Lecture 23: Binary Search Trees

CS 62

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BST

A binary tree is a binary search tree iff

- it is empty or
- if the value of every node is both greater than or equal to every value in its left subtree and less than or equal to every value in its right subtree.

Interface

```
public class BinarySearchTree> {
      protected BinaryTree root;
      public void add(E value);
      public void contains(E value);
      public void remove(E value);
      protected BinaryTree locate(BinaryTree root, E val);
      protected BinaryTree predecessor(BinaryTree node);
      protected BinaryTree removeTop(BinaryTree topNode);
}
```

Locating a Value

- Useful for add, contains, and remove
- Returns a pointer to the node with a given value
  - …or to a node where that exact value could be added
- Recursive implementation (could be iterative)

Locating a Value

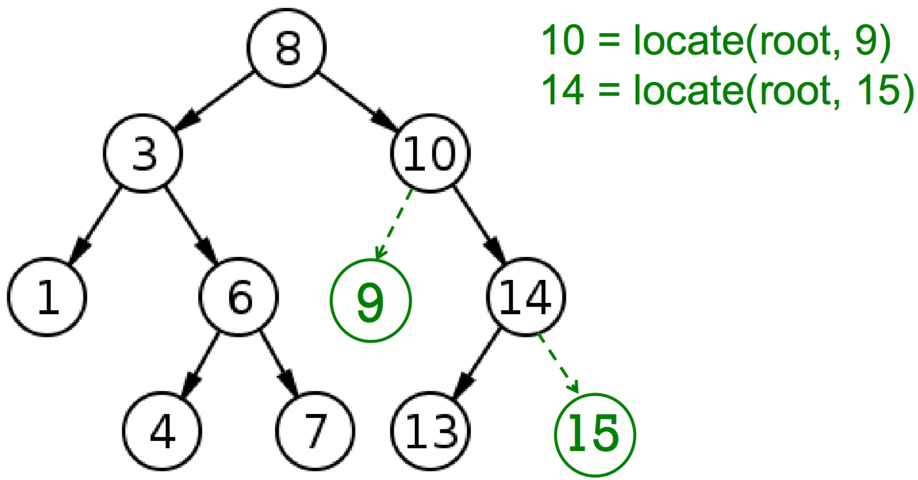
- Check current value vs. the search value
  - If equal, return this node
  - If smaller, locate within left subtree
  - Else within right subtree
  - If the appropriate subtree is empty, return this node

```java
// @pre root and value are non-null
// @post returned: 1 - existing tree node with the desired value, or
//                 2 - the node to which value should be added
    protected BinaryTree<E> locate(BinaryTree<E> root, E value){
        E rootValue = root.value();
        BinaryTree<E> child;
        if (rootValue.equals(value)) return root; // found at root
        // look left if less-than, right if greater-than
        if (ordering.compare(rootValue,value) < 0) {
            child = root.right();
        } else {
            child = root.left();
        }
        // no child there: not in tree, return this node,
        // else keep searching
        if (child.isEmpty()) {
            return root;
        } else {
            return locate(child, value);
        }
    }
```

Using locate to add a node

Case One: Locate returns pointer to where node should be added

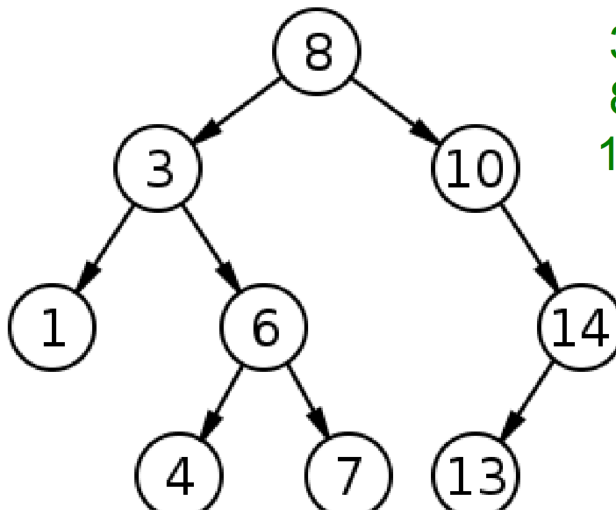
- If value less than returned node, create new left child
- If value greater than returned node, create new right child



Using locate to add a node

Case Two: Locate returns pointer to node with same value

- Duplicates go in left subtree (could have chosen right)
- Where in the left subtree?

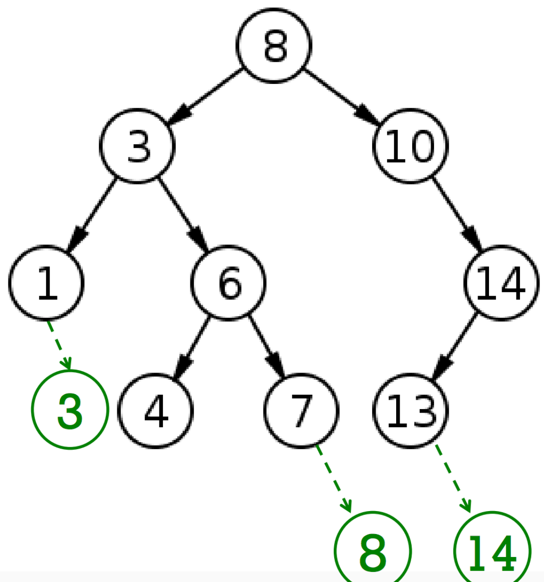


3 = locate(root, 3)
8 = locate(root, 8)
14 = locate(root, 14)

Using locate to add a node

Case Two: Locate returns pointer to node with same value

- Duplicates go in left subtree (could have chosen right)
- *Should be the rightmost descendant*



Predecessor

Finds the rightmost descendent in left subtree

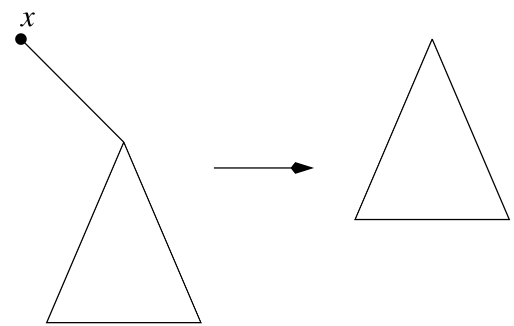
- The next-smallest value in the tree
- What’s the big-O runtime?

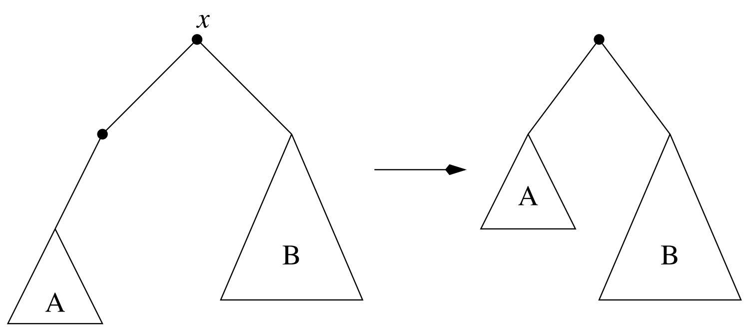
```java
protected BinaryTree<E> predecessor(BinaryTree<E> root) {
        BinaryTree<E> result = root.left();
        while (!result.right().isEmpty()) {
            result = result.right();
        }
        return result;
   }

   protected BinaryTree<E> successor(BinaryTree<E> root) {
        BinaryTree<E> result = root.right();
        while (!result.left().isEmpty()) {
            result = result.left();
        }
        return result;
   }
```

```java
public void add(E value) {
     BinaryTree<E> newNode = new BinaryTree<E>(value,EMPTY,EMPTY);
     // add value to binary search tree
     // if there's no root, create value at root
     if (root.isEmpty()) {
         root = newNode;
     } else {
         BinaryTree<E> insertLocation = locate(root,value);
         E nodeValue = insertLocation.value();
         // The location returned is the successor or predecessor
         // of the to-be-inserted value
         if (ordering.compare(nodeValue,value) < 0) {
             insertLocation.setRight(newNode);
         } else {
             if (!insertLocation.left().isEmpty()) {
                 // if value is in tree, we insert just before
                 predecessor(insertLocation).setRight(newNode);
             } else {
                 insertLocation.setLeft(newNode);
             }
         }
     }
     count++;
 }
```

Removing nodes – Easy cases

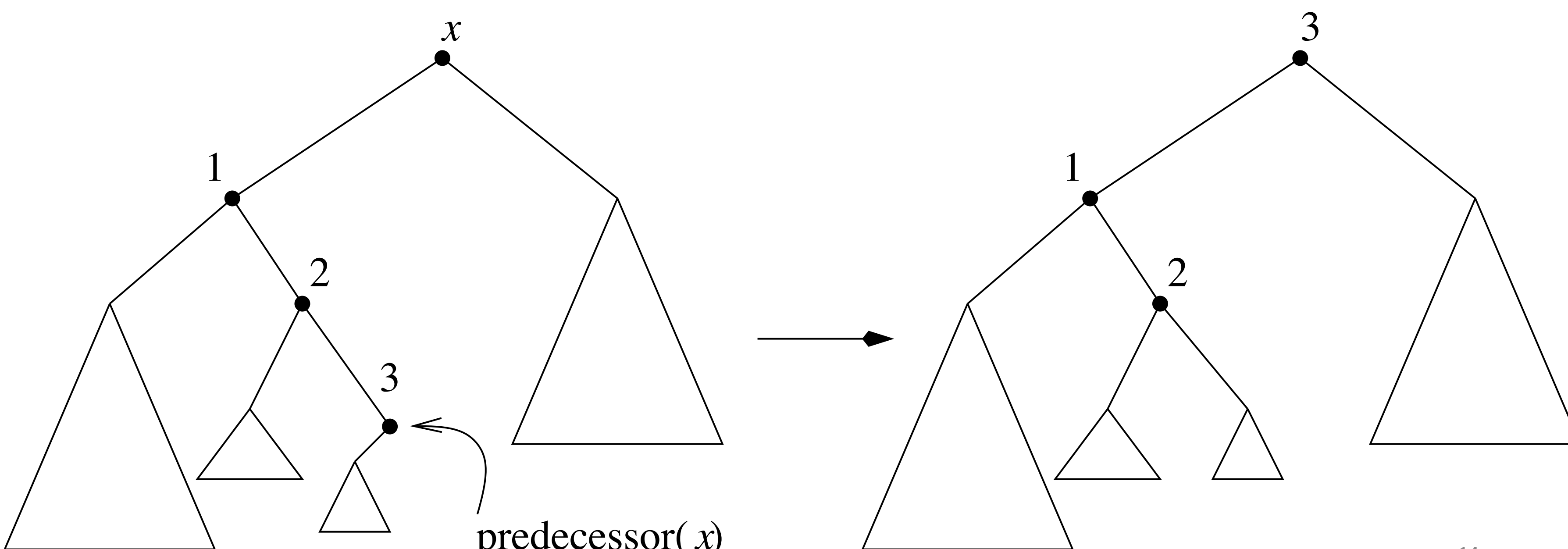
- Node is a leaf
- Node has only one child





Removing nodes – General Case

Left Child has a right subtree:



Removing nodes

- Calling `remove(E val)` removes node with value `val`
- Predecessor of root becomes new root
  - Predecessor is in left subtree
  - Predecessor has no right subtree

  - Complexity is O(h) where h is height of tree
    - Worst-case O(h) to locate
    - Worst-case O(h) to find predecessor

Complexity

- `locate`, `add`, `contains`, `remove` are all O(h)
- Can we guarantee that h is O(log n)?
  - Only if tree stays balanced!!
- Binary search trees that stay balanced
  - AVL trees
  - Red-black trees
- We'll do splay tree, which doesn't guarantee balance
    - but guarantees good average behavior
    - easier to understand than alternatives
    - better than others if likely to go back to recent nodes