Lecture 7: Analysis of Algorithms

CS 62

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Alexandra Papoutsaki & William Devanny

Order of Magnitude

- Definition: We say that f(n) is $O(g(n))$ iff there exist two constants C and k such that
$$|f(n)| <= C\ |g(n)|, for\ all\ n\ >\ k.$$

- Used to measure time and space complexity of algorithms on data structures of size n.

- Examples:
  - $2n + 1$ is $O(n)$
  - $n^3 - n^2 + 83$ is $O(n^3)$
  - $2^n + n^2$ is $O(2^n)$

Order of Magnitude

- Most common are:
  - $O(1)$ - constant
  - $O(\log n)$ – logarithmic
  - $O(n)$ – linear
  - $O(n^2)$ – quadratic
  - $O(n^c)$ – polynomial
  - $O(c^n)$ – exponential
  - $O(n!)$ – factorial

  - Growth:
    - $O(1), O(\log n), O(n), O(n \log n), O(n^2), O(2^n), O(n!), O(n^n)$

Complexity

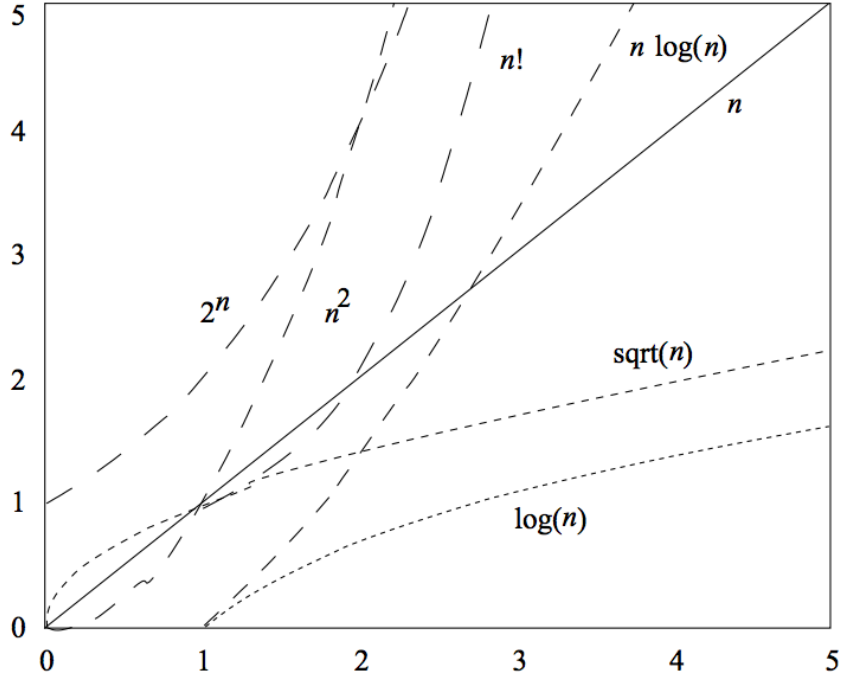


Figure 5.2 Near-origin details of common curves. Compare with Figure 5.3.

Asymptotic Analysis

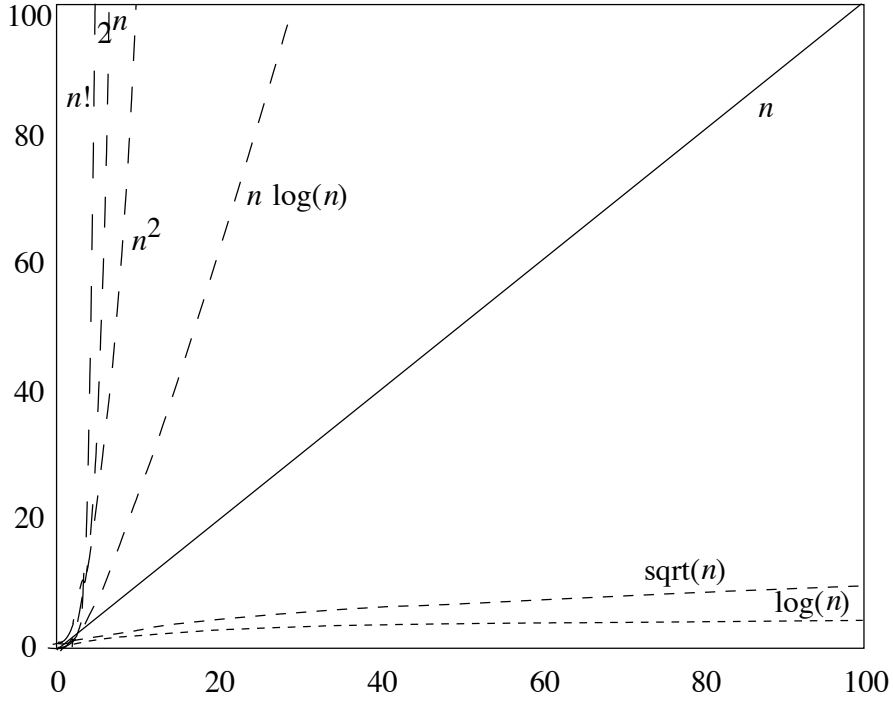


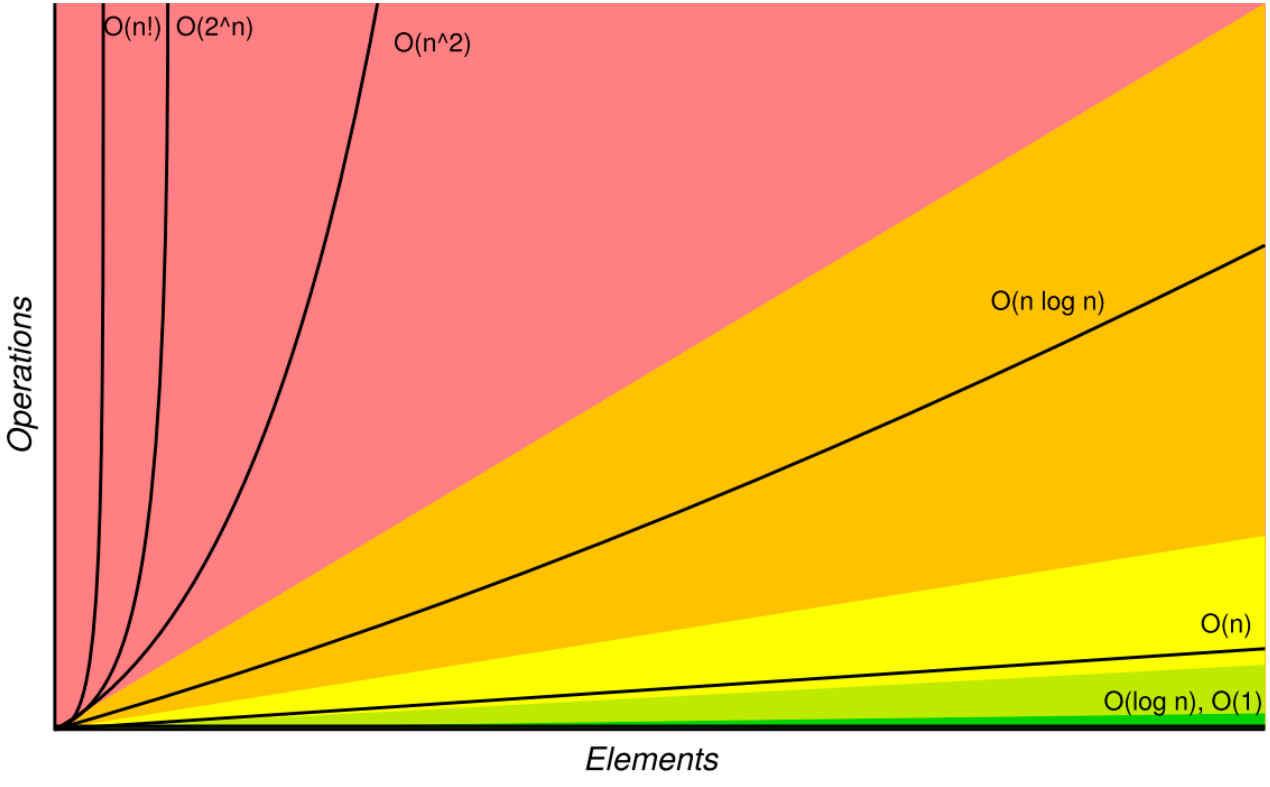
Figure 5.3 Long-range trends of common curves. Compare with Figure 5.2.

Comparing Orders of Magnitude

- Suppose we have the operations with complexities given and that a problem of size n takes time t.
- How long would it take if we increase size of problem?

Problem Size:	*10 n*	*100n*	*1000n*
$O(\log n)$	3+t	7 + t	10+ t
$O(n)$	10 t	100 t	1000 t
$O(n \log n)$	> 10 t	> 100 t	> 1000 t
$O(n^2)$	100 t	10,000 t	1,000,000 t
$O(2^n)$	~ t^{10}	~ t^{100}	~ t^{1000}

Rule of thumb



bigocheatsheet.com

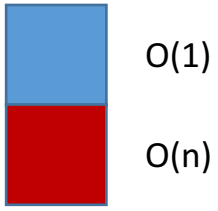
Adding to `ArrayList`

- Suppose there are n elements in `ArrayList` and you want to add one more. What is the cost of this operation?

- If enough space (size<capacity):
  - Add to end is $O(1)$
  - Add to beginning is $O(n)$

- If not space:
  - What is the cost of `ensureCapacity`?
  - $O(n)$ because n elements in array

Amortized Time Analysis



As the arraylist increases in size, the doubling happens half as often but costs twice as much

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Insertion Cost	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Copying Cost	0	1	2	0	4	0	0	0	8	0	0	0	0	0	0	0	16
Total Cost	1	2	3	1	5	1	1	1	9	1	1	1	1	1	1	1	17

Amortized Time Analysis

We will use the aggregate method – the simplest method for amortized time analysis

Others: accounting (banker's) and potential (physicist's).

Think of it as an average. For a total of n operations: O(total cost) / (number of operations) (in this case additions)

O(total cost of operations)= $\sum$ cost of insertions + $\sum$ cost of copying

Total cost of insertions: n of them, each $O(1)$ cost, therefore $nO(1) = O(n)$

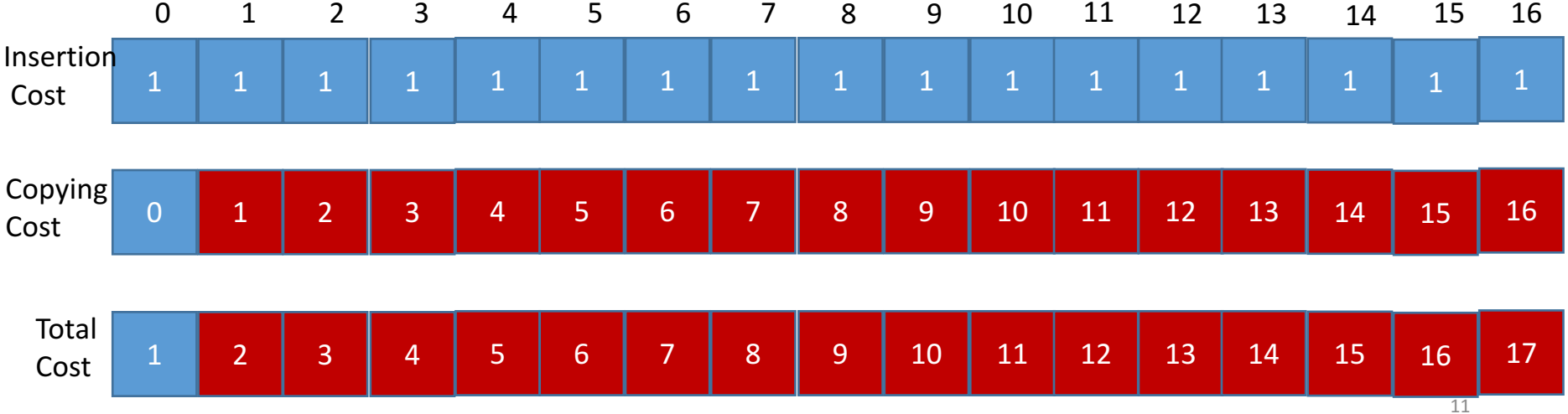
Total cost of copying: $1 + 2 + 2^2 + \cdots + 2^{\lfloor \log_2 n \rfloor} \leq 2n$ which is $O(n)$

O(total cost) = $O(n) + O(n) = O(n)$

Amortized time = $O(n)/n = O(1)$ but "lumpy"

EnsureCapacity

- What if we only increase the capacity by 1 element each time?
  - Adding n elements one at a time to end
    - Total cost of n insertions: $1 + 2 + 3 + \cdots + (n-1) = n(n-1)/2$
    - Total cost of $O(n^2)$
  - Average cost of *each* is $O(n)$



`ArrayList` Operations

- Best case:
  - $O(1)$: `size()`, `isEmpty()`, `get(int i)`, `set(int i, E elet)`, `remove()`, `add()`
- Worst case:
  - $O(1)$: `size()`, `isEmpty()`, `get(int i)`, `set(int i, E elet)`
  - $O(n)$: `remove`, `add()`
- `add()` runs in amortized constant time: adding n elements requires $O(n)$ time.

Assignment

- **`WordStream`**: Reads text word by word
  - Use **`nextToken()`** but make sure **`hasMoreTokens()`**
- **`Pair`**: of two elements
- **`StringPair`**
  - Pair of Strings. Extends **`Pair`**
- Assume two associations <k,v>, <k’,v’>.
  - Useful methods: `get(i)` and `getValue()`
  - the equals method will return true iff the k and k’ are equal
- **`List`**
  - **`indexOf`**`(Object o)` finds index of **`o`** in a list
  - Return -1 if `o` not in list

FreqList

- list of associations holding words and their frequencies
- Instance variable `List<Association<String, Integer>> flist`
- Start with **`toString()`**
- Continue with **`add()`**
  - What to check when adding?

In general…

- Work on paper first!
- More demanding than assignment 1. Start early!
- Come to office hours
- Don’t forget Friday’s quiz