MINIMUM SPANNING TREES

David Kauchak
CS62– Fall 2020

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Quiz!

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Admin

Assignment 10 due Tuesday

Tuesday's class
- Recap
- Bring questions!

Final exam 12/4, 2-5pm (PST)
- More details on Tuesday

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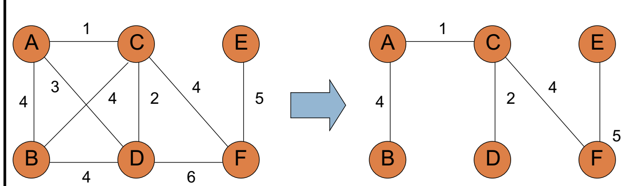
Minimum spanning trees (MST)

The lowest weight set of edges that connects all vertices of an undirected graph with positive weights



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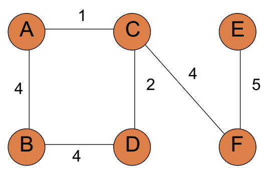
MSTs

Can an MST have a cycle?



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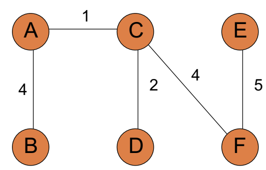

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Applications?

Connectivity

- Networks (e.g. communications)
- Circuit design/wiring

hub/spoke models (e.g. flights, transportation)

approximate solutions to some "hard" problems

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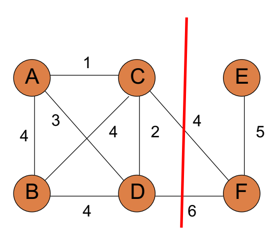
Cuts

A cut is a partitioning of the vertices into two sets S and V-S

An edge "crosses" the cut if it connects a vertex u∈V and v∈V-S



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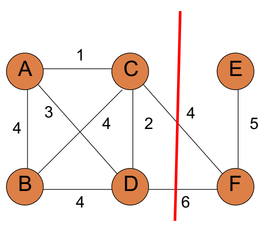
Minimum cut property

Given a partition S, let edge e be the minimum cost edge that **crosses** the partition. *Every* minimum spanning tree contains edge e.



Why?

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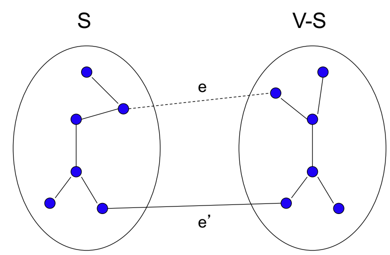

Consider an MST with edge e' that is not the minimum edge

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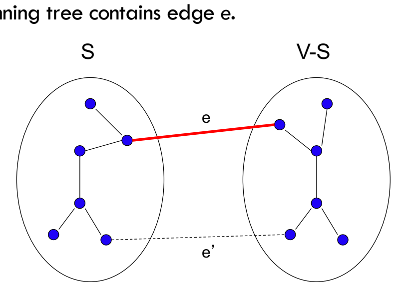

Using e instead of e', still connects the graph, but produces a tree with smaller weights

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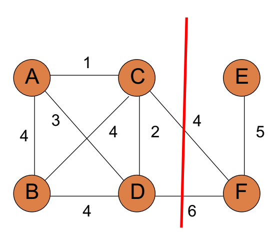
Minimum cut property

If the minimum cost edge that **crosses** the partition is not unique, then *some* minimum spanning tree contains edge e.



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Kruskal's algorithm

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Kruskals:
- Sort edges by increasing weight
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  - check if adding edge to MST creates a cycle
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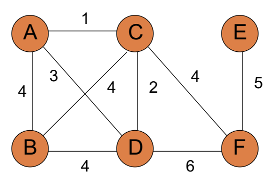
Kruskal's algorithm

Add smallest edge that doesn't create a cycle



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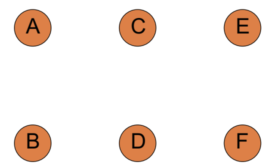
MST



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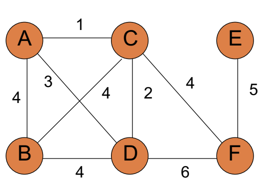

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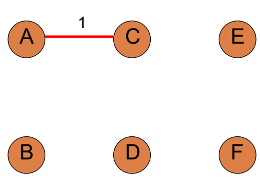

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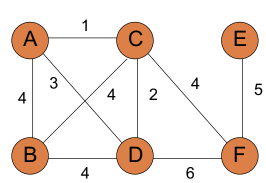

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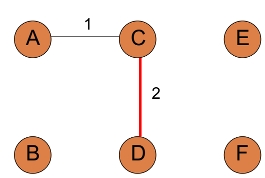

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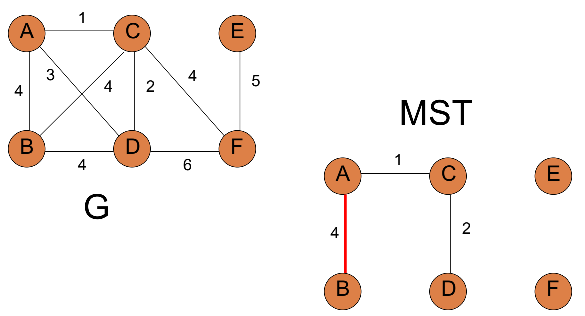

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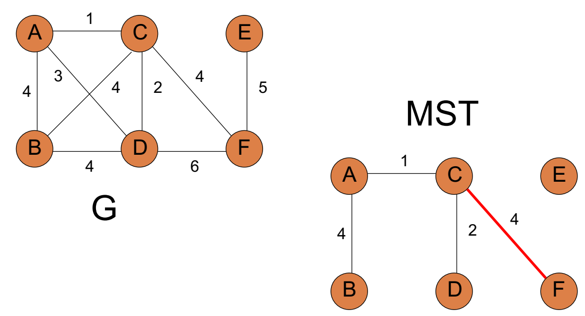

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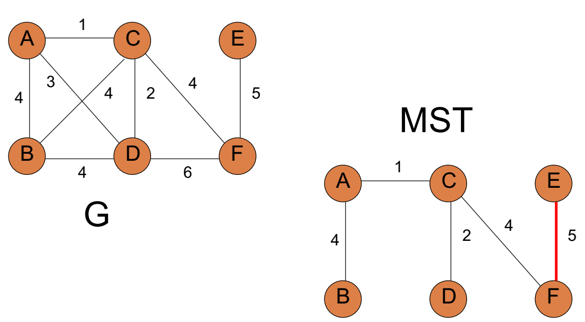

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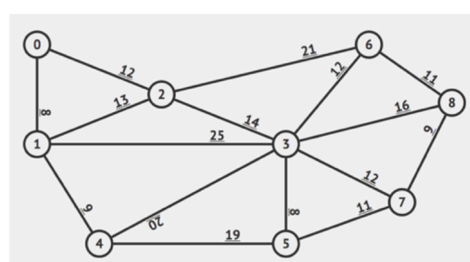
Practice



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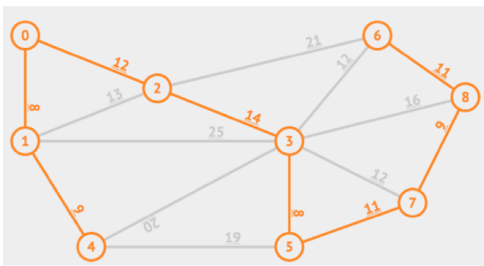
Solution



Sum = 8 + 8 + 9 + 9 + 11 + 11 + 12 + 14 = 82

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Why does Kruskal's work?

Never adds an edge that creates a cycle

Therefore, always adds lowest cost edge to connect two connected components. By min cut property, that edge must be part of the MST

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Run-time?

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Run-time: O(V+E) to do a DFS/BFS

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Overall run-time?

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Run-time: $O(VE+E^2)$ do this E times

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Kruskal's details

We can do better!

Uses a data structure called "disjoint set" to efficiently check whether adding an edge creates a cycle

Run-time: O(E log E) (bounded by the sort)

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