Lecture 17: Binary Trees

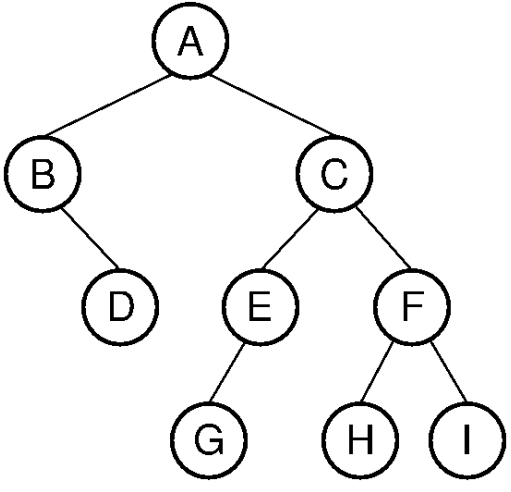
CS 62

Fall 2017

Kim Bruce & Alexandra Papoutsaki

Tree

- A tree is either:
  - Empty or
  - consists of a node, called the root node, together with a collection of trees, called its subtrees. These trees are disjoint from each other and the root.

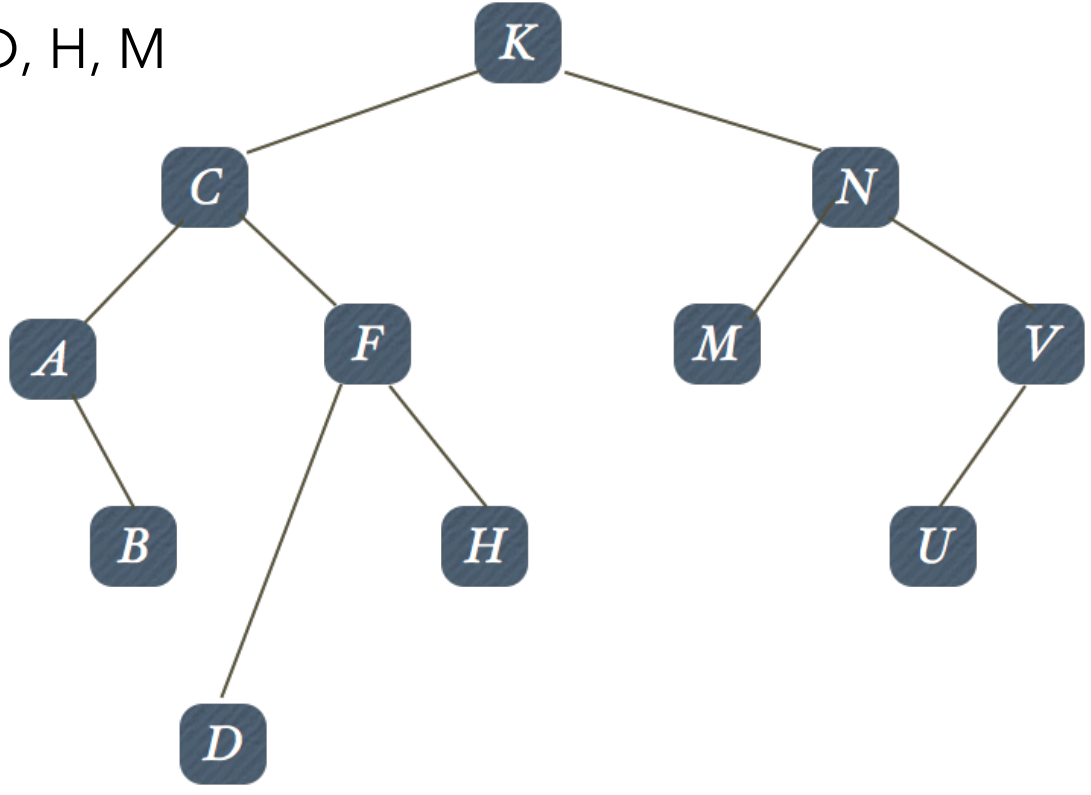


Definitions

- An ***edge*** connects a node to its subtrees.
- The roots of the subtrees of a node are said to be the ***successors*** or ***descendants*** of the node.
- Nodes without successors are called ***leaves***. The others are called ***interior*** *nodes*.
- All nodes except root have unique predecessor.
- A collection of trees is called a *forest*.

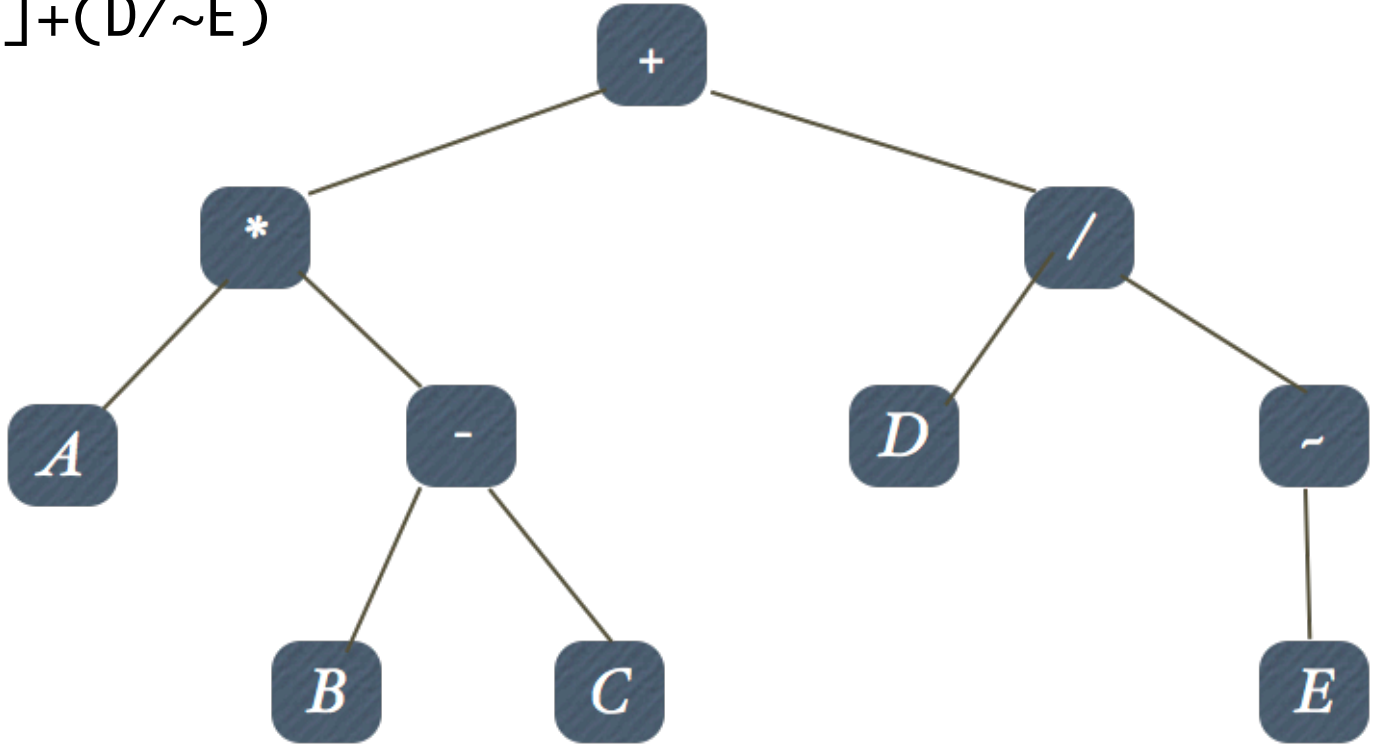
Example: Binary Search Tree

K, C, A, N, B, V, F, U, D, H, M



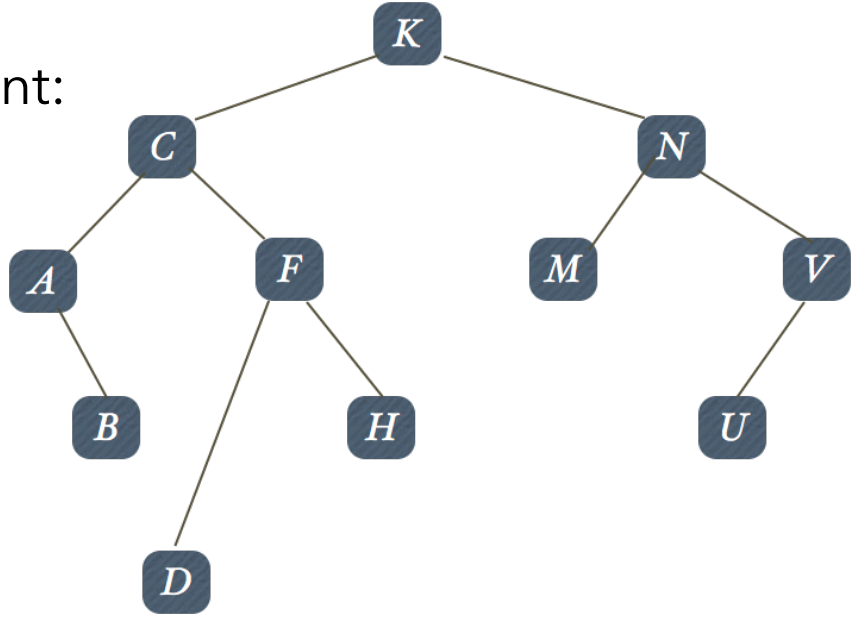
Expression Tree

[A*(B-C)]+(D/~E)



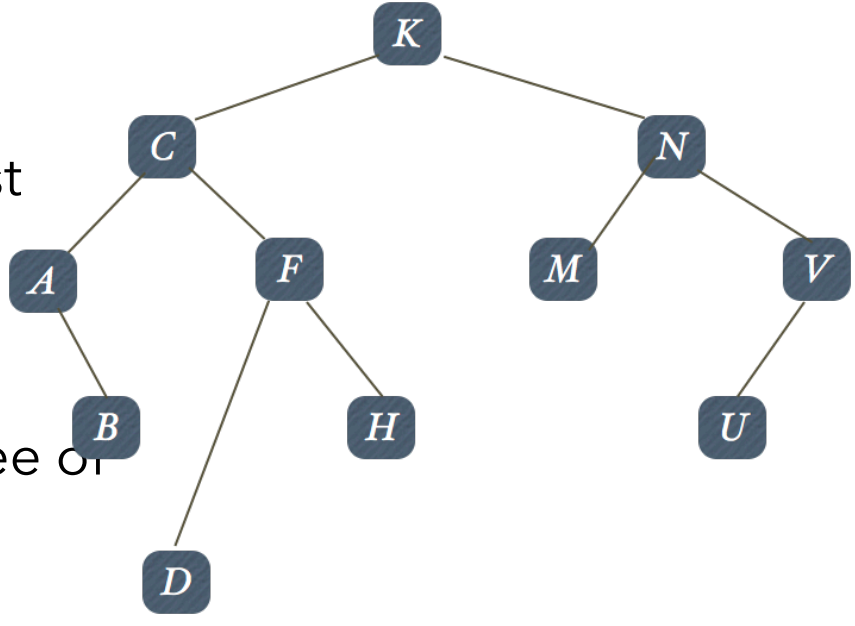
Family Tree Terminology

- ***Parent*** node is directly above ***child*** node:
  - K is parent to C, N.
- ***Sibling*** node has same parent:
  - A, F
- K is ***ancestor*** of B
- B is ***descendant*** of K
- Node plus all descendants gives ***subtree***



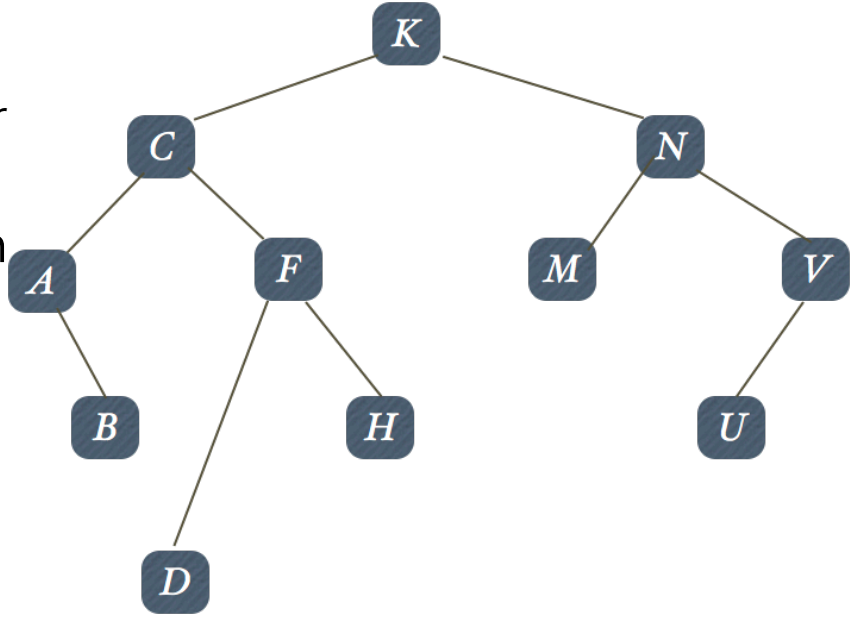
More Terminology

- **Simple path** is series of distinct nodes s.t. there is edge between successive nodes.
- ***Path length*** = # edges in path
- ***Height of node*** = length of longest path to a leaf
- ***Height of tree*** = height of root
- ***Degree of node*** is # of children
- ***Degree of tree (arity)*** = max degree of any its nodes
- Binary tree has arity ≤ 2.



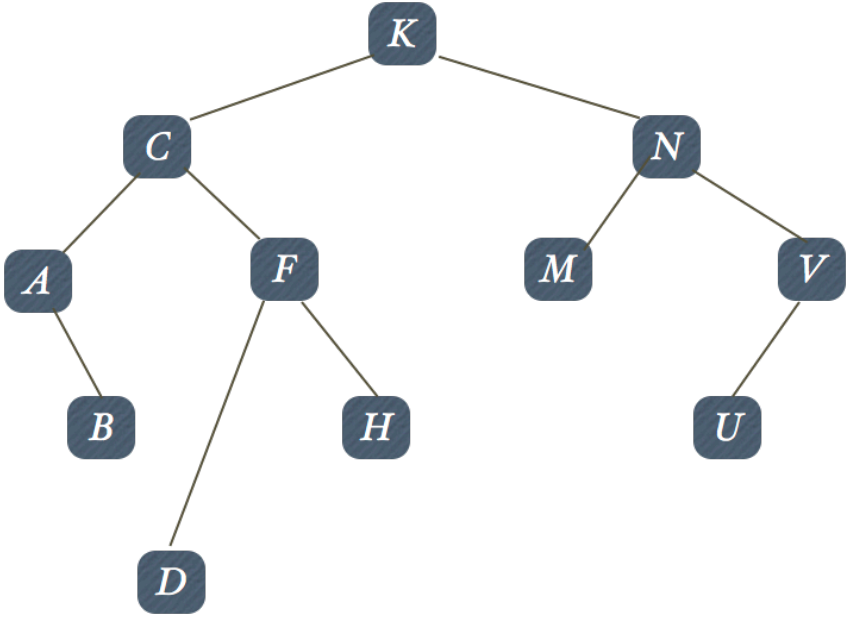
Even More Terminology

- **Level/depth** of node defined recursively:
  - Root is at level 0
  - Level of any other node is one greater than level of parent
- Level of node is also length of path from root to the node.



Counting

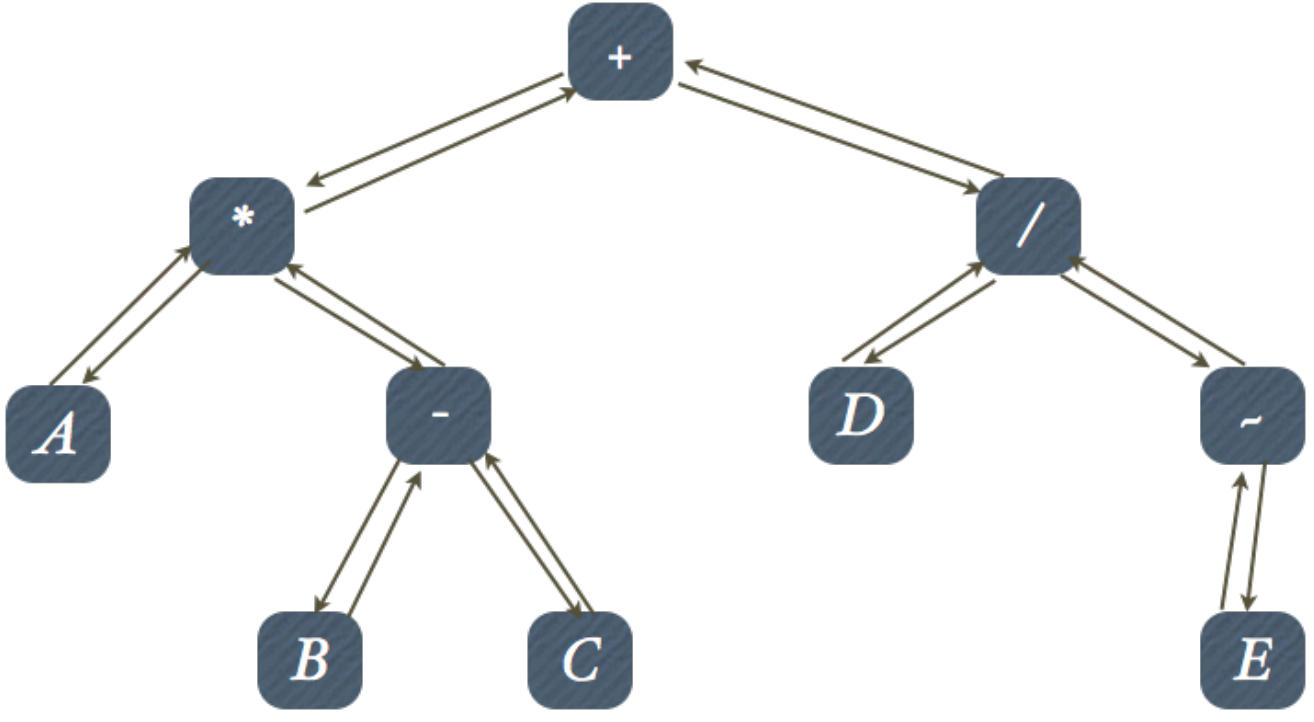
- Lemma: if T is a binary tree, then at level k, T has $\leq 2^k$ nodes.
- Theorem: If T has height h, then # nodes in $T \leq 2^{h+1} - 1$.
- Equivalently, if T has n nodes then $n - 1 \geq h \geq \log(n+1) - 1$



Binary Trees in Java

- No implementation in standard Java libraries
- Structure5 has BinaryTree<E> class, but no interface (*though we provide one!*).
- Like doubly-linked list:
  - value: E
  - parent, left, right: BinaryTree<E>

Linked Representation



Tree Traversals

- Traversals:
  - Pre-Order: root, left subtree, right subtree
  - In-Order: left subtree, root, right subtree
  - Post-Order: left subtree, right subtree, root
- Most algorithms have two parts:
  - Build tree
  - Traverse tree, performing operations on nodes

Evaluate Expression Tree

- Evaluate left subtree, right subtree, perform operation at root.
- Generate stack-based code to evaluate: post-order