Lecture 34: Graphs I

CS 62

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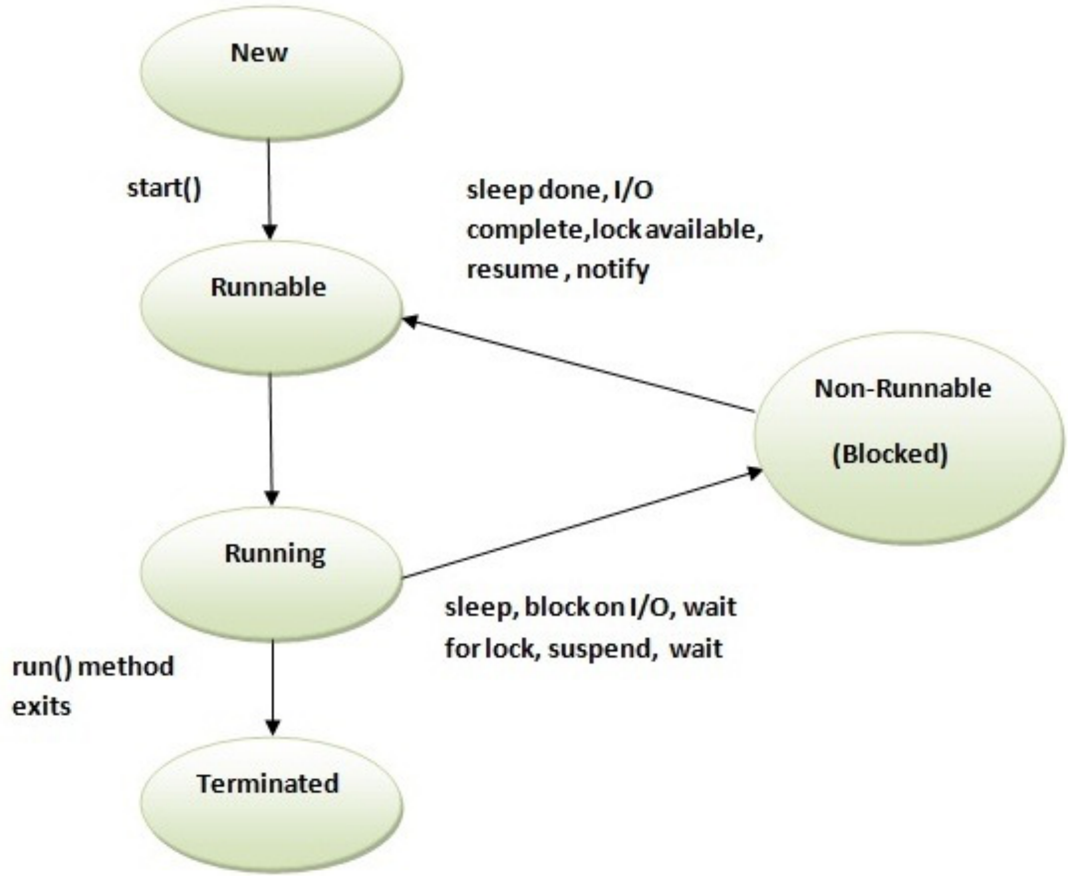
Assignment


New
start()
Runnable
sleep done, I/O
complete, lock available,
resume , notify
Non-Runnable
(Blocked)
Running
sleep, block on I/O, wait
for lock, suspend, wait
run() method
exits
Terminated

Graphs

- Represent relationships that exist between pairs of objects
- Nothing to do with charts and function plots!
- Extremely versatile, can be used to represent many problems

The Graph ADT

A graph $G = (V, E)$

- V is a finite, non-empty set of vertices (or nodes)
- E is a binary relation on V
  (that is, E is a collection of edges that connects pairs of vertices)
- Edges are either *directed* or *undirected*

Applications

- transportation networks (flights, roads, etc.)
  - flights and flight patterns.
  - what sort of questions might we ask? What sort of application might we be interested in having a graph?
  - booking flights, picking shortest time? shortest distance?
  - airlines save fuel, number of people who use the route
- Google maps
  - driving directions, mapping out sightseeing

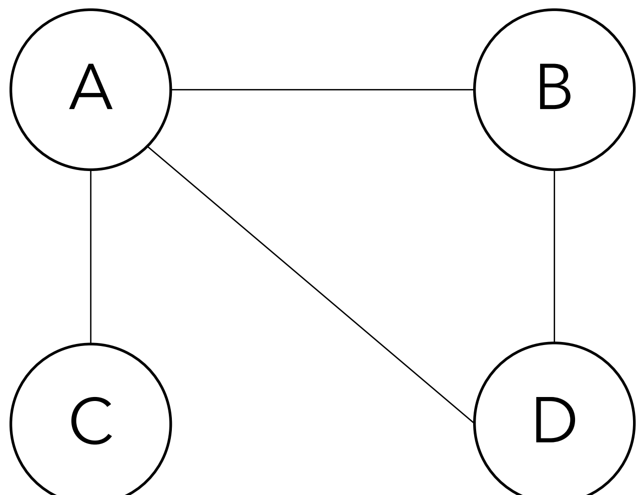
More Applications

- communications networks/utility networks
  - electrical grid, phone networks, computer networks
  - minimize cost for building infrastructure
  - minimize losses, route packets faster
- social networks
  - Does this person know that person.
  - Can this person introduce me to that person – job opportunities

Undirected Graphs

Example: $G = (V, E)$, where

- $V = \{A, B, C, D\}$
- $E = \{\{A, C\}, \{A, B\}, \{A, D\}, \{B, D\}\}$



Definitions for UnDirected Graphs

- **subgraph**: is a subset of a graph's edges (and associated vertices) that constitutes a graph.
- **path**: a sequence of connected vertices.
  - **simple path** – a path where all vertices occur only once.
- **path length**: number of edges in the path.
  - Example: path C-A-D-B has length 3.
- **cycle**: path of length ≥ 1 that begins and ends with the same vertex.
  - Example: path A-D-B-A is a cycle.
- **simple cycle**: a simple path that begins and ends with the same vertex.

More Definitions for UnDirected Graphs

- **self loop**: Cycle consisting of one edge and one vertex.
- **adjacent vertices**: when connected by an edge.
- **incident edge**: the edge that is incident on two adjacent vertices
    - Edge (A,B) above is incident on adjacent vertices A and B
- **degree**: number of incident edges on a vertex.
- **simple graph**: a graph with no self loops.
- **acyclic graph**: a graph with no cycles.

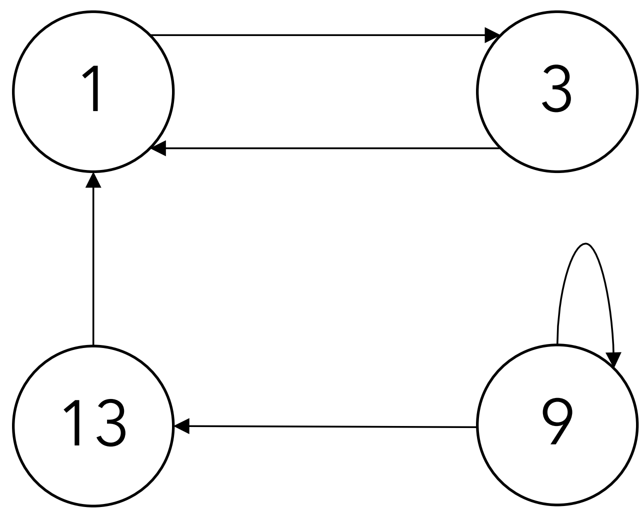
Even More Definitions for UnDirected Graphs

- **connected vertices**: if path that connects them exists
- **connected graph**: a graph where every pair of vertices is connected by a path.
- **tree**: acyclic connected graph
- **forest**: disjoint set of trees

Directed Graphs (Digraphs)

Example: $G = (V, E)$, where

- $V = \{1, 3, 9, 13\}$
- $E = \{(1,3), (3, 1), (13,1), (9,9)(9,13)\}$



Definitions for Digraphs

- **subgraph**: subset of a digraph's edges (and associated vertices) that constitutes a digraph.
- **path**: sequence of vertices with a (directed) edge pointing from each vertex to its successor
  - **simple path** – a path where all vertices occur only once.
- **length**: number of edges in the path.
- **cycle**: directed path of length ≥ 1 that begins and ends with the same vertex.
  - **simple cycle**: a simple path that begins and ends with the same vertex.

More Definitions for Digraphs

- **self loop**: Cycle consisting of one edge and one vertex.
  - Example: 9
- **outdegree**: number of edges pointing from it.
- **indegree**: number of edges pointing to it.
- **directed acyclic graph (DAG)**: a digraph with no directed cycles.

Even More Definitions for Digraphs

- **reachable vertices**: when there is a directed path from one to another.
- **strongly connected vertices**: if mutually reachable
- **strongly connected digraph**: directed path from every vertex to every other vertex
- **weakly connected graph**: a digraph that would be connected if all of its directed edges were replaced by undirected edges.