots of left and right subtrees
        private int size;         // #nodes in subtree rooted at this

        public Node(Key key, Value val, int size) {
            this.key = key;
            this.val = val;
            this.size = size;
        }
    }
```

Binary search trees - search

- Compare key with root node. Smaller? Go left. Larger? Go right.
- Search hit: If found node with key you're looking for. Return associated value.
- Search miss: reached a null node. Return null.

-
  ```
  public Value get(Key key) {
        Node x = root;
        while (x != null) {
              int cmp = key.compareTo(x.key);
              if (cmp < 0)
                      x = x.left;
              else if (cmp > 0)
                      x = x.right;
              else if (cmp == 0)
                      return x.val;
         }
         return null;
  }
  ```

Binary search trees - search

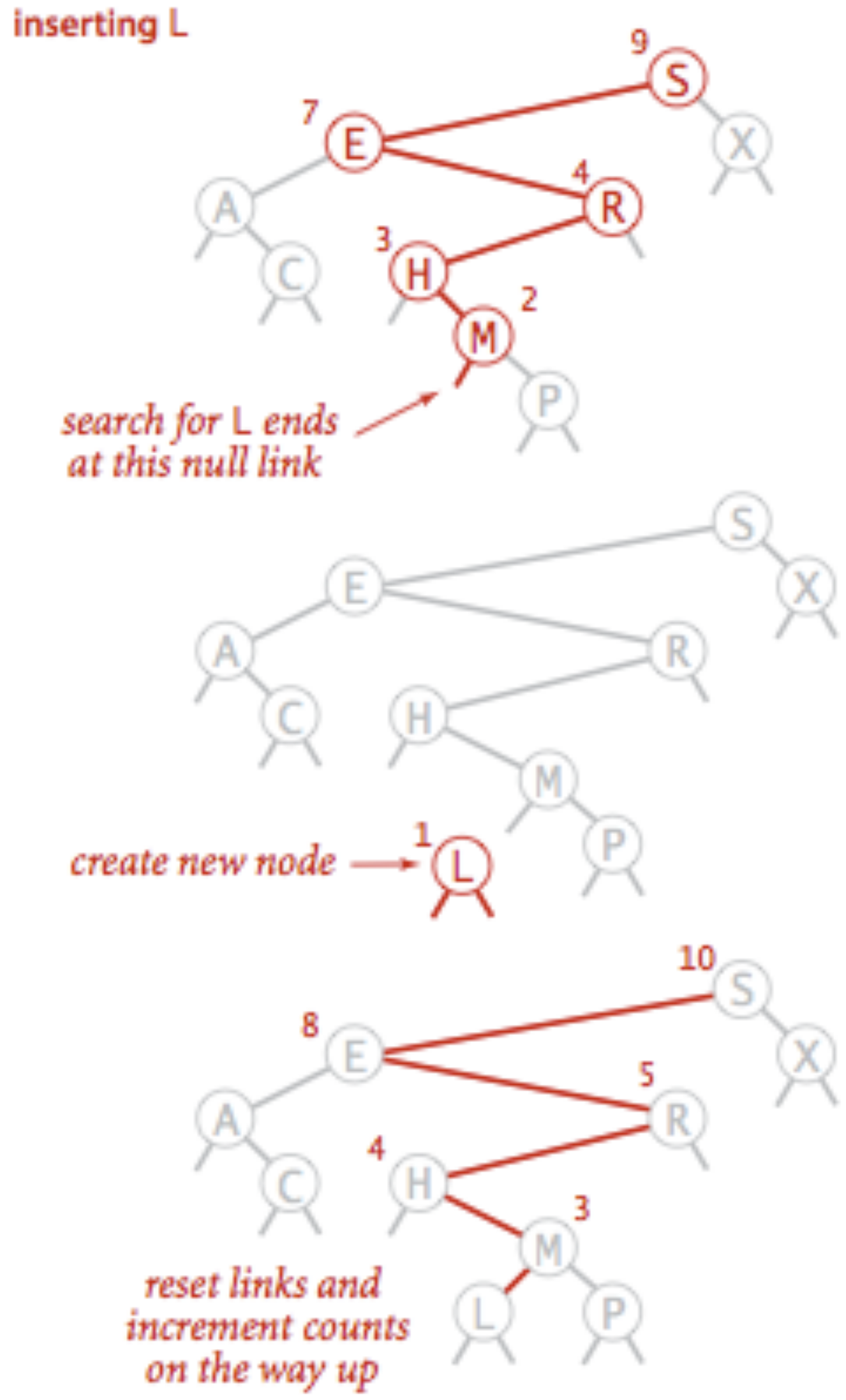

successful search for R
R is less than S so look to the left
black nodes could match the search key
R is greater than E so look to the right
gray nodes cannot match the search key
found R (search hit) so return value
unsuccessful search for T
T is greater than S so look to the right
T is less than X so look to the left
link is null so T is not in tree (search miss)
S
E
X
A
R
C
H
M

Successful (left) and unsuccessful (right) search in a BST

Binary search trees - insertion

- Compare key with root node. Smaller? Go left. Larger? Go right.
- If found node with same key, update value.
- If reached a null node, insert (key,value) pair.
-
```java
public void put(Key key, Value val) {
     root = put(root, key, val);
}
private Node put(Node x, Key key, Value val) {
     if (x == null)
           return new Node(key, val, 1);
     int cmp = key.compareTo(x.key);
     if (cmp < 0)
         x.left = put(x.left, key, val);
     else if (cmp > 0)
         x.right = put(x.right, key, val);
     else
         x.val = val;
     x.size = 1 + size(x.left) + size(x.right);
     return x;
}
```

Binary search trees - insertion


inserting L
9
7
4
3
2
S
X
E
R
A
H
C
M
P
search for L ends at this null link
create new node
1
L
10
8
5
4
3
reset links and increment counts on the way up

Insertion into a BST

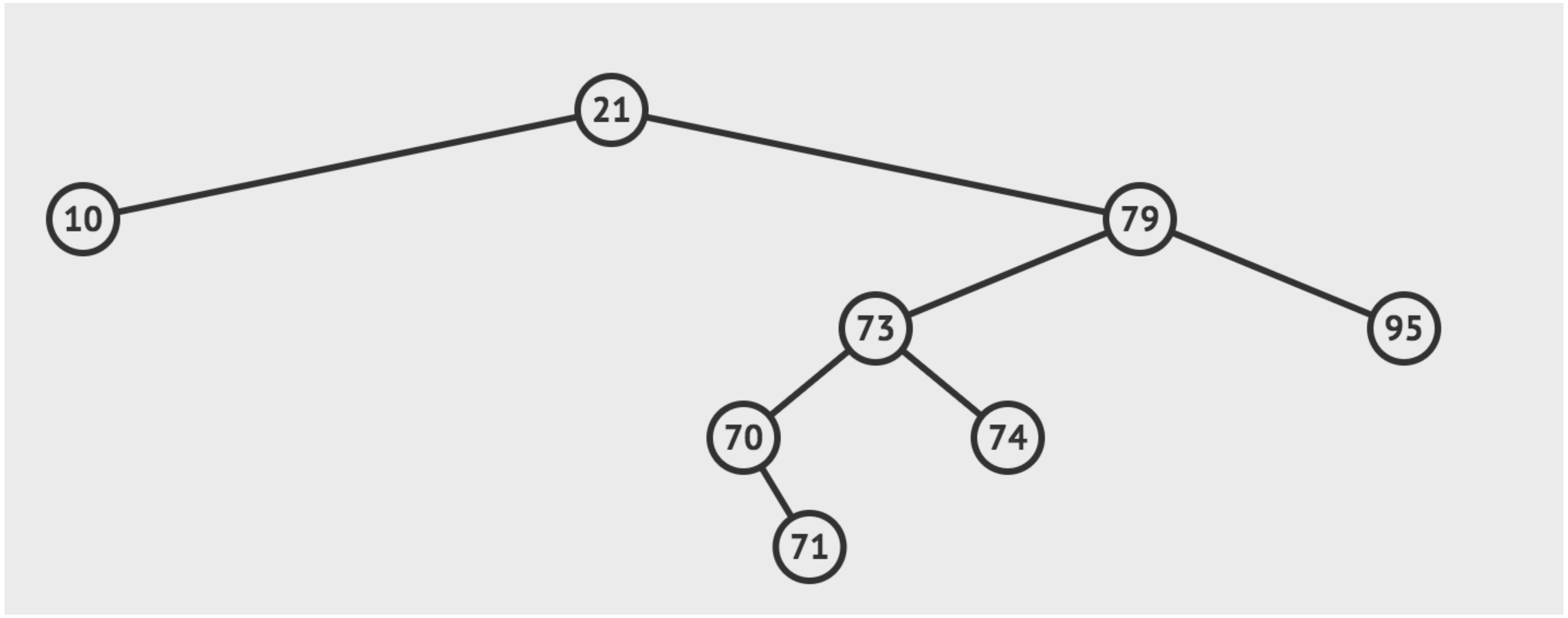
Binary search trees - Hibbard's deletion

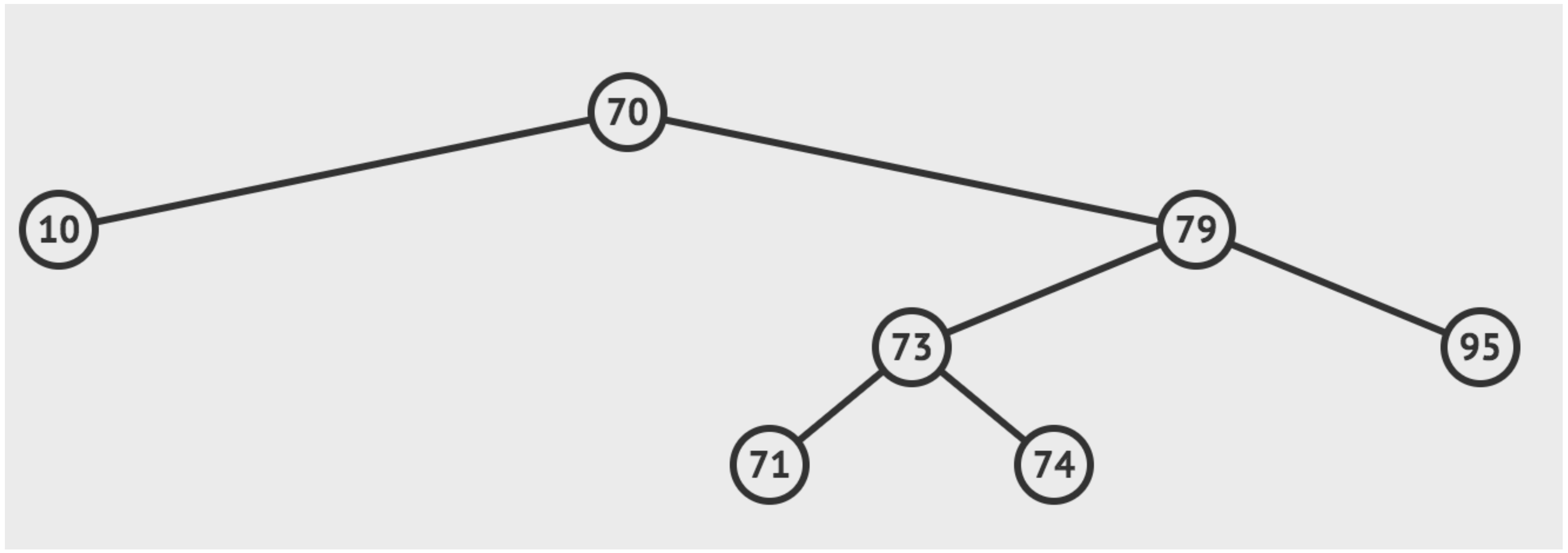
- Search for node:
  - Leaf? Just delete it.
  - Node with one child? Delete it and replace with child.
  - Node with two children? Delete and replace with successor (smallest of the larger keys). If successor has a (right) child, pass it to parent.

Binary search trees - Hibbard's deletion

```
public void delete(Key key) {
    root = delete(root, key);
}

 private Node delete(Node x, Key key) {
     if (x == null) return null;

     Int cmp = key.compareTo(x.key);
     if (cmp < 0)
         x.left  = delete(x.left,  key);
     else if (cmp > 0)
         x.right = delete(x.right, key);
     else {
         if (x.right == null)
             return x.left;
         if (x.left  == null)
             return x.right;
         Node t = x; //replace with successor
         x = min(t.right);
         x.right = deleteMin(t.right);
         x.left = t.left;
     }
     x.size = size(x.left) + size(x.right) + 1;
     return x;
 }
```

Binary search trees - delete node with key 21





Misc

- Comparable/Comparator Interfaces
- Iterable/Iterator Interfaces
- BT Traversals

Comparable Interface

- Interface with a single method that we need to implement: `public int compareTo(T that)`
- Implement it so that `v.compareTo(w)`:
  - Returns >0 if v is greater than `w`.
  - Returns <0 if v is smaller than `w`.
  - Returns 0 if v is equal to `w`.
- Corresponds to natural ordering.

Comparator Interface

- Sometimes the natural ordering is not the type of ordering we want.
- Comparator is an interface which allows us to dictate what kind of ordering we want by implementing the method:
  `public int compare(T this, T that)`
- Implement it so that `compare(v, w)`:
  - Returns >0 if v is greater than `w`.
  - Returns <0 if v is smaller than `w`.
  - Returns 0 if v is equal to `w`.
-
```
public static Comparator<ClassName> reverseComparator(){
   return (ClassName a, ClassName b)->{return -a.compareTo(b)};
 }
```

Misc

- Comparable/Comparator Interfaces
- Iterable/Iterator Interfaces
- BT Traversals

`Iterable<T>` Interface

- Interface with a single method that we need to implement: `Iterator<T> iterator()`
- Class becomes iterable, that is it can be traversed with a for-each loop.
-
```
for (String student: students){
        System.out.println(student);
}
```

`Iterator<T>` Interface

- Interface with two methods that we need to implement: `boolean hasNext()` and `T next()`.
- `hasNext()` checks whether there is any element we have not seen yet.
- `next()` returns the next available element.
- Always check if there are any available elements before returning the next one.
- Typically a comparable class, has an inner class that implements Iterator. Outer class's `iterator` method returns an instance of inner class.
- Can also be implemented in a standalone class where collection to iterate over is passed in the constructor.

Misc

- Comparable/Comparator Interfaces
- Iterable/Iterator Interfaces
- BT Traversals

BT traversals

- Pre-order: mark root visited, left subtree, right subtree.
- In-order: left subtree, mark root visited, right subtree.
- Post-order: left subtree, right subtree, mark root visited.
- Level-order: start at root, mark each node as visited level by level, from left to right.

Practice Problems

- Problem 1 - Sorting
- Problem 2 - Heaps
- Problem 3 - Tree traversals
- Problem 4 - Binary Trees
- Problem 5 - Binary Search Trees
- Problem 6 - Iterators

Problem 1 - Sorting

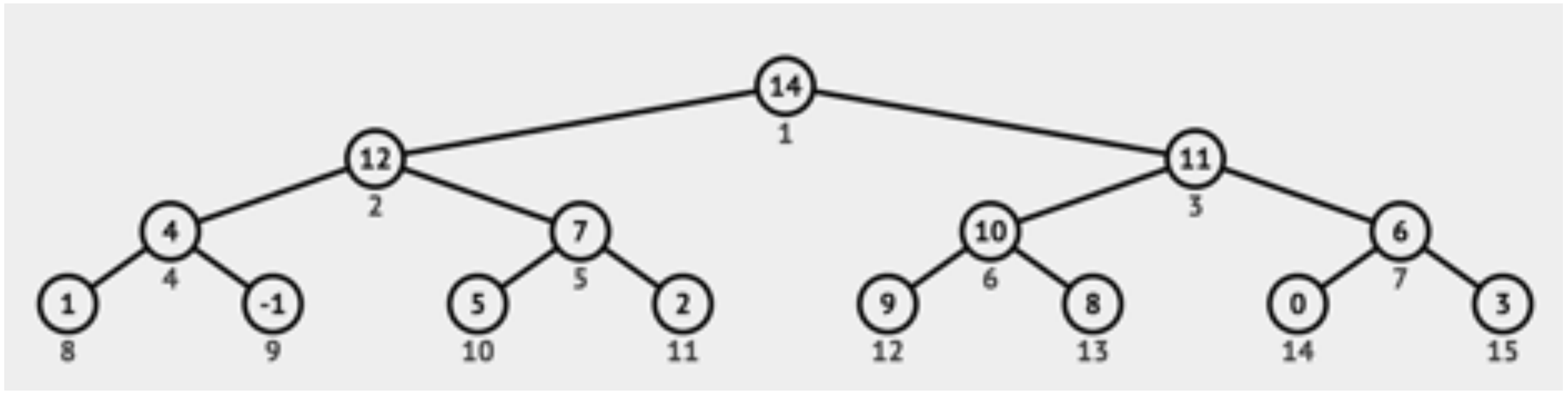
- In the next slide, you can find a table whose first row (last column 0) contains an array of 18 unsorted numbers between 1 and 50. The last row (last column 6) contains the numbers in sorted order. The other rows show the array in some intermediate state during one of these five sorting algorithms:
    - 1-Selection sort
    - 2-Insertion sort
    - 3-Mergesort
    - 4-Quicksort (one partition only)
    - 5-Heapsort
- Match each algorithm with the right row by writing its number (1-5) in the last column.

Problem 1 - Sorting

12	11	35	46	20	43	42	47	44	32	16	10	40	18	41	21	28	15	**0**
11	12	20	35	42	43	46	47	44	32	16	10	40	18	41	21	28	15	
12	11	10	15	20	43	42	47	44	32	16	35	40	18	41	21	28	46	
10	11	12	15	16	43	42	47	44	32	20	35	40	18	41	21	28	46	
43	32	42	28	20	40	41	21	15	11	16	10	35	18	12	44	46	47	
11	12	20	35	46	43	42	47	44	32	16	10	40	18	41	21	28	15	
10	11	12	15	16	18	20	21	28	32	35	40	41	42	43	44	46	47	**6**

Problem 2 - Heaps

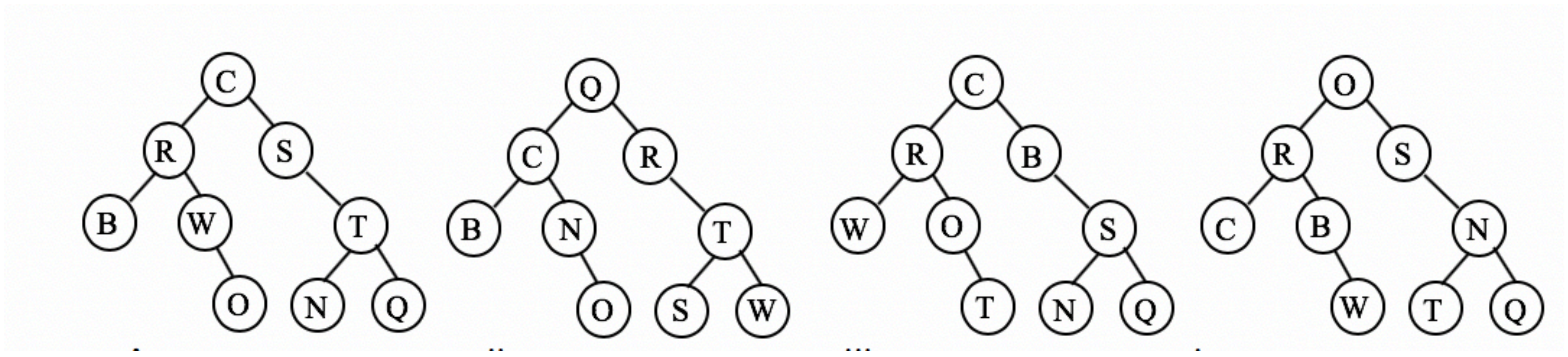
- Consider the following max-heap:



- Draw the heap after you insert key 13.
- Suppose you delete the maximum key from the original heap. Draw the heap after you delete 14.

Problem 3 - Tree Traversals

- Circle the correct binary tree(s) that would produce both of the following traversals:
  - Pre-order: C R B W O S T N Q
  - In-order: B R W O C S N T Q



Problem 4 - Binary Trees

- You are extending the functionality of the `BinaryTree` class that represents binary trees with the goal of counting the number of leaves. Remember that `BinaryTree` has a pointer to a `root` Node and the inner class Node has two pointers, `left` and `right` to the root nodes that correspond to its left and right subtrees.
- You are given the following public method:

```
public int sumLeafTree()
  return sumLeafTree(root);
}
```

- Please fill in the body of the following recursive method

```
private int sumLeafTree(Node x){…}
```

Problem 5 - Binary Search Trees

- You are extending the functionality of the BST class that represents binary search trees with the goal of counting the number of nodes whose keys fall within a given `[low, high]` range. That is you want to count how many nodes have keys that are equal or larger than `low` and equal or smaller than `high`. Remember that BST has a pointer to a `root` Node and the inner class Node has two pointers, `left` and `left` to the root nodes that correspond to its left and right subtrees and a `Comparable Key key` (please ignore the value).
- You are given the following public method:

```
public int countRange(Key low, Key high)
  return countRange(root, Key low, Key high);
}
```

- Please fill in the body of the following recursive method

```
private int countRange(Node x, Key low, Key high){…}
```

Problem 6 - Iterators

- A programmer discovers that they frequently need only the odd numbers in an arraylist of integers. As a result, they decided to write a class `OddIterator` that implements the `Iterator` interface. Please help them implement the constructor and the `hasNext()` and `next()` methods so that they can retrieve the odd values, one at a time. For example, if the arraylist contains the elements [7, 4, 1, 3, 0], the iterator should return the values 7, 1, and 3. You are given the following public class:

```
public class OddIterator implements Iterator<Integer> {

// The array whose odd values are to be enumerated
private ArrayList<Integer> myArrayList;

//any other instance variables you might need

//An iterator over the odd values of myArrayList
public OddIterator(ArrayList<Integer> myArrayList){…}

//runs in O(n) time
public boolean hasNext(){…}

//runs in O(1) time
public Integer next(){…}
}
```

Answers

- Solution to Problem 1 - Sorting
- Solution to Problem 2 - Heaps
- Solution to Problem 3 - Tree traversals
- Solution to Problem 4 - Binary Trees
- Solution to Problem 5 - Binary Search Trees
- Solution to Problem 6 - Iterators

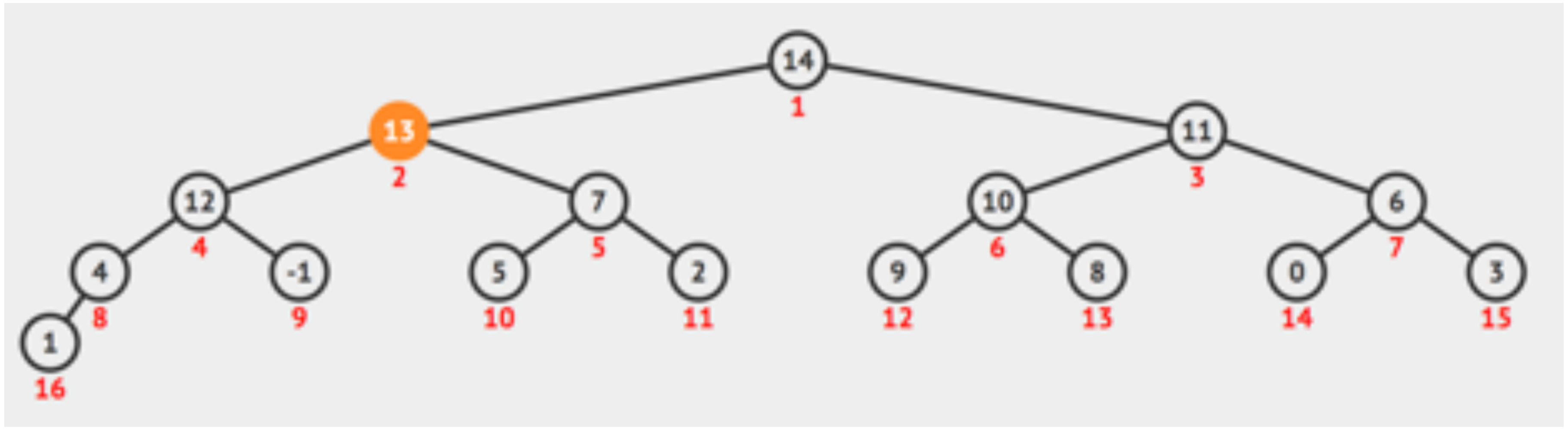
Solution to Problem 1 - Sorting

- 0-Starting point
- 1-Selection sort
- 2-Insertion sort
- 3-Mergesort
- 4-Quicksort (one partition only)
- 5-Heapsort
- 6-Final sorted result

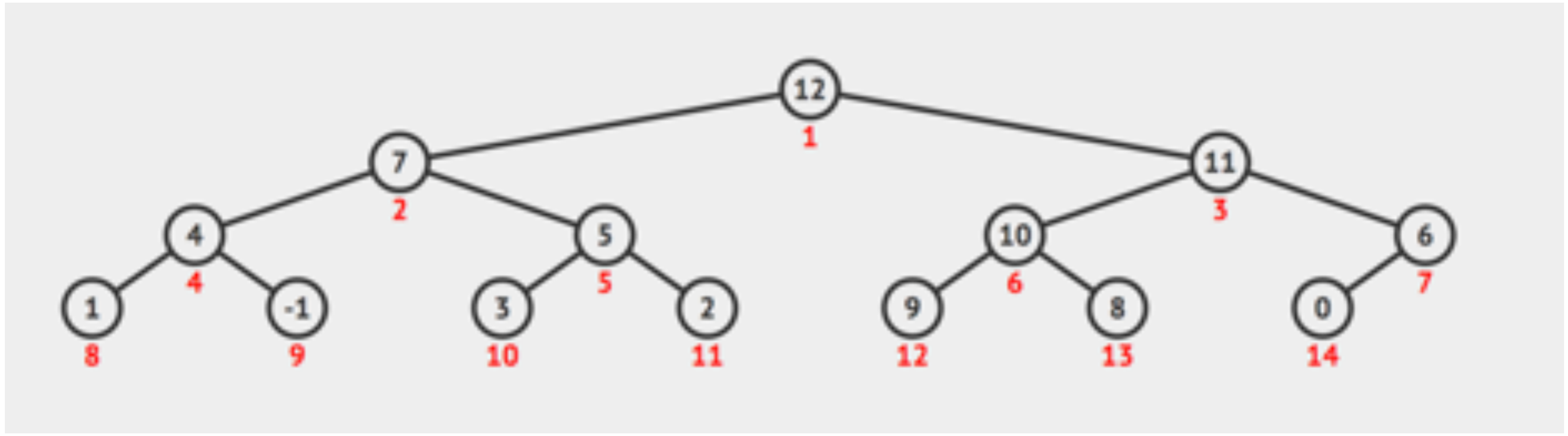
12	11	35	46	20	43	42	47	44	32	16	10	40	18	41	21	28	15	**0**
11	12	20	35	42	43	46	47	44	32	16	10	40	18	41	21	28	15	**2**
12	11	10	15	20	43	42	47	44	32	16	35	40	18	41	21	28	46	**4**
10	11	12	15	16	43	42	47	44	32	20	35	40	18	41	21	28	46	**1**
43	32	42	28	20	40	41	21	15	11	16	10	35	18	12	44	46	47	**5**
11	12	20	35	46	43	42	47	44	32	16	10	40	18	41	21	28	15	**3**
10	11	12	15	16	18	20	21	28	32	35	40	41	42	43	44	46	47	**6**

Solution to Problem 2 - Heaps

- Insert key 13:

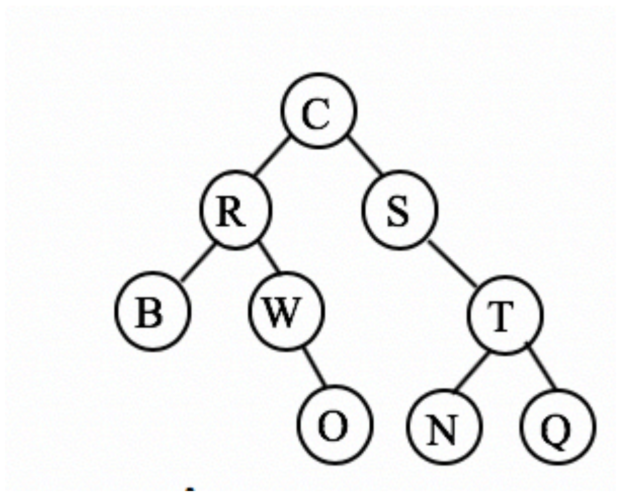


- Delete max-key (14):



Solution to Problem 3 - Tree traversals

- Pre-order: C R B W O S T N Q
- In-order: B R W O C S N T Q



Solution to Problem 4 - Binary Trees

```
private int sumLeafTree(Node x){
    if (x == null){
        return 0;
    }
    else if (x.left == null && x.right == null){
          return 1;
    }
    else{
        return sumLeafTree(x.left) + sumLeafTree(x.right);
    }
}
```

Solution to Problem 5 - Binary Search Trees

```
private int countRange(Node x, Key low, Key high){
    if (x == null){
        return 0;
    }
    if (x.key.compareTo(low)>=0 && x.key.compareTo(high)<=0){
        return 1 + countRange(x.left, low, high) + countRange(x.right, low, high);
    }
    else if (x.key.compareTo(low)<0){
        return countRange(x.right, low, high);
    }
    else{
        return countRange(x.left, low, high);
    }
}
```

Solution to Problem 6 - Iterators

```
public class OddIterator implements Iterator<Integer> {

    private ArrayList<Integer> myArrayList;
    int counter;

    public OddIterator(ArrayList<Integer> myArrayList){
        this.myArrayList = myArrayList;
        counter = 0;
    }

    //runs in O(n) time
    public boolean hasNext(){
        for (int i=counter; i<myArrayList.size(); i++){
            if(myArrayList.get(i)%2 == 1){
                counter = i;
                return true;
            }
        }
        return false;
    }


    //runs in O(1) time
    public Integer next(){
        return myArrayList.get(counter++);
    }
}
```

GOOD LUCK! YOU CAN DO THIS!