Lecture 21: Heaps & Heapsort

CS 62
Fall 2017
Kim Bruce & Alexandra Papoutsaki

Lab Today

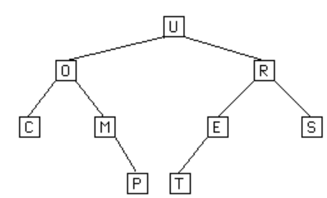
- Build binary search trees *Different from heap!*
- A binary tree is a binary search tree iff
  - it is empty or
  - if the value of every node is both greater than or equal to every value in its left subtree and less than or equal to every value in its right subtree.
- How do you build binary search tree?
  - Insert by following from root until find empty slot

Quiz Friday

- Array representations of trees
- Priority queues
- Heapsort

Array Representation of Trees

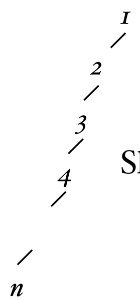
- data[0..n-1] can hold values in trees
  - left subtree of node i in 2*i+1, right in 2*i+2,
  - parent in (i-1)/2



Indices:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
data[]:	U	O	R	C	M	E	S	–	–	–	P	T	–	–	–

How bad can it be?

- What if long stringy tree (e.g. only single left-most branch)?
  - How much space to hold n elements.
  - If complete what is height?



Slots: 0, 1, 3, 7, 15, 31, ..., $2^{n-1}-1$

$O(2^n)$ space in worst case!

Min-Heap

- Min-Heap H is complete binary tree s.t.
  - H is empty, or
  - Both of the following hold:
    - The value in root position is smallest value in H
    - The left and right subtrees of H are also heaps.
      Equivalent to saying parent ≤ both left and right children
- Excellent implementation for priority queue
  - Dequeue elements w/lowest priority values before higher

Implementations

- As regular queue (array or linked) where either keep in order or search for lowest to remove:
  - One of add or remove will be O(n)
- Heap representation (in arraylist) is more efficient: O(log n) for both add and remove.
  - Insert into heap:
    - Place in next free position,
    - "Percolate" it up.
  - Delete:
    - remove root,
    - move last element in array up to root. "Push" it down.

See VectorHeap code!

Called PriorityQueue class in standard Java

Sorting with Trees

Tree Sort

- Build Binary search tree (later)
- Do Inorder traversal, adding elts to array
  - Inorder traversal: O(n)
  - Building tree:
    - log 1 + log 2 + ... + log n = O(n log n) in best (& average) case
    - $O(n^2)$ in worst case
- O(n log n) in best & average case
- $O(n^2)$ in worst case :-(*What is worst case?*
- Heapsort is always better!

Heapsort

- Make vector into a heap:
  - n add operations = O(n log n)
- Remove elements in order
  - n remove operations = O(n log n)
- Total: O(n log n)
  - If clever can make into heap in O(n)
  - ... but still O(n log n) total.
  - O(1) extra space (for swaps)

Comparing Sorts

- Quicksort: fastest on average O(n log n), but worst case is $O(n^2)$ & takes O(log n) extra space
- Heapsort: O(n log n) in average & worst case. No extra space.
  - Bit slower on average than quick & mergesorts.
- Mergesort: O(n log n) in average and worst case. O(n) extra space.
  - Performs well on external files where not all fit in memory.