CS062

DATA STRUCTURES AND ADVANCED PROGRAMMING

27: 2-3 Search Trees

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Lecture 27: 2-3 Search Trees

- 2-3 Search Trees
- Search
- Insertion
- Construction
- Performance

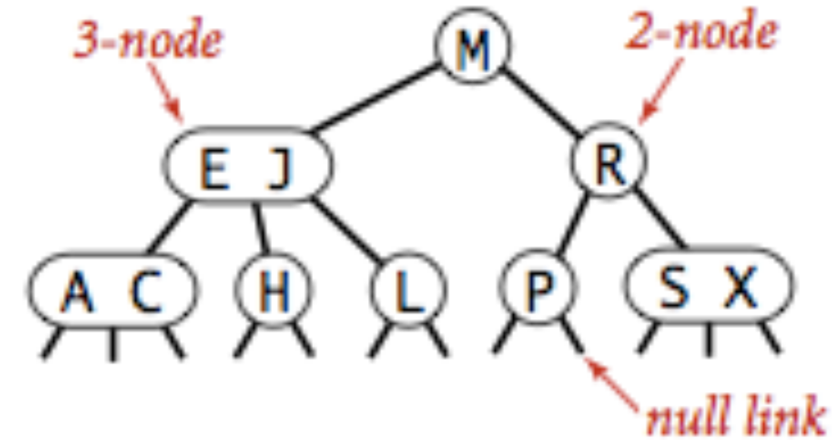
Some slides adopted from Algorithms 4th Edition or COS226

The story so far

- The symbol table is a fundamental data type.
- Naive implementations (arrays/linked lists sorted or unsorted) are way too slow.
- Binary search trees work well in the average case, but can grow too tall and imbalanced in the worst case.
- Question of the day: How to balance search trees?

Order of growth for symbol table operations

	Worst case			Average case		
	Search	Insert	Delete	Search	Insert	Delete
Sequential search (unordered	n	n	n	n	n	n
Binary search (ordered array)	$\log n$	n	n	$\log n$	n	n
BST	n	n	n	$\log n$	$\log n$	$\sqrt{n}$
Goal	$\log n$	$\log n$	$\log n$	$\log n$	$\log n$	$\log n$



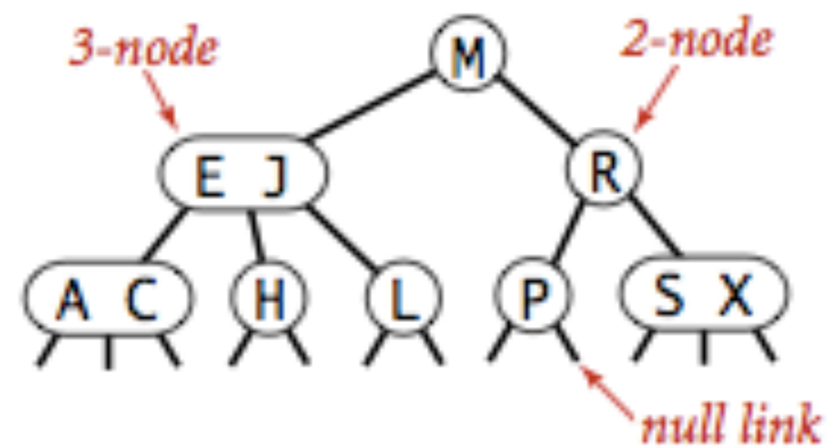
Anatomy of a 2-3 search tree

2-3 tree

- Definition: A 2-3 tree is either empty or a
  - 2-node: one key (and associated value) and two links, a left to a 2-3 search tree with smaller keys, and a right to a 2-3 search tree with larger keys (similarly to standard BSTs), or a
  - 3-node: two keys (and associated values) and three links, a left to a 2-3 search tree with smaller keys, a middle to a 2-3 search tree with keys between the node's keys, and a right to a 2-3 search tree with larger keys.
- Symmetric order: Inorder traversal yields keys in ascending order.
- Perfect balance: Every path from root to null link (empty tree) has the same length.

Example of a 2-3 tree

- 2-node, business as usual with BSTs.
  - (e.g., EJ are smaller than M and R is larger than M).
- In 3-node,
  - left link points to 2-3 search tree with smaller keys than first key,
    - (e.g., AC are smaller than E.)
  - middle link points to 2-3 search tree with keys between first and second key,
    - (e.g. H is between E and J.)
  - right link points to 2-3 search tree with keys larger than second key.
    - (e.g, L is larger than J).



Anatomy of a 2-3 search tree

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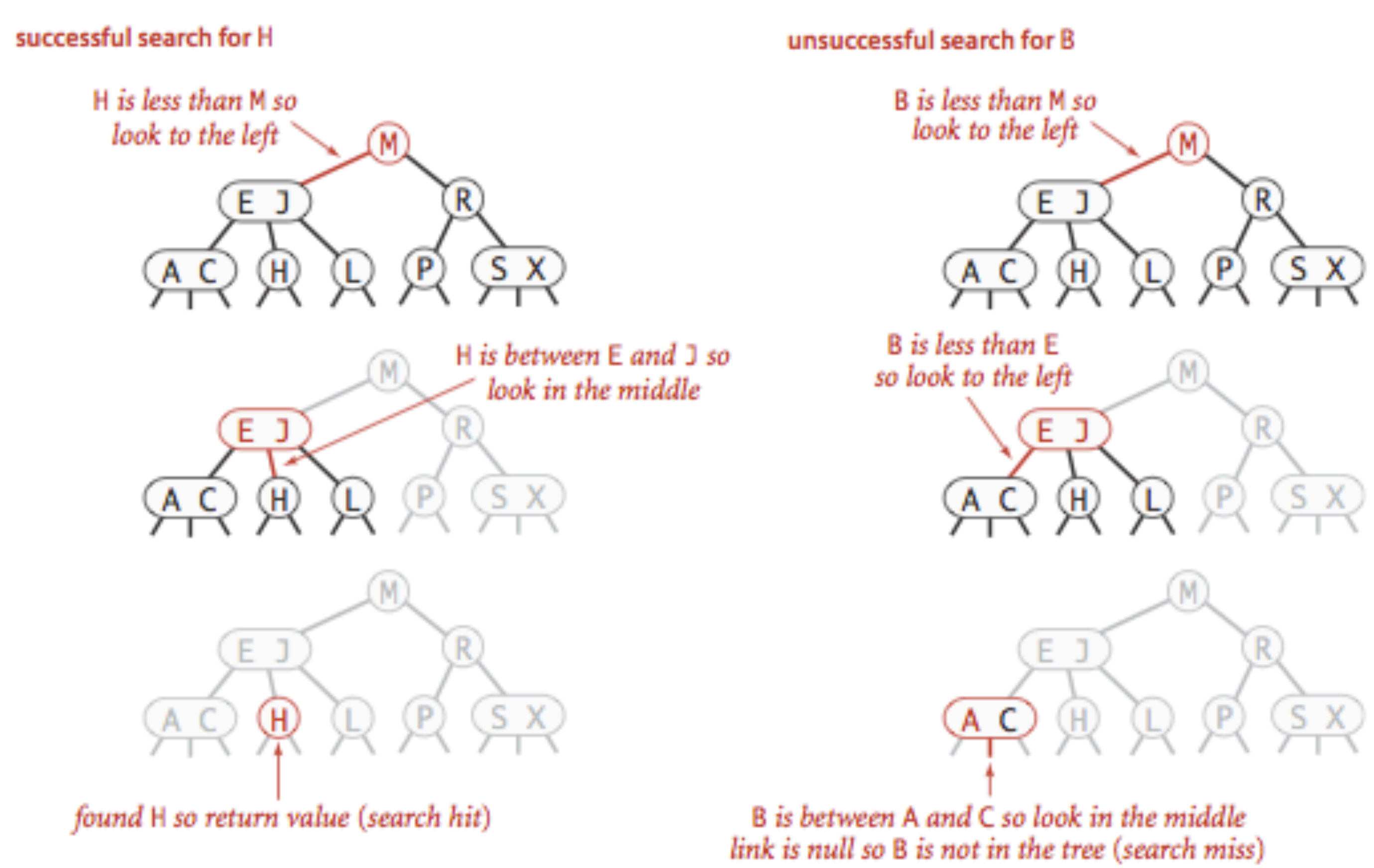
How to search for a key

- Compare search key against (every) key in node.
- Find interval containing search key (left, potentially middle, or right).
- Follow associated link, recursively.



Search hit (left) and search miss (right) in a 2-3 tree

Algorithms

Robert Sedgewick | Kevin Wayne

http://algs4.cs.princeton.edu

3.3 2–3 Tree Demo

- search
- insertion
- construction

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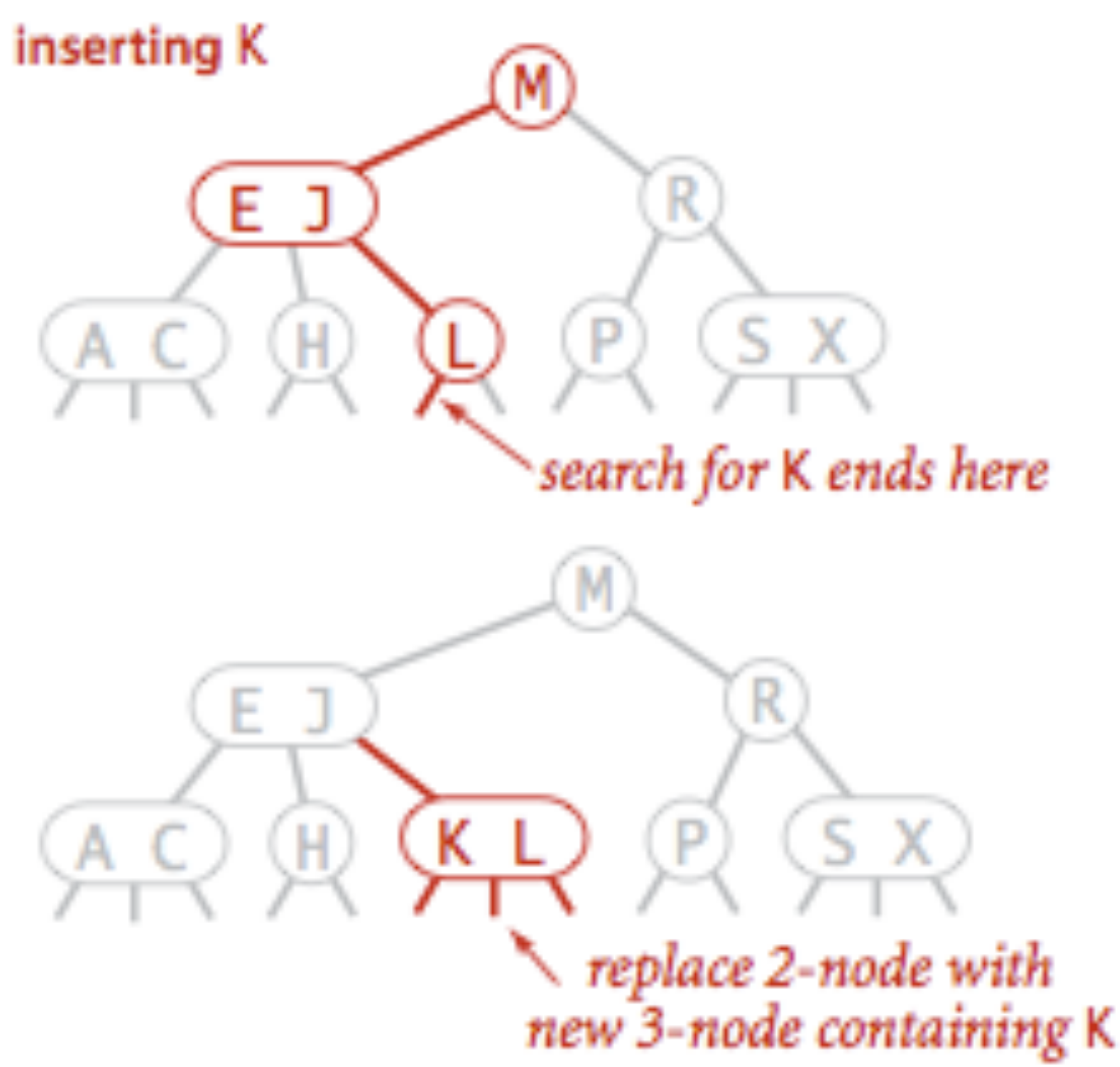
How to insert into a 2-node

- Add new key to 2-node to create a 3-node.



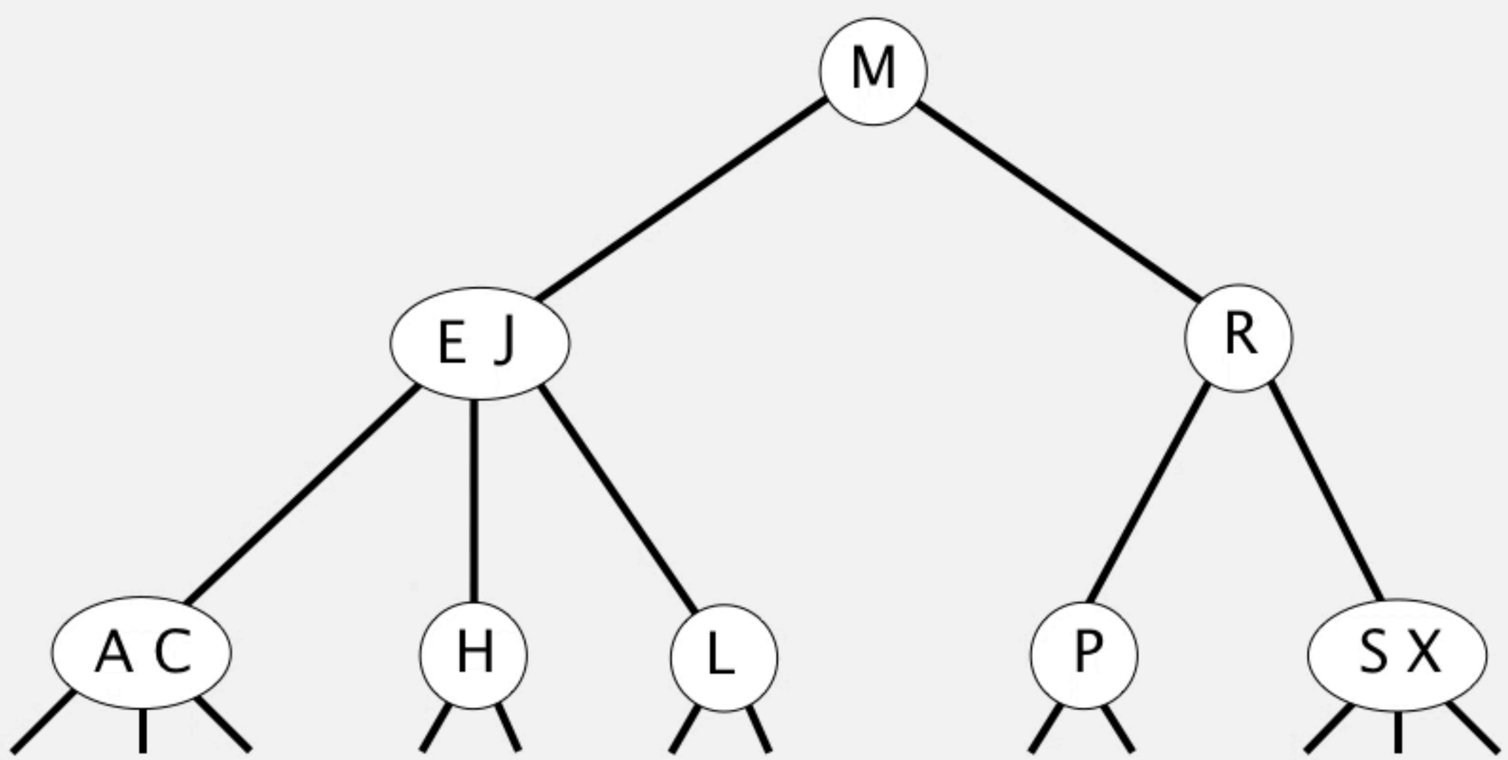
Insert into a 2-node

2–3 tree demo: insertion

Insert into a 2-node at bottom.

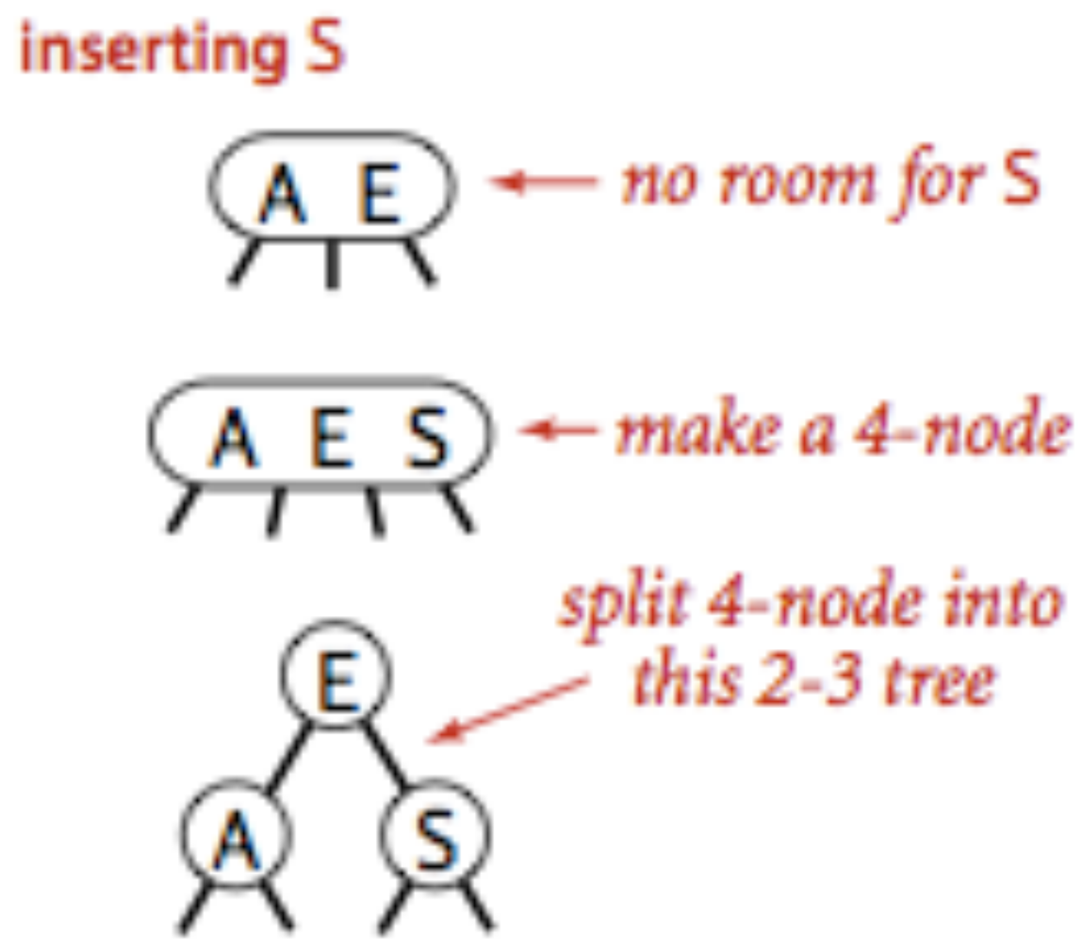
- Search for key, as usual.
- Replace 2-node with 3-node.

insert K



How to insert into a tree consisting of a single 3-node

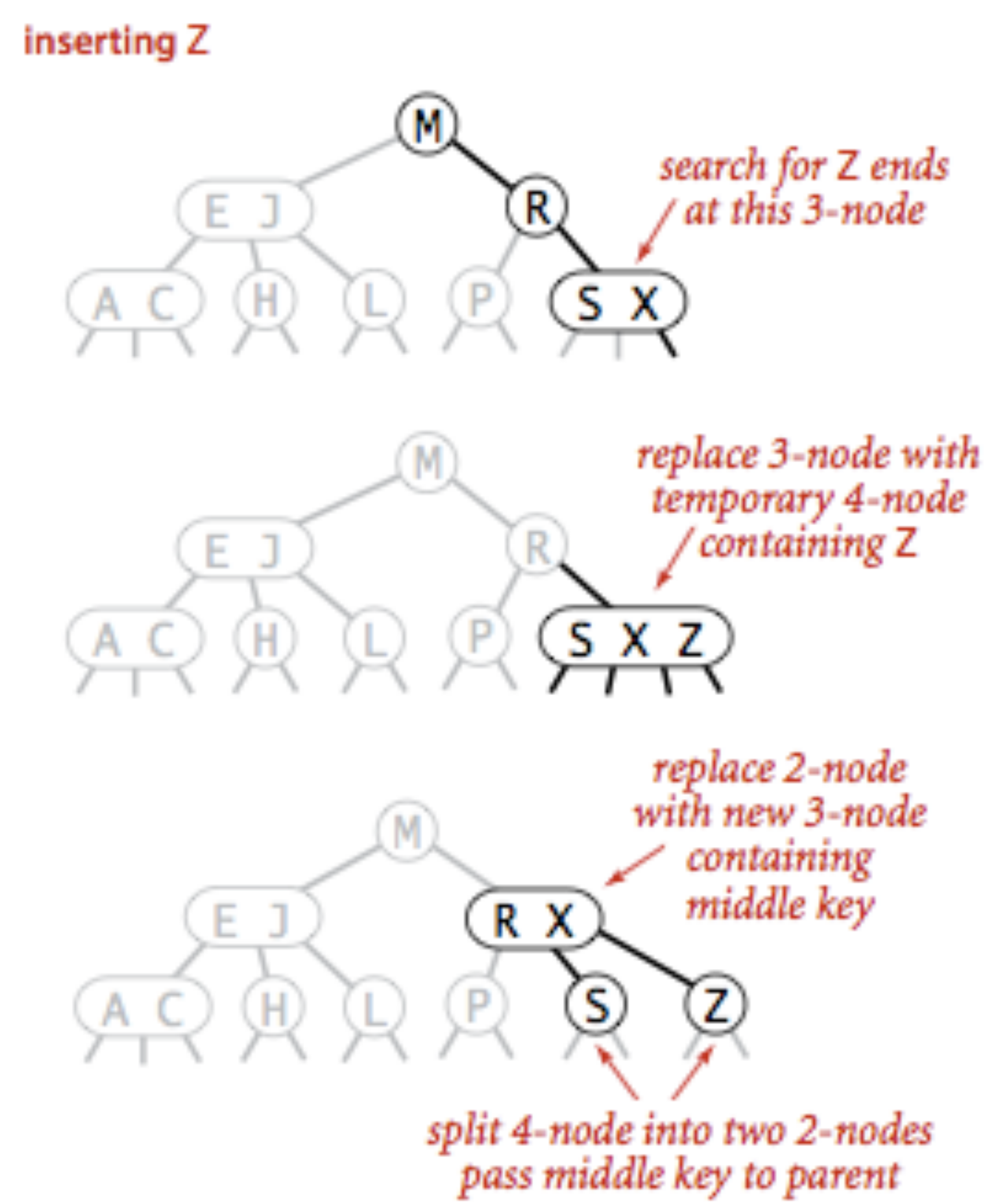
- Add new key to 3-node to create a temporary 4-node.
- Move middle key in 4-node into parent.
- Split 4-node into two 2-nodes.
- Height went up by 1.



Insert into a single 3-node

How to insert into a 3-node whose parent is a 2-node

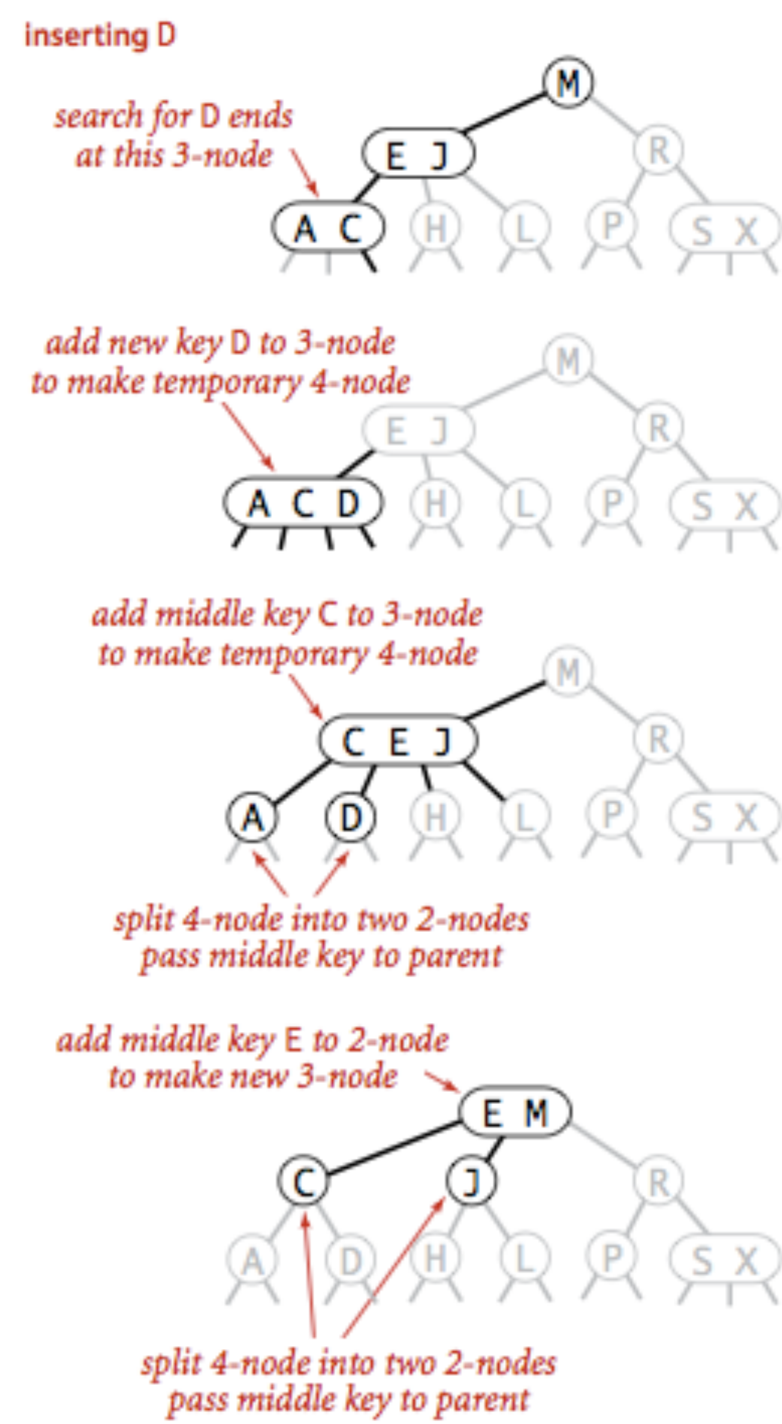
- Add new key to 3-node to create a temporary 4-node.
- Split 4-node into two 2-nodes and pass middle key to parent.
- Replace 2-node parent with 3-node.



Insert into a 3-node whose parent is a 2-node

How to insert into a 3-node whose parent is a 3-node

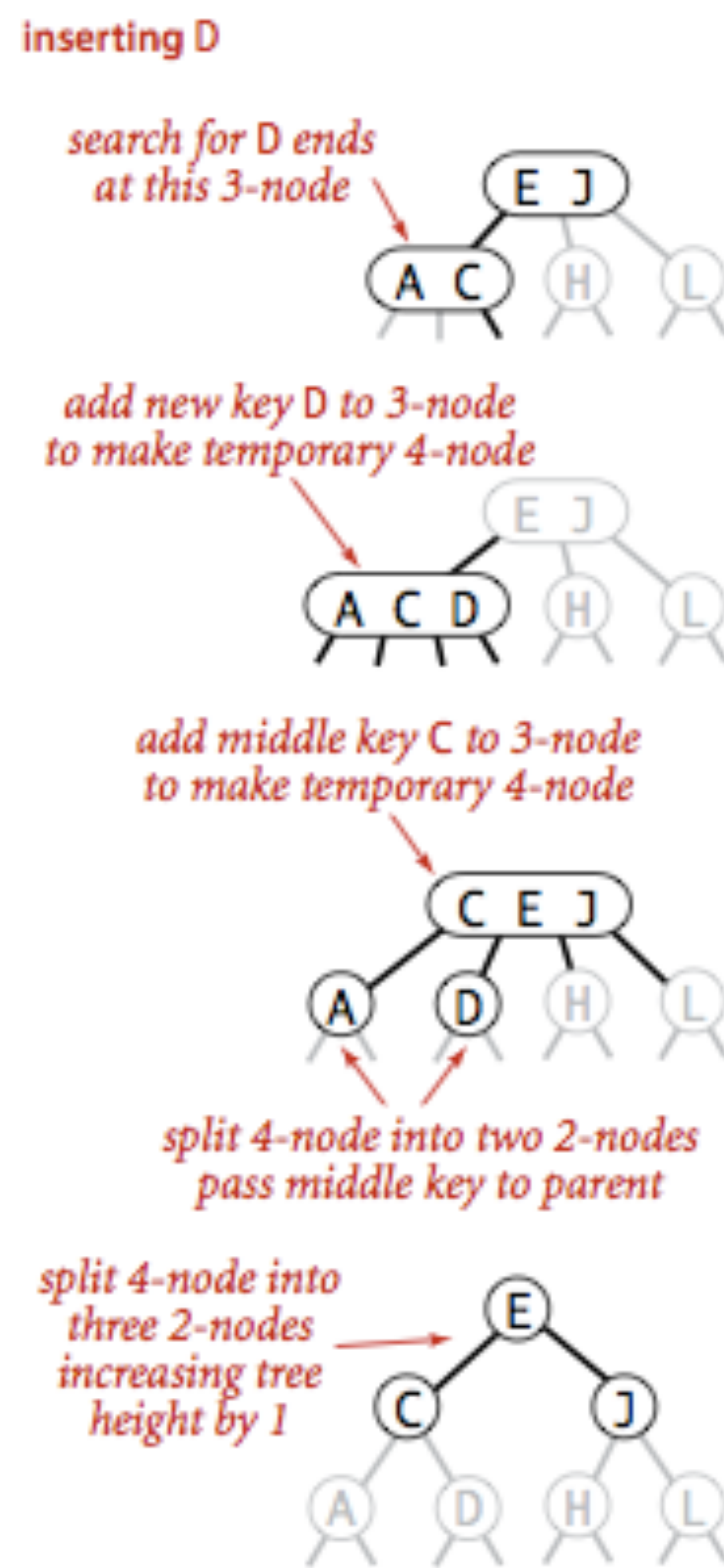
- Add new key to 3-node to create a temporary 4-node.
- Split 4-node into two 2-nodes and pass middle key to parent creating a temporary 4-node.
- Split 4-node into two 2-nodes and pass middle key to parent.
- Repeat up the tree, as necessary.



Insert into a 3-node whose parent is a 3-node

Splitting the root

- If end up with a temporary 4-node root, split into three 2-nodes.
- Increases height by 1 but perfect balance is preserved.



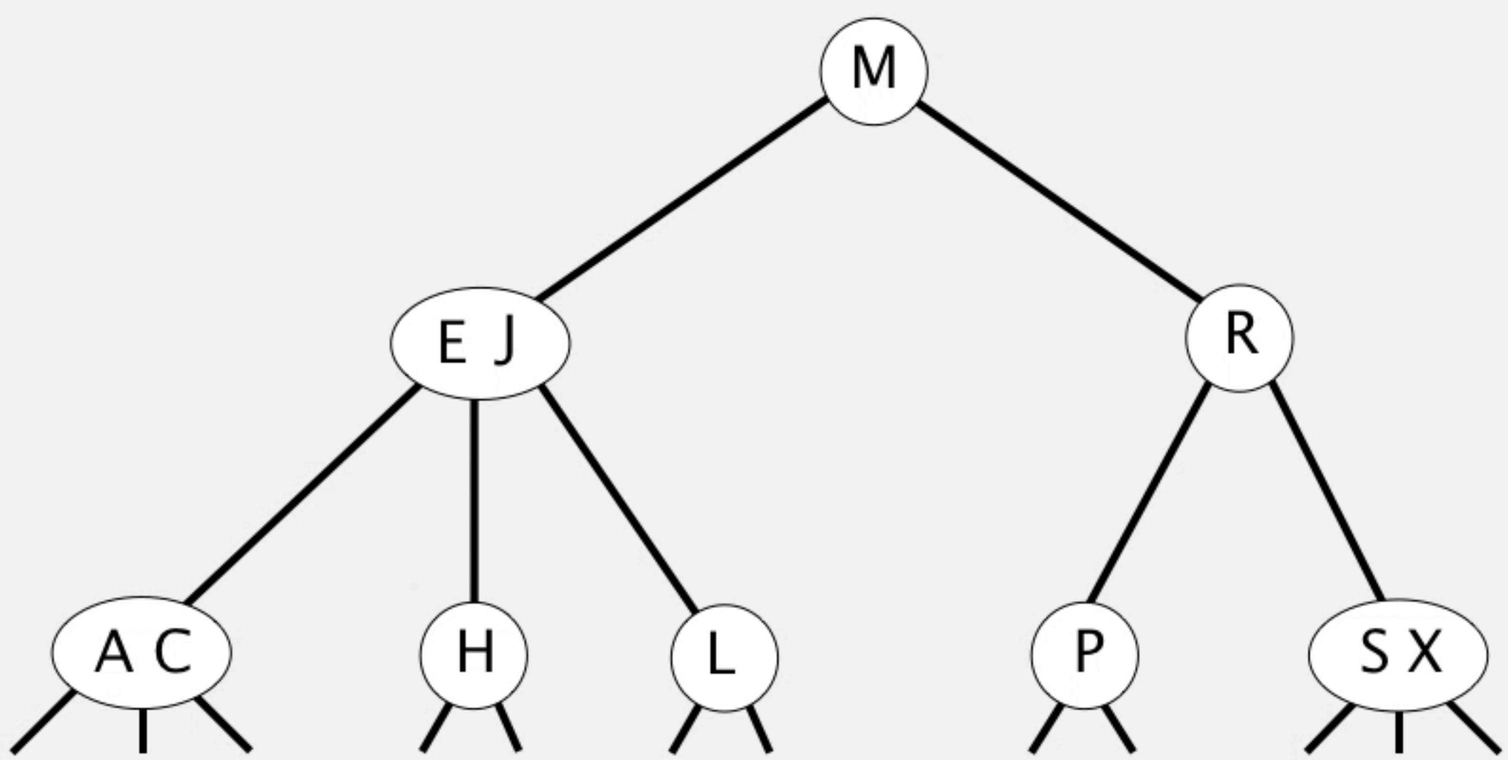
Splitting the root

2-3 tree demo: insertion

Insert into a 2-node at bottom.

- Search for key, as usual.
- Replace 2-node with 3-node.

insert K



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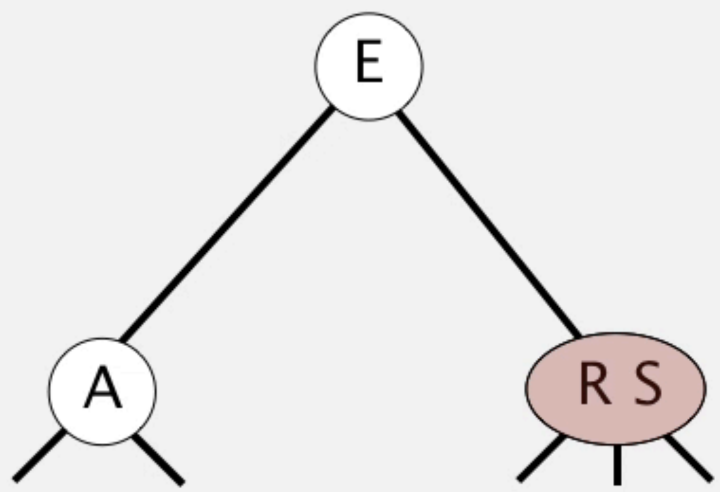
2-3 tree demo: construction

insert R

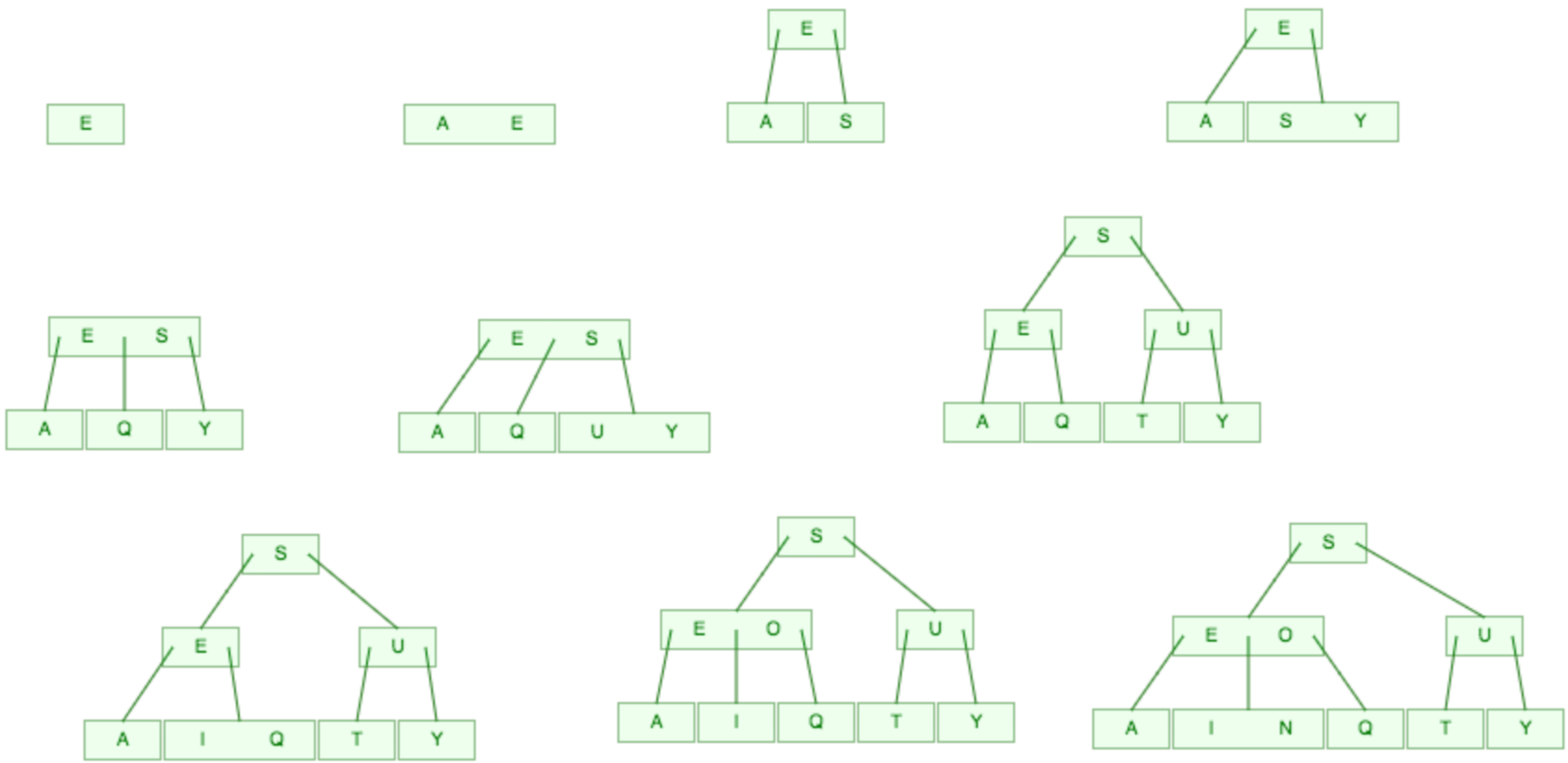


Practice Time

- Draw the 2-3 tree that results when you insert the keys: E A S Y Q U T I O N in that order in an initially empty tree.

Answer

- EASYQUTION



https://www.cs.usfca.edu/~galles/visualization/BTree.html

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Height of 2-3 search trees

- Worst case: $\log n$ (all 2-nodes).
- Best case: $\log_3 n = 0.631 \log n$ (all 3-nodes)
  - That means that storing a million nodes will lead to a tree with height between 12 and 20, and storing a billion nodes to a tree with height between 18 and 30 (not bad!).
- Search and insert are $O(\log n)$!
- But implementation is a pain and the overhead incurred could make the algorithms slower than standard BST search and insert.
- We did provide insurance against a worst case but we would prefer the overhead cost for that insurance to be low. Stay tuned!

Summary for symbol table operations

	Worst case			Average case		
	Search	Insert	Delete	Search	Insert	Delete
Sequential search (unordered	n	n	n	$n/2$	n	$n/2$
Binary search (ordered array)	$\log n$	n	n	$\log n$	$n/2$	$n/2$
BST	n	n	n	$1.39 \log n$	$1.39 \log n$	?
2-3 search tree	$c \log n$	$c \log n$	$c \log n$	$c \log n$	$c \log n$	$c \log n$

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Readings:

- Textbook: Chapter 3.3 (Pages 424-431)
- Website:
  - https://algs4.cs.princeton.edu/33balanced/

Practice Problems:

- 3.3.2-3.3.5