GRAPHS

CS062

DATA STRUCTURES AND ADVANCED PROGRAMMING

33: Intro to Undirected Graphs

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LECTURES

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LABS

Lecture 33: Intro to Undirected Graphs

- Undirected Graphs

Some slides adopted from Algorithms 4th Edition or COS226

Graphs

- Graphs: mathematical abstractions that model a set of vertices connected pairwise by edges.
- Why study graphs?

- Thousands of practical applications.
- Hundreds of graph algorithms.
- Interesting and widely applicable abstraction.
- Core branch of computer science and discrete math.

Example: (Fake) LA subway map

- Vertices: stations.
  Edges: route.

- Source: LA Weekly

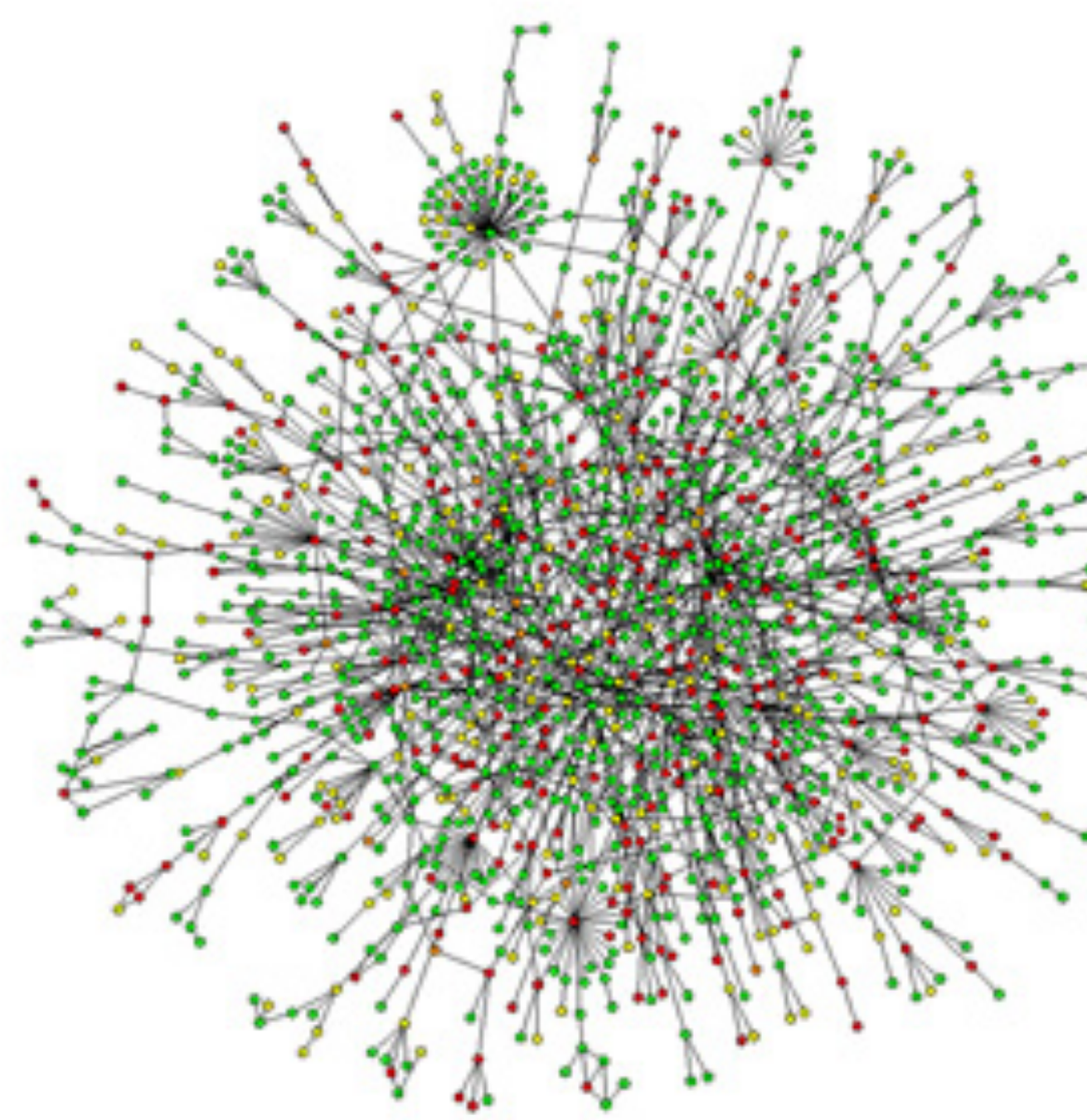
Example: Social networks

- Vertices: people. Edges: “friendships”.

Source: Paul Butler

Example: Protein-protein networks

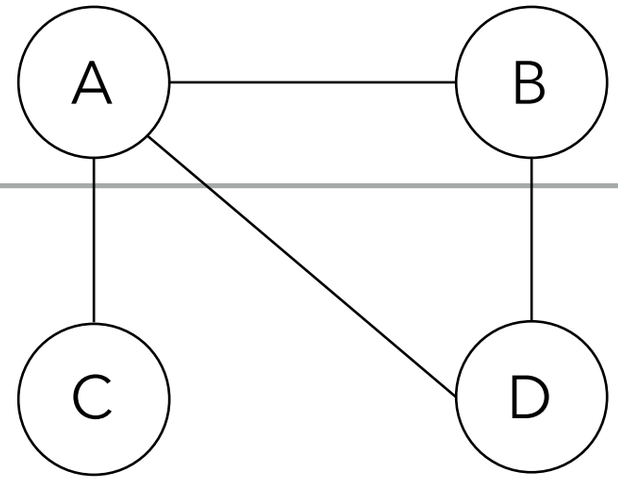
- Vertices: proteins.
- Edges: interactions.



- Source: Macmillan Magazines Ltd.

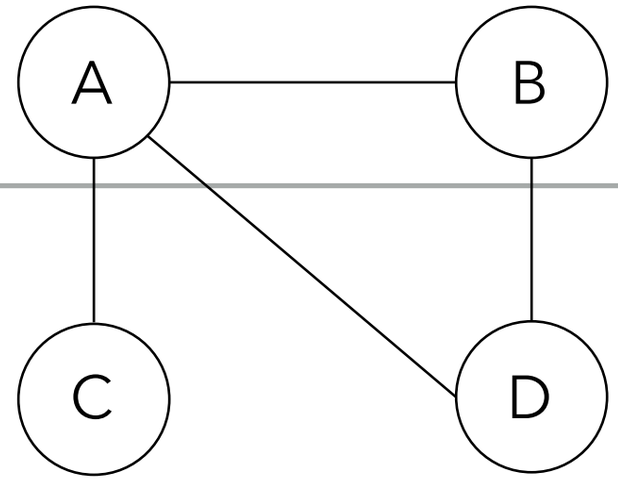
Graph Applications

Graph	Vertex	Edge
Communication	Telephone, computer	Cable
Circuit	Gate, register, processor	Wire
Financial	Stock	Transaction
Transportation	Intersection	Street
Game	Board	Legal move
Neural network	Neuron	Synapse
Molecule	Atom	Bond
Schedule	Job	Constraint



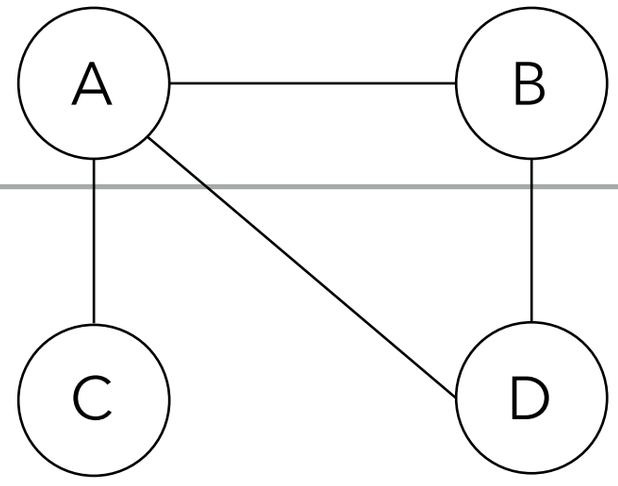
Graph Terminology

- Graph: set of vertices V connected pairwise by a set of edges E.
  - E.g., V = {A, B, C, D}, E = {{A,B}, {A,C}, {A,D}, {B,D}}.
- Path: sequence of vertices connected by edges, with no repeated edges.
  - A simple path is a path with no repeated vertices.
- Cycle: Path with at least one edge whose first and last vertices are the same.
  - A simple cycle is a cycle with no repeated vertices (other than the first and last).
- The length of a cycle or a path is its number of edges.



Graph Terminology

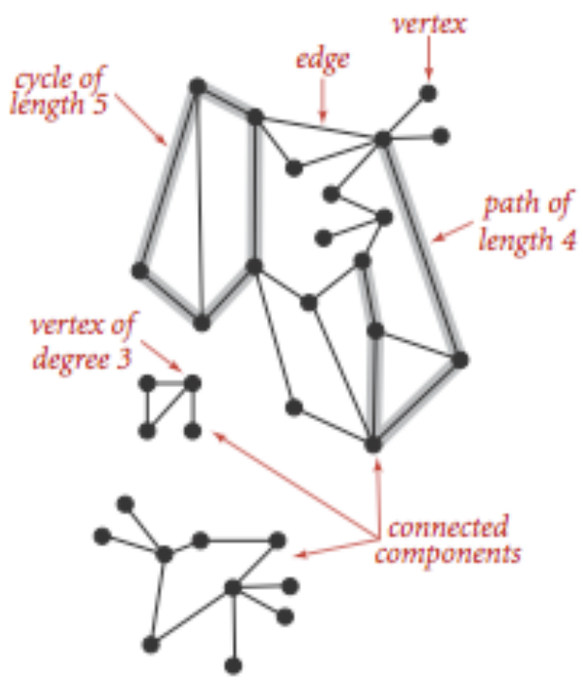
- Self-loop: an edge that connects a vertex to itself.
- Two vertices are connected if there is a path between them.
- Two edges are parallel if they connect the same pair of vertices.
- When an edge connects two vertices, we say that the vertices are adjacent to one another and that the edge is incident on both vertices.
- The degree of a vertex is the number of edges incident on it.
- A subgraph of a graph is a subset of a graph's edges and their associated vertices.



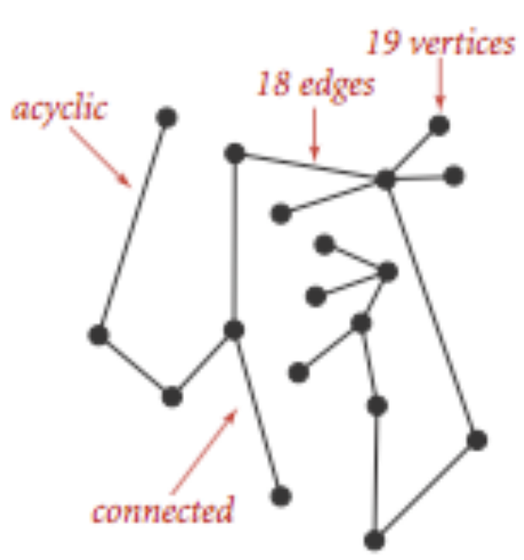
Graph Terminology

- A graph is connected if there is a path from every vertex to every other vertex.
- A graph that is not connected consists of a set of connected components, which are maximal connected subgraphs.
- An acyclic graph is a graph with no cycles.
- A tree is an acyclic connected graph.
- A forest is a disjoint set of trees.

Graph Terminology



Anatomy of a graph



A tree

Popular graph problems

Problem	Description
s-t path	Is there a path between s and t?
Shortest s-t path	What is the shortest path between s and t?
Cycle	Is there a cycle in the graph?
Euler cycle	Is there a cycle that uses each edge exactly once?
Hamilton cycle	Is there a cycle that uses each vertex exactly once?
Connectivity	Is there a path between every pair of vertices?
Biconnectivity	Is there an vertex whose removal disconnects the graph?

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Readings:

- Textbook: Chapter 4.1 (Pages 515-521)
- Website:
  - https://algs4.cs.princeton.edu/41graph/