CS062

DATA STRUCTURES AND ADVANCED PROGRAMMING

16–17: Sorting Basics

Alexandra Papoutsaki
LECTURES

Mark Kampe
LABS

Lecture 16-17: Sorting Basics

- Introduction
- Selection sort
- Insertion sort

Some slides adopted from Algorithms 4th Edition or COS226

Why study sorting?

- Analyzing sorting algorithms is a good example of how to compare the performance of different algorithms for the same problem.
- Many of the techniques used here can be found in different problems.
- Sorting your input will often be a good starting point when solving other problems.

Definitions

- Sorting: the process of arranging n items of a collection in some logical order, typically numerically or alphabetically.
  - Examples: sorting students by names, purchases by price, neighborhoods by zipcode, flights by departure time, etc.
- Key: assuming that an item (also known as record, tuple, etc) consists of multiple components, sort key is the property based on which we sort items.
  - Examples: items could be books and potential keys are the title or the author which can be sorted alphabetically.

Total order

- Sorting is well defined if and only if there is total order.
- Total order: a binary relation $\leq$ that satisfies:
  - Totality: for all v and w, if both $v \leq w$ or $w \leq v$ or both.
  - Transitivity: for all v and w, if both $v \leq w$ or $w \leq x$ then $v \leq x$.
  - Antisymmetry: for all v and w, if both $v \leq w$ and $w \leq v$ then $v = w$.

Rules of the game

- We will be sorting arrays of n items, where each item contains a key.
- In Java, objects are responsible in telling us how to *naturally* compare their keys.
- This is achieved by making our class T implement the `Comparable` interface (more on this in a few lectures). We will need to `compareTo` to satisfy a total order:
- `public int compareTo(T that)`
- Implement it so that `v.compareTo(w)`:
    - Returns >0 if v is greater than `w`.
    - Returns <0 if v is smaller than `w`.
    - Returns 0 if v is equal to `w`.
- Java classes such as `Integer`, `Double`, `String`, `File` all implement `Comparable`.

https://docs.oracle.com/javase/8/docs/api/java/lang/Comparable.html

Two useful abstractions

- We will refer to data only through comparisons and exchanges.
- Less: Is v less than w?

```
private static boolean less(Comparable v, Comparable w) {
     return v.compareTo(w) < 0;
}
```

- Exchange: swap item in array a[] at index i with the one at index j.

```
private static void exch(Comparable[] a, int i, int j) {
      Comparable swap = a[i];
      a[i]=a[j];
      a[j]=swap;
}
```

Rules of the game

- Sorting cost model: we count compares and exchanges. If a sorting algorithm does not use exchanges, we count array accesses.
- Extra memory: often as important as running time. Sorting algorithms are divided into two categories:
  - In place: use constant or logarithmic extra memory.
  - Not in place: use linear auxiliary memory.

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Selection sort

- First, find the smallest item in the array.
- Exchange it with the first entry.
- Then, find the next smallest item.
- Exchange it with the second entry.
- Continue until the entire array is sorted.

Selection sort

a[]

i	min	0	1	2	3	4	5	6	7	8	9	10
		S	O	R	T	E	X	A	M	P	L	E
0	6	S	O	R	T	E	X	A	M	P	L	E
1	4	A	O	R	T	E	X	S	M	P	L	E
2	10	A	E	R	T	O	X	S	M	P	L	E
3	9	A	E	E	T	O	X	S	M	P	L	R
4	7	A	E	E	L	O	X	S	M	P	T	R
5	7	A	E	E	L	M	X	S	O	P	T	R
6	8	A	E	E	L	M	O	S	X	P	T	R
7	10	A	E	E	L	M	O	P	X	S	T	R
8	8	A	E	E	L	M	O	P	R	S	T	X
9	9	A	E	E	L	M	O	P	R	S	T	X
10	10	A	E	E	L	M	O	P	R	S	T	X
		A	E	E	L	M	O	P	R	S	T	X

entries in black are examined to find the minimum

entries in red are a[min]

entries in gray are in final position

Trace of selection sort (array contents just after each exchange)

http://algs4.cs.princeton.edu

2.1 SELECTION SORT DEMO

Selection sort

```
public static void sort(Comparable[] a) {
        int n = a.length;
        for (int i = 0; i < n; i++) {          ← In iteration i
            int min = i;
            for (int j = i+1; j < n; j++) {    ← Find the index min of the
                if (less(a[j], a[min]))            smallest remaining array
                    min = j;
            }
            exch(a, i, min);                   ← swap a[i] and a[min]
        }
    }
```

- Invariants: At the end of each iteration `i`:
    - the array a is sorted in ascending order for the first `i+1` elements `a[0…i]`
    - no entry in `a[i+1…n-1]` is smaller than any entry in `a[0…i]`

Selection sort: mathematical analysis for worst-case

```
public static void sort(Comparable[] a) {
        int n = a.length;
        for (int i = 0; i < n; i++) {
            int min = i;
            for (int j = i+1; j < n; j++) {
                if (less(a[j], a[min]))
                    min = j;
            }
            exch(a, i, min);
       }
   }
```

- Comparisons: $1 + 2 + \ldots + (n-2) + (n-1) \sim n^2/2$, that is $O(n^2)$.
- Exchanges: n or $O(n)$
- Running time is quadratic, even if input is sorted.
- In-place, requires almost no additional memory.

Practice Time

- Using selection sort, sort the array with elements [12,10,16,11,9,7].
- Visualize your work for every iteration of the algorithm.

Answer

https://subscription.packtpub.com/book/application_development/9781785888731/13/ch13lvl1sec89/selection-sort

Lecture 16: Sorting Basics I

- Introduction
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Insertion sort

- Move from left to right through the array.
- Look at one element at a time and move it before the larger items on its left.
- Everything before the current time is sorted.
- Everything after the current time has not been examined yet.

Insertion sort

		a[]										
i	j	0	1	2	3	4	5	6	7	8	9	10
		S	O	R	T	E	X	A	M	P	L	E
1	0	O	S	R	T	E	X	A	M	P	L	E
2	1	O	R	S	T	E	X	A	M	P	L	E
3	3	O	R	S	T	E	X	A	M	P	L	E
4	0	E	O	R	S	T	X	A	M	P	L	E
5	5	E	O	R	S	T	X	A	M	P	L	E
6	0	A	E	O	R	S	T	X	M	P	L	E
7	2	A	E	M	O	R	S	T	X	P	L	E
8	4	A	E	M	O	P	R	S	T	X	L	E
9	2	A	E	L	M	O	P	R	S	T	X	E
10	2	A	E	E	L	M	O	P	R	S	T	X
		A	E	E	L	M	O	P	R	S	T	X

entries in gray do not move

entry in red is a[j]

entries in black moved one position right for insertion

Trace of insertion sort (array contents just after each insertion)

http://algs4.cs.princeton.edu

2.1 INSERTION SORT DEMO

In case you didn't get this…

- https://www.youtube.com/watch?v=ROalU379l3U

Insertion sort

```
public static void sort(Comparable[] a) {
        int n = a.length;
        for (int i = 0; i < n; i++) {
           for (int j = i; j > 0; j--) {
               if(less(a[j], a[j-1]))
                  exch(a, j, j-1);
               else
                  break;
           }
       }
   }
```

- Invariants: At the end of each iteration `i`:
  - the array *a* is sorted in ascending order for the first `i+1` elements `a[0...i]`

Insertion sort: mathematical analysis for worst-case

```
public static void sort(Comparable[] a) {
        int n = a.length;
        for (int i = 0; i < n; i++) {
           for (int j = i; j > 0; j--) {
               if(less(a[j], a[j-1]))
                  exch(a, j, j-1);
               else
                  break;
           }
       }
   }
```

- Comparisons: $0 + 1 + 2 + \ldots + (n-2) + (n-1) \sim n^2/2$, that is $O(n^2)$.
- Exchanges: $0 + 1 + 2 + \ldots + (n-2) + (n-1) \sim n^2/2$, that is $O(n^2)$.
- Worst-case running time is quadratic.
- In-place, requires almost no additional memory.

Insertion sort: average and best case

```
public static void sort(Comparable[] a) {
        int n = a.length;
        for (int i = 0; i < n; i++) {
           for (int j = i; j > 0; j--) {
               if(less(a[j], a[j-1]))
                  exch(a, j, j-1);
               else
                  break;
           }
       }
   }
```

- Average case: quadratic for both comparisons and exchanges $\sim n^2/4$ when sorting a randomly ordered array.

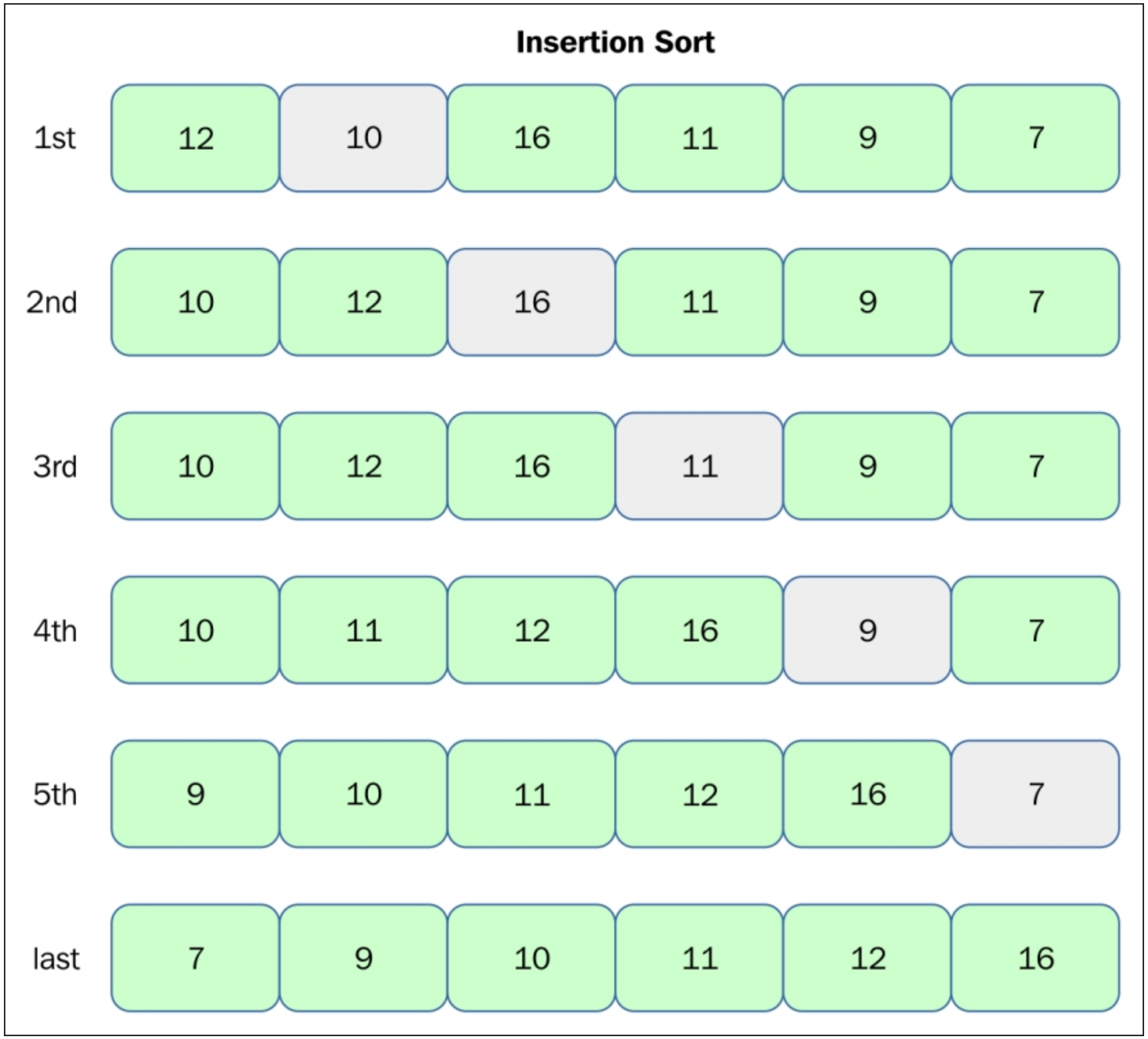
- Best case: $n - 1$ comparisons and 0 exchanges for an already sorted array.

https://www.toptal.com/developers/sorting-algorithms/insertion-sort

Practice Time

- Using insertion sort, sort the array with elements [12,10,16,11,9,7].
- Visualize your work for every iteration of the algorithm.

Answer



https://subscription.packtpub.com/book/application_development/9781785888731/13/ch13lvl1sec90/insertion-sort?query=insertion%20sort

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Readings:

- Textbook:
  - Chapter 2.1 (pages 244–262)
- Website:
  - Elementary sorts: https://algs4.cs.princeton.edu/21elementary/
  - Code: https://algs4.cs.princeton.edu/21elementary/Selection.java.html and https://algs4.cs.princeton.edu/21elementary/Insertion.java.html

Practice Problems:

- 2.1.1-2.1.8