GRAPHS

CS062

DATA STRUCTURES AND ADVANCED PROGRAMMING

24: Summary

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she/her/hers

Information

- Exam: Tuesday Dec 12th, 2-5pm in the same room we always meet.
- You can bring two hand-written (ok *hand-written* on tablets and then printed) sheets of papers (i.e. four pages).
- Cumulative with a bigger emphasis on topics we covered since midterm 2.
- Review midterm 1 and 2 and quizzes. Use the practice questions in this presentation.

Java Basics

- Chapter 1.1 (Pages 8–35).
- Chapter 1.2 (Pages 64–77, 84–88, 96–99, 107).
- Quick overview of Java tutorials.
  - https://docs.oracle.com/javase/tutorial/java/
- In general, review the basics of OOP and of Java so that you are comfortable reading and writing code.

Analysis of Algorithms

- Chapter 1.4 (Pages 172-205).
- Experimental analysis including doubling hypothesis.
- Mathematical analysis including reviewing (not memorizing) useful approximations of sums.
  - Use midterm review slides for practice.
- Order of growth classifications.
- Review of running time of operations on array lists, linked lists, stacks and queues.

ArrayLists

- Chapter 1.3 (Pages 136-137).
- Textbook API and code.
  - https://github.com/pomonacs622021fa/LectureCode/blob/main/Lecture6/ArrayList.java
- Java Oracle API https://github.com/pomonacs622023fa/code/blob/main/Lecture6/ArrayList.java
- Amortized and worst-case time analysis.

Singly Linked Lists

- Chapter 1.3 (Pages 142-146).
- Textbook API and code.
  - https://github.com/pomonacs622023fa/code/blob/main/Lecture7/SinglyLinkedList.java
- Worst-case time analysis for standard operations.

Doubly Linked Lists

- Chapter 1.3 (Pages 126-157).
- Textbook API and code.
  - https://github.com/pomonacs622023fa/code/blob/main/Lecture8/DoublyLinkedList.java
- Java Oracle API.
  - https://docs.oracle.com/javase/7/docs/api/java/util/LinkedList.html
- Worst-case time analysis for standard operations.

Stacks, Queues, and Iterators

- Chapter 1.3 (Pages 142-146).
- Code for alternative implementations
  - https://github.com/pomonacs622023fa/code/tree/main/Lecture9
- Java Oracle Iterator and Iterable.
  - https://docs.oracle.com/javase/8/docs/api/java/util/Iterator.html
  - https://docs.oracle.com/javase/8/docs/api/java/lang/Iterable.html
- Worst-case time analysis for standard operations based on the underlying implementation

Comparators

- Comparable vs Comparator interface.
  - https://docs.oracle.com/javase/8/docs/api/java/lang/Comparable.html
  - https://docs.oracle.com/javase/8/docs/api/java/util/Comparator.html
  - https://github.com/pomonacs622023fa/code/tree/main/Lecture10_11

Sorting

- Chapter 2 (Pages 244-296).
- Selection sort and Insertion sort.
  - https://github.com/pomonacs622023fa/code/blob/main/Lecture10_11/BasicSorting.java
- Mergesort.
  - https://github.com/pomonacs622023fa/code/blob/main/Lecture12/MergeSort.java
- Quicksort.
  - https://github.com/pomonacs622023fa/code/blob/main/Lecture13/QuickSort.java
  - We have seen Lomuto's partition scheme
- Know how to apply them, best and worst case running times, in-place or not, stability

Binary Trees and Heaps

- Definitions, basic properties, and traversals.
- Binary Heaps and operations.

Priority Queues and Heapsort

- Chapter 2.4 (Pages 308-325), 2.5 (336-344).
- Different implementations along with complexities.
- Heapsort. Know how to apply, running time analysis.
- Textbook code.
  - https://algs4.cs.princeton.edu/code/edu/princeton/cs/algs4/MaxPQ.java.html

What you need to remember about sorting

	In place	Stable	Best	Average	Worst	Remarks
Selection	X		n^2	n^2	n^2	n exchanges
Insertion	X	X	n	n^2	n^2	Use for small arrays or partially ordered
Merge		X	$n \log n$	$n \log n$	$n \log n$	Guaranteed performance; stable
Quick	X		$n \log n$	$n \log n$	n^2	$n \log n$ probabilistic guarantee; fastest in practice
Heap	X		$n \log n$	$n \log n$	$n \log n$	$n \log n$ guarantee; in place

Dictionaries and Binary Search Trees

- Chapter 3.1 (Pages 362-386).
- Different implementations along with complexities.
- Chapter 3.2 (Pages 396-414).
- Textbook code.
  - https://algs4.cs.princeton.edu/code/edu/princeton/cs/algs4/BST.java.html
- Addition, Search, Deletion.

2-3 Search Trees

- Chapter 3.3 (Pages 424-291).
- Definitions, Search, Insertion, Construction.
- Performance.

Left-leaning red-black trees

- Chapter 3.3 (Pages 292-447).
- Definitions, Operations, Insertion.
- Performance.

Hash tables

- Chapter 3.3 (Pages 458-477).
- Hashing, separate chaining, open addressing.

Summary for Dictionary operations

	Worst case			Average case		
	Search	Insert	Delete	Search	Insert	Delete
Sequential search (unordered list)	n	n	n	n	n	n
Binary search (ordered array)	$\log n$	n	n	$\log n$	n	n
BST	n	n	n	$\log n$	$\log n$	$\log n$
2-3 search tree	$\log n$	$\log n$	$\log n$	$\log n$	$\log n$	$\log n$
Red-black BSTs	$\log n$	$\log n$	$\log n$	$\log n$	$\log n$	$\log n$
Separate chaining	n	n	n	1	1	1
Linear probing	n	n	n	1	1	1

Undirected Graphs

- Chapter 4.1 (Pages 515-556).
- Definitions, representations, APIs.
- BFS.
- DFS.
- Textbook code.
  - https://algs4.cs.princeton.edu/code/edu/princeton/cs/algs4/Graph.java.html
  - https://algs4.cs.princeton.edu/code/edu/princeton/cs/algs4/DepthFirstSearch.java.html
  - https://algs4.cs.princeton.edu/code/edu/princeton/cs/algs4/BreadthFirstPaths.java.html

Directed Graphs

- Chapter 4.2 (Pages 566-594).
- Definitions, representations, APIs.
- BFS.
- DFS.
- Textbook code.
  - https://algs4.cs.princeton.edu/code/edu/princeton/cs/algs4/Digraph.java.html
  - https://algs4.cs.princeton.edu/code/edu/princeton/cs/algs4/DirectedDFS.java.html

Shortest Paths

- Chapter 4.4 (Pages 638-676).
- Dijkstra's algorithm.
- Textbook code.
  - https://algs4.cs.princeton.edu/code/edu/princeton/cs/algs4/EdgeWeightedDigraph.java.html
  - https://algs4.cs.princeton.edu/code/edu/princeton/cs/algs4/DirectedEdge.java.html
  - https://algs4.cs.princeton.edu/code/edu/princeton/cs/algs4/DijkstraSP.java.html

Problem 1: Balanced Binary Search Trees

- a. Starting with an empty 2-3 search tree, what is the resulting 2-3 search tree after you insert the keys 2, 7, 17, 1, 90, 3, 36, 47?
- b. Starting with an empty LLRB search tree, what is the resulting LLRB search tree after you insert the keys 15, 6, 23, 4, 7, 5, 50, 71?

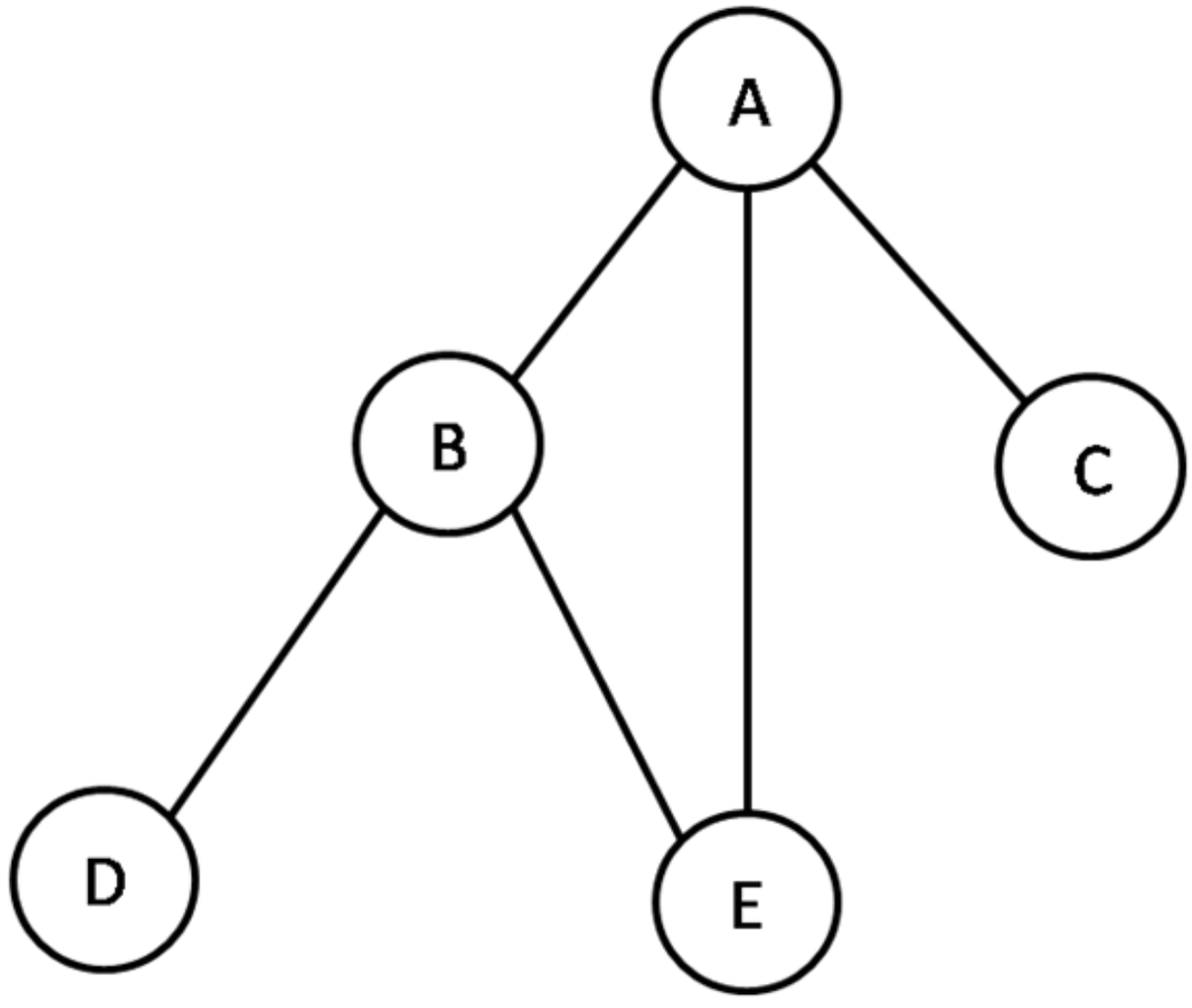
Problem 2: Hashtables

- Consider inserting the keys 25, 17, 12, 26, 27, 5, 9, 29, 11, 23 into a hash table of size $m = 11$ using the hash function $h(k) = k \,\%\, m$. For each of the following questions, fill in the following hash table:
  - a. using open addressing and linear probing with $h(k, i) = (h(k) + i) \,\%\, m$.
  - b. using open addressing and quadratic probing with $h(k, i) = (h(k) + i^2) \,\%\, m$.
  - c. using external chaining.

0	1	2	3	4	5	6	7	8	9	10

Problem 3: Traversing Graphs

- Show the adjacency matrix and adjacency list representations of the undirected graph above.
- Run a.recursive and b.iterative DFS and c.BFS starting at vertex A assuming adjacent vertices are returned in lexicographic order. Fill in the table below:

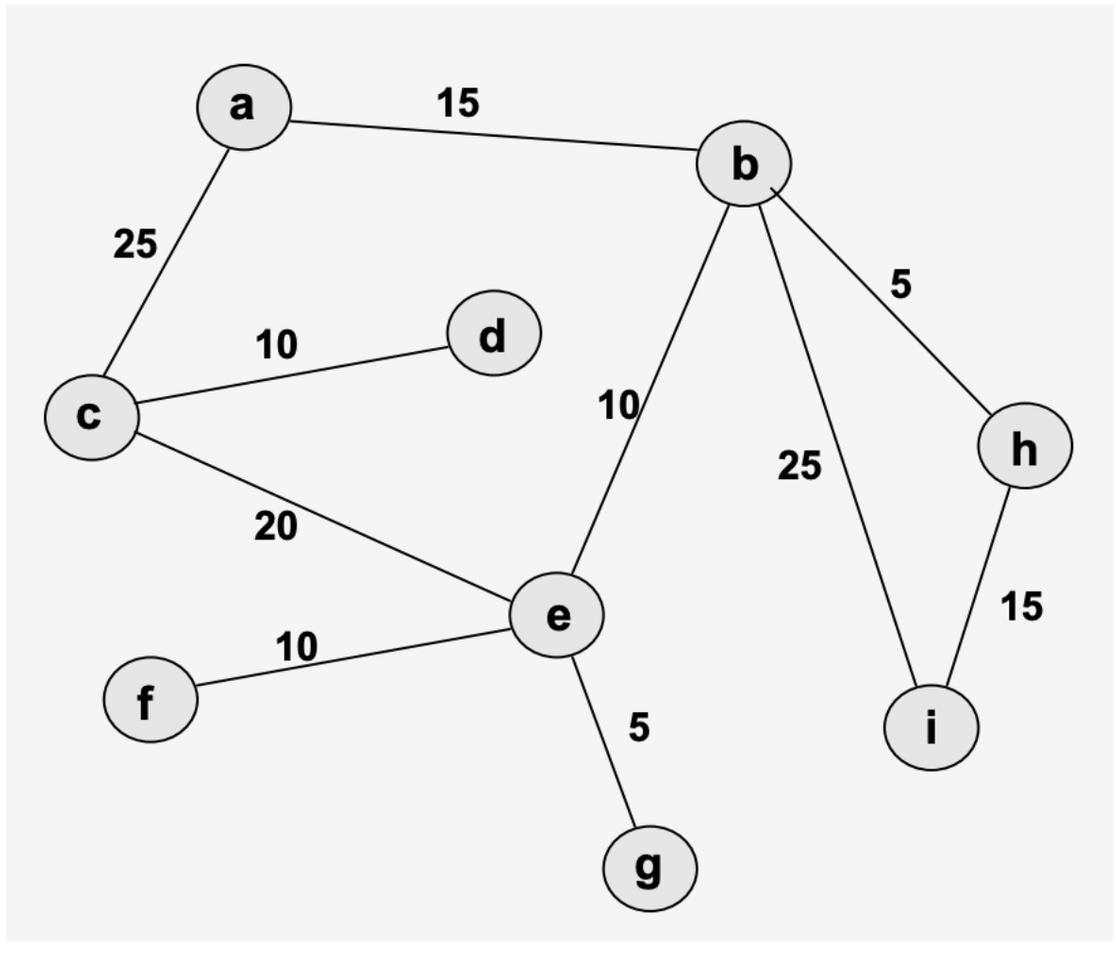


v	marked	distTo	edgeTo
A			
B			
C			
D			
E			

Problem 4: Shortest Paths

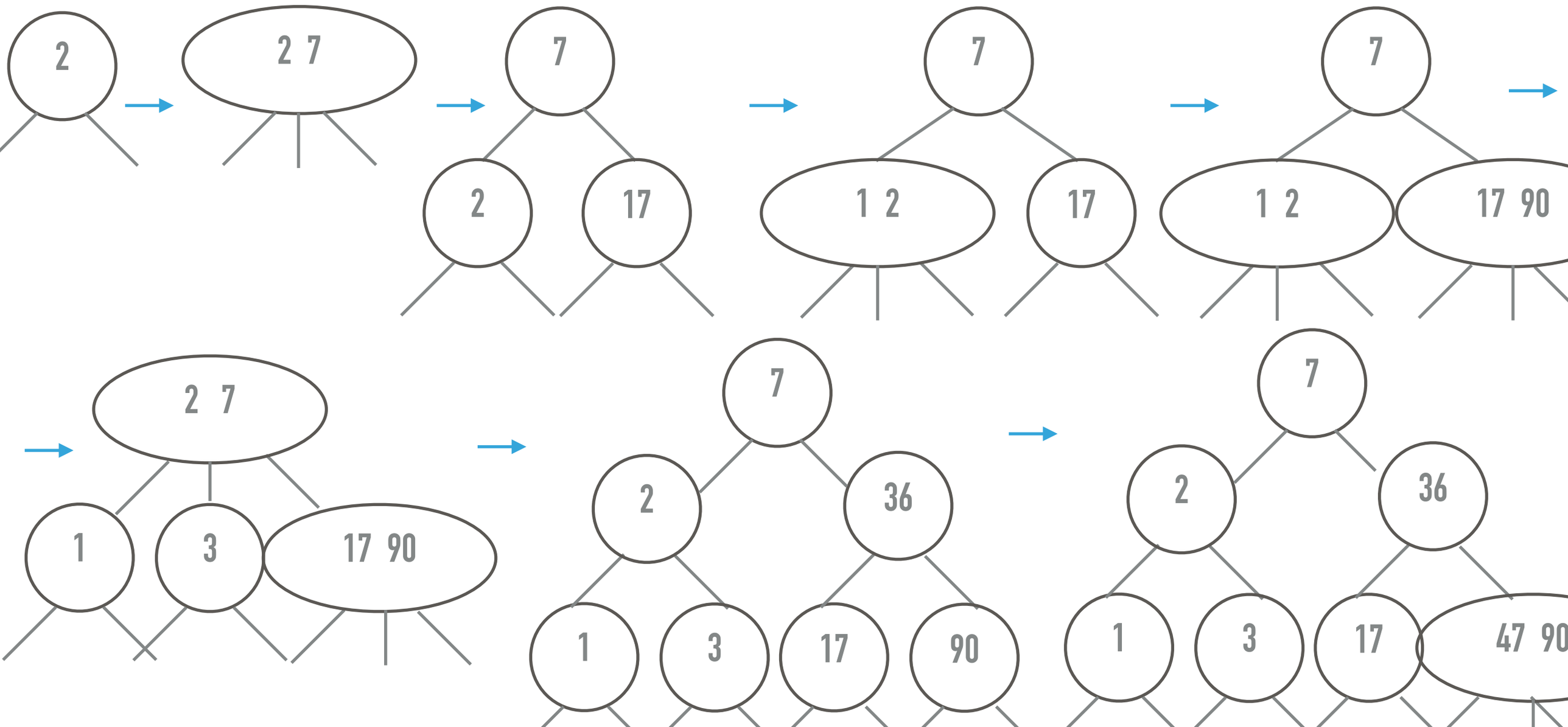
- Run Dijkstra's algorithm on this graph, starting at vertex a. Fill in the resulting distTo[] and edgeTo[] arrays below. In the edgeTo[] column, please indicate the last edge in the shortest path from a to every other vertex and mark the shortest path tree.

v	distTo	edgeTo
a		
b		
c		
d		
e		
f		
g		
h		
i		



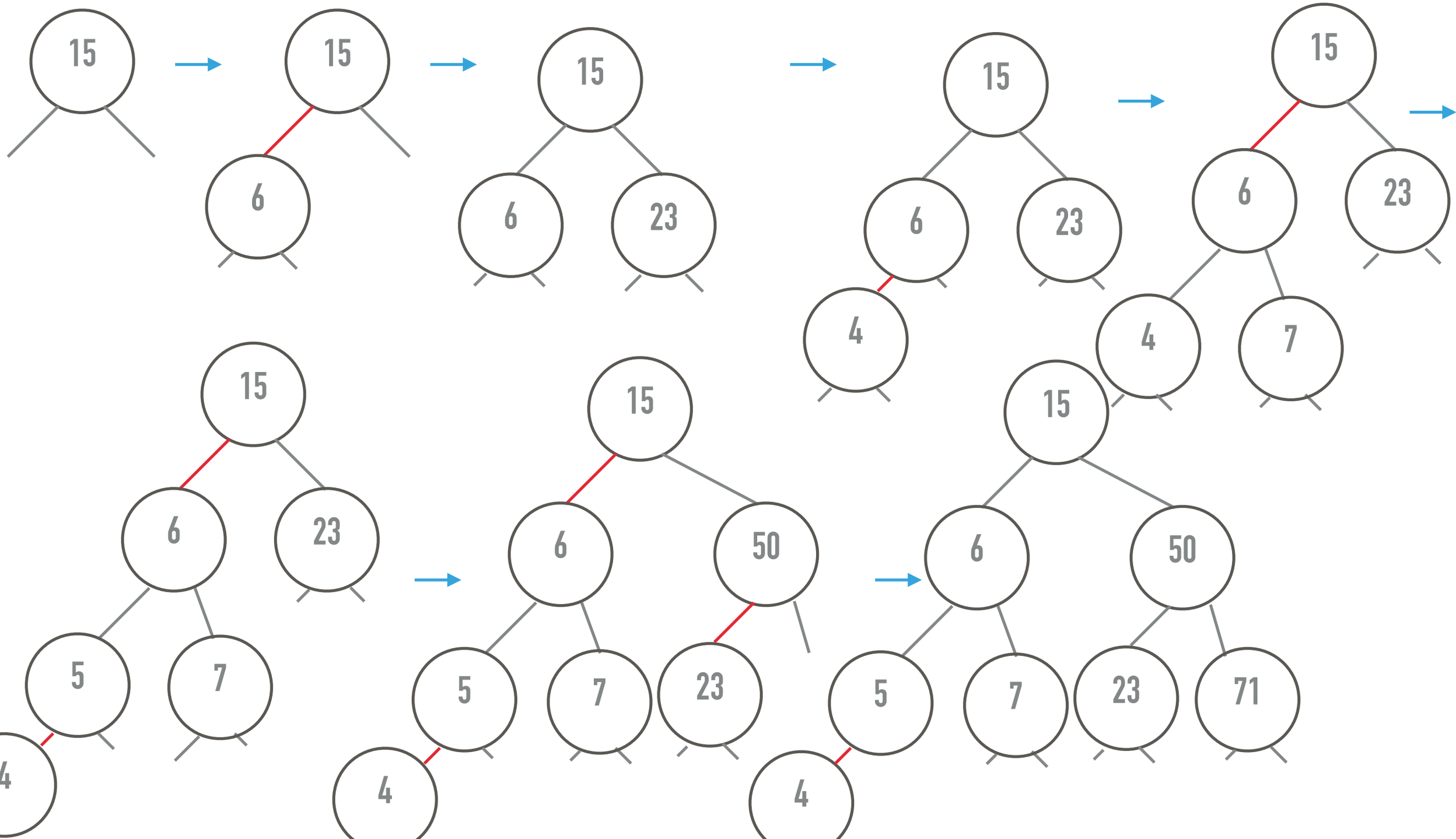
Solution to Problem 1a: Balanced Binary Search Trees

- Starting with an empty 2-3 search tree, what is the resulting 2-3 search tree after you insert the keys 2, 7, 17, 1, 90, 3, 36, 47?



Solution to Problem 1b: Balanced Binary Search Trees

- Starting with an empty LLRB search tree, what is the resulting LLRB search tree after you insert the keys 15, 6, 23, 4, 7, 5, 50, 71?



Solution to Problem 2a: Hashtables

- Consider inserting the keys 25, 17, 12, 26, 27, 5, 9, 29, 11, 23 into a hash table of size $m = 11$ using the hash function $h(k) = k \,\%\, m$. For each of the following questions, fill in the following hash table using open addressing and linear probing with $h(k, i) = (h(k) + i) \,\%\, m$.

11	12	23	25	26	27	17	5	29	9	
0	1	2	3	4	5	6	7	8	9	10

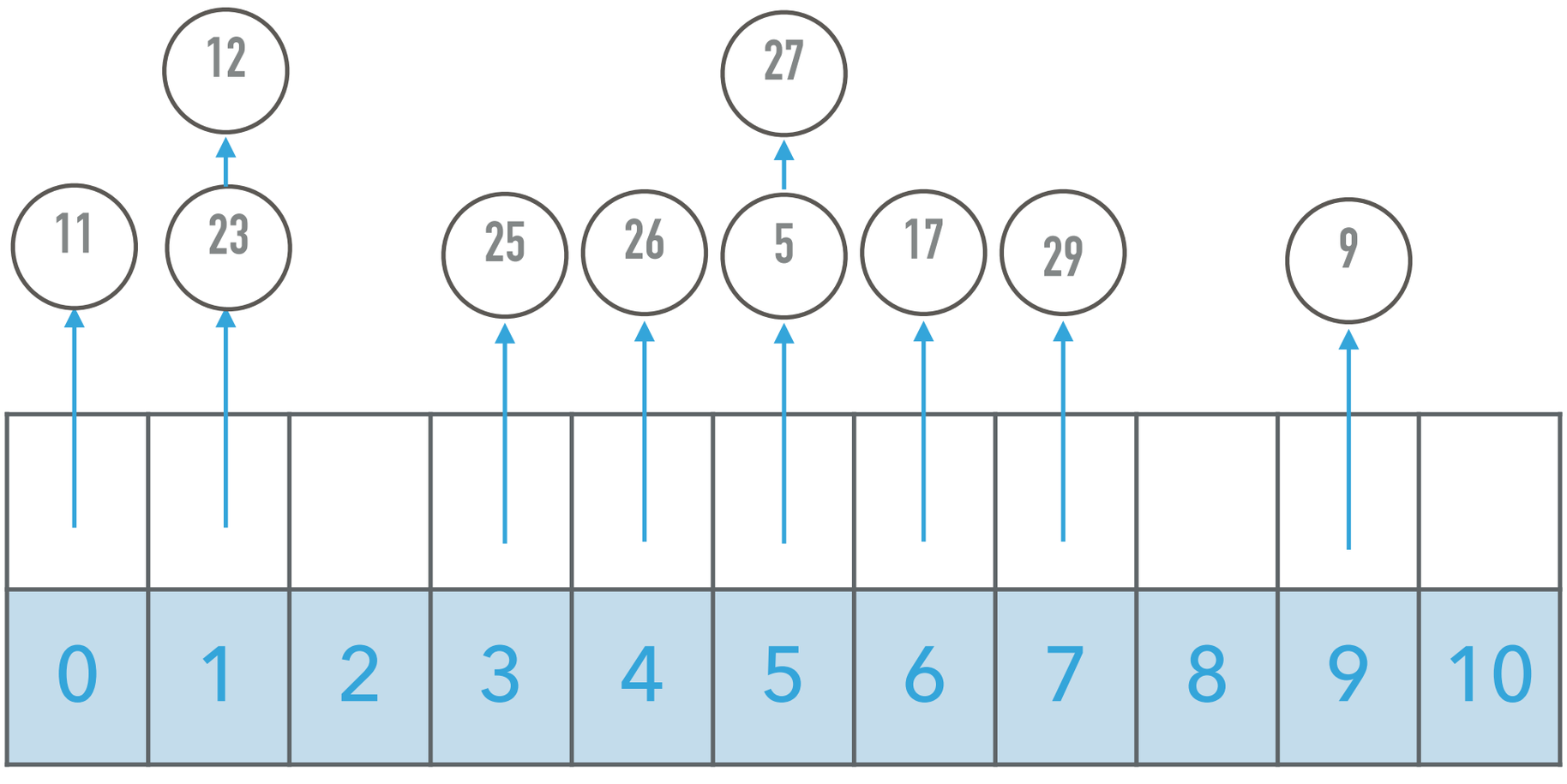
Solution to Problem 2b: Hashtables

- Consider inserting the keys 25, 17, 12, 26, 27, 5, 9, 29, 11, 23 into a hash table of size $m = 11$ using the hash function $h(k) = k \,\%\, m$. For each of the following questions, fill in the following hash table using open addressing and linear probing with $h(k, i) = (h(k) + i^2) \,\%\, m$.

11	12	23	25	26	27	17	29		5	9
0	1	2	3	4	5	6	7	8	9	10

Solution to Problem 2c: Hashtables

- Consider inserting the keys 25, 17, 12, 26, 27, 5, 9, 29, 11, 23 into a hash table of size $m = 11$ using the hash function $h(k) = k \,\%\, m$. For each of the following questions, fill in the following hash table using external chaining.



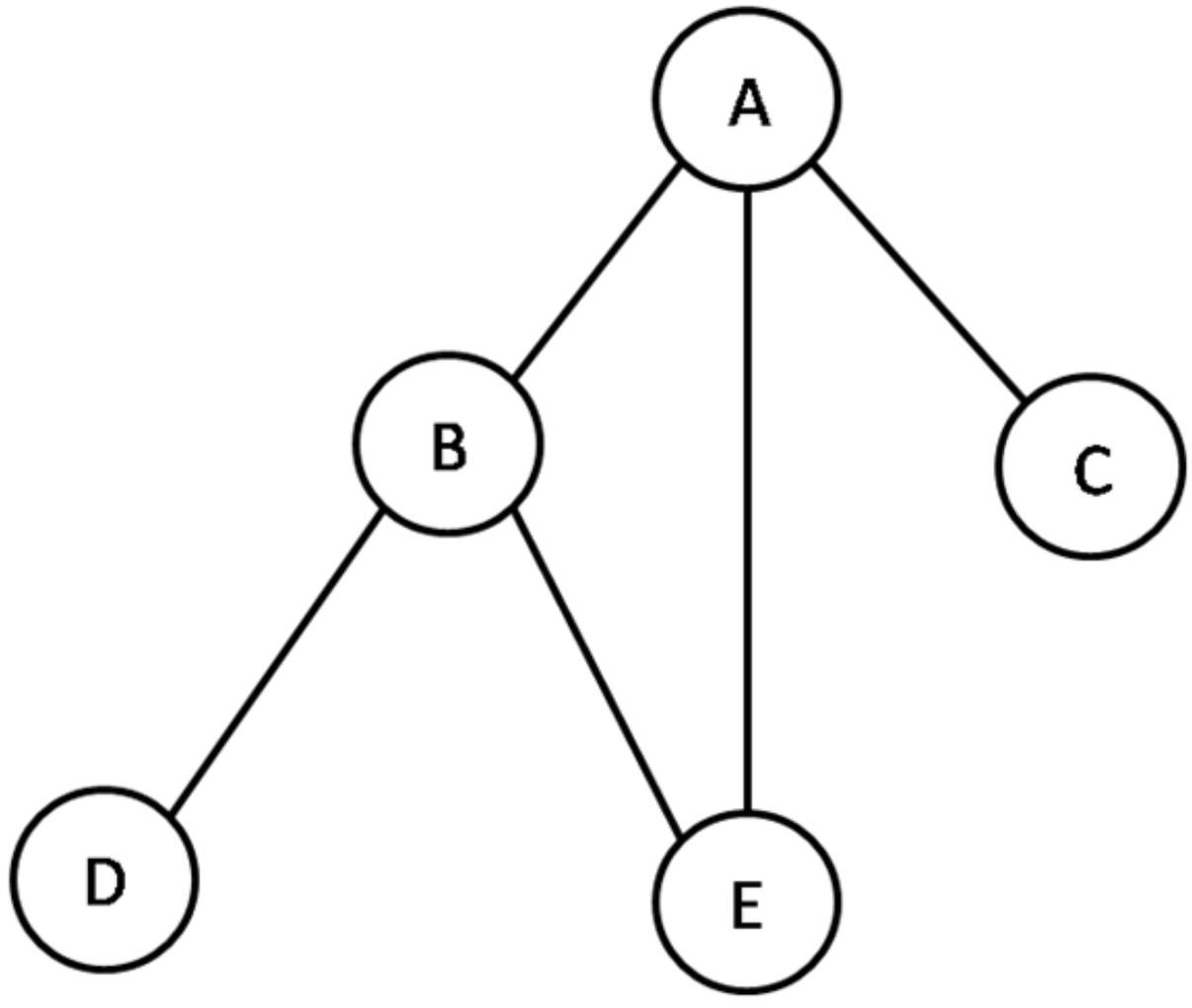
Solution to Problem 3a: Traversing Graphs

- Show the adjacency matrix and adjacency list representations of the undirected graph above.

- Run a.recursive DFS assuming adjacent vertices are returned in lexicographic order.

- Order of visit: A, B, D, E, C

v	marked	edgeTo
A	**T**	**-**
B	T	A
C	T	A
D	T	B
E	T	B



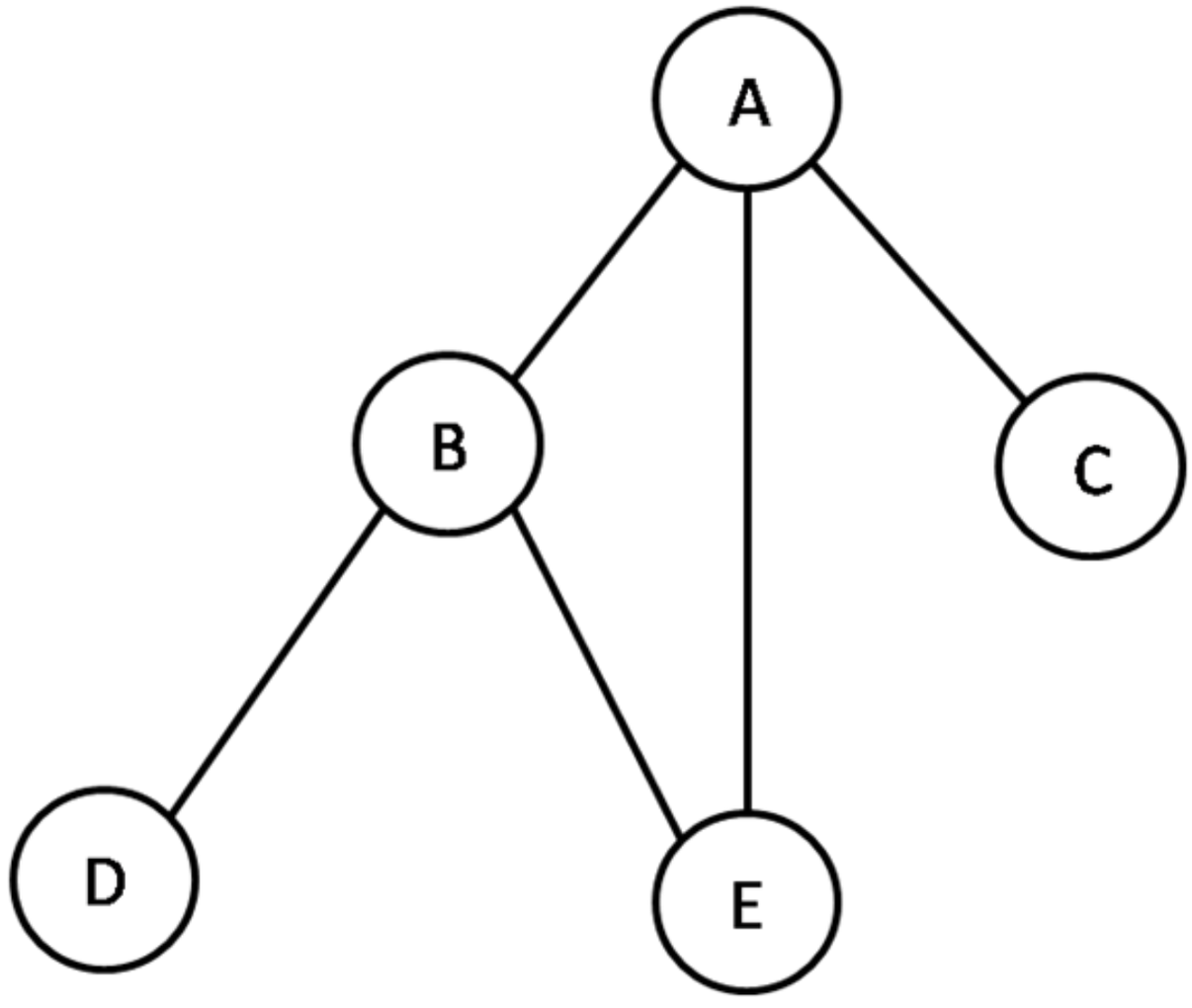
Solution to Problem 3b: Traversing Graphs

- Show the adjacency matrix and adjacency list representations of the undirected graph above.

- Run b. iterative DFS assuming adjacent vertices are returned in lexicographic order.

- Order of visit: A, E, B, D, C

v	marked	edgeTo
A	T	-
B	T	E
C	T	A
D	T	B
E	T	A



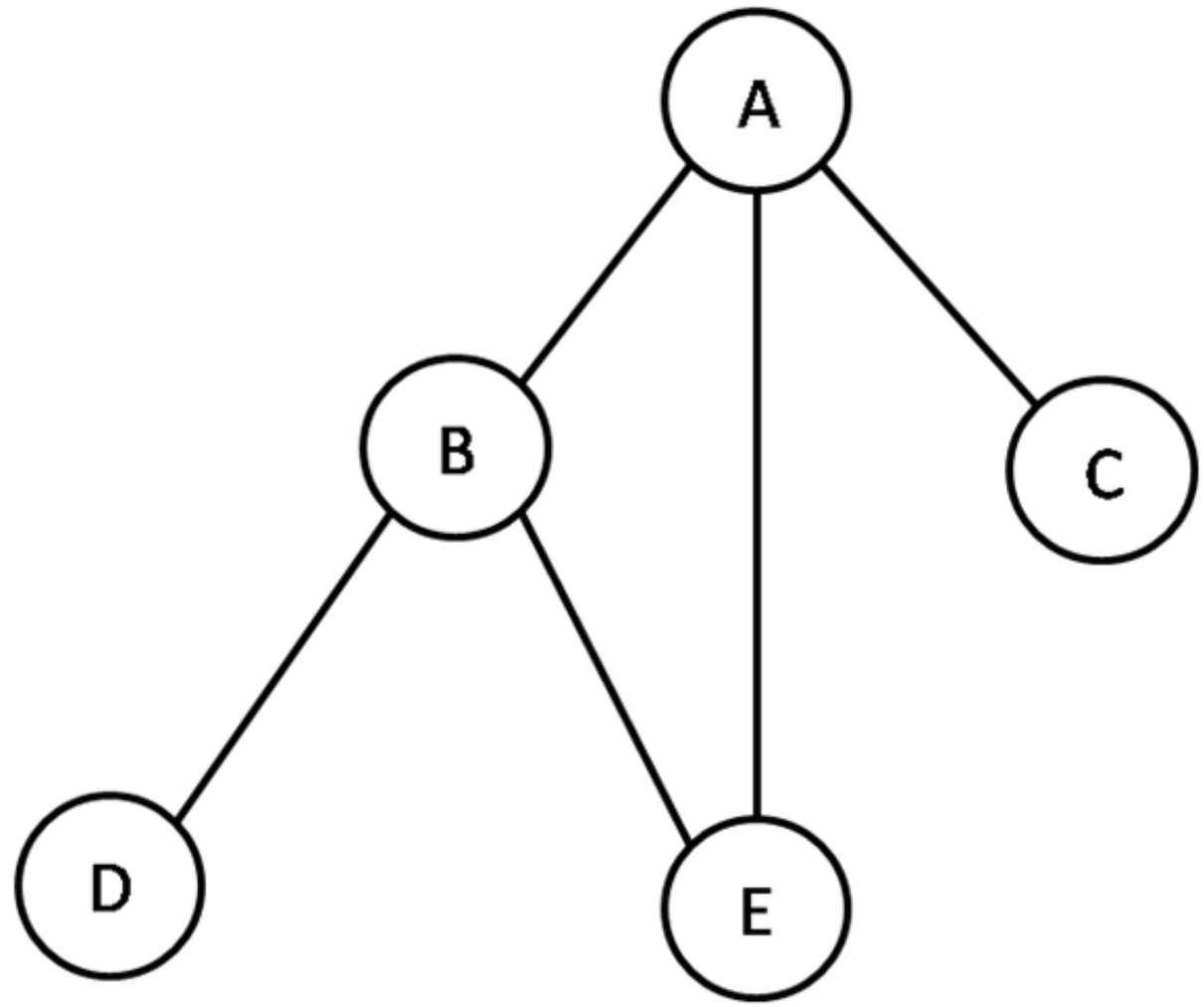
Solution to Problem 3c: Traversing Graphs

- Show the adjacency matrix and adjacency list representations of the undirected graph above.

- Run c. BFS assuming adjacent vertices are returned in lexicographic order.

- Order of visit: A, B, C, E, D

v	marked	distTo	edgeTo
A	T	0	-
B	T	1	A
C	T	1	A
D	T	2	B
E	T	1	A



Solution to Problem 4: Shortest Paths

- Run Dijkstra's algorithm on this graph, starting at vertex a. Fill in the resulting distTo[] and edgeTo[] arrays below. In the edgeTo[] column, please indicate the last edge in the shortest path from a to every other vertex and mark the shortest path tree.

v	distTo	edgeTo
a	0	-
b	15	a
c	25	a
d	35	c
e	25	b
f	35	e
g	30	e
h	20	b
i	35	h