csci54 – discrete math & functional programming counting, probability

- ► A deck of cards consists of 52 cards, each with a rank (2-10, J, Q, K, A) and a suit (♣♦♥♥♠).
- If you draw a card from a perfectly-shuffled deck of cards, what is the probability that the card is a heart?
- If you draw a card from a perfectly-shuffled deck of cards, what is the probability that it is either the Queen of Hearts or the 9 of clubs?
- If you draw two cards (without replacement) from a perfectlyshuffled deck of cards, what is the probability that both cards are hearts?

Probability

- ► A deck of cards consists of 52 cards, each with a rank (2-10, J, Q, K, A) and a suit (ふ◇♡ふ).
 - If you draw a card from a perfectly-shuffled deck of cards, what is the probability that the card is a heart?

One process:

- define a sample space S, which is a set containing all possible outcomes
- determine the subset of outcomes that defines the event
- calculate the probability of the outcomes in the event
- sum those probabilities

Probability – some definitions

Definition 10.1: Outcomes and sample space.

An *outcome* of a probabilistic process is the sequence of results for all randomly determined quantities. (An outcome can also be called a *realization* of the probabilistic process.) The *sample space S* is the set of all outcomes.

Definition 10.2: Probability function.

Let S be a sample space. A probability function $Pr : S \to \mathbb{R}$ describes, for each outcome $s \in S$, the fraction of the time that s occurs. (We denote probabilities using square brackets, so the probability of $s \in S$ is written Pr[s].) We insist that the following two conditions hold of the probability function Pr:

$\sum_{s\in S} { t Pr}\left[s ight] = 1$
$\Pr[s] \ge 0 \text{ for all } s \in S.$

Definition 10.3: Event.

Let S be a sample space with probability function Pr. An event is a subset of S. The probability of an

event E is the sum of the probabilities of the outcomes in E, and it is written $\Pr[E] = \sum_{s \in E} \Pr[s]$.

If you have equally likely outcomes, then the probability of a particular event is the number of outcomes in that event divided by the total number of possible outcomes

In practice

- ► A deck of cards consists of 52 cards, each with a rank (2-10, J, Q, K, A) and a suit (ふ◇♡ふ). For each of the following specify your sample space befeore answering the question.
- If you draw a card from a perfectly-shuffled deck of cards, what is the probability that:
 - the card is a heart?
- If you draw two cards (without replacement) from a perfectlyshuffled deck of cards, what is the probability that
 - both cards are hearts?
 - the two cards have different suits?
 - the two cards sum to 3 (i.e. you draw an Ace and a 2)

Definition 10.9: Independent and dependent events.

Two events A and B are *independent* if and only if $\Pr[A \cap B] = \Pr[A] \cdot \Pr[B]$. The events A and B are

called *dependent* if they are not independent.

- I randomly choose a number 1, 2, ..., 10. Consider the following 3 events. Are any pair of them independent?
 - A: I choose an odd number
 - B: I choose a prime number
 - C: I choose a number (strictly) less than 5.

Definition 10.11: Conditional probability.

The *conditional probability of A given B*, written $\Pr[A|B]$, is given by $\Pr[A|B] = \frac{\Pr[A \cap B]}{\Pr[B]}$.

- I randomly choose a number 1, 2, ..., 10. Consider the following two events. What are the conditional probabilities Pr[A|B] and Pr[B|A]?
 - A: I choose an odd number
 - B: I choose a prime number

Theorem 10.14: The Law of Total Probability.

Let A and B be arbitrary events. Then $\Pr[A] = \Pr[A|B] \cdot \Pr[B] + \Pr[A|\overline{B}] \cdot \Pr[\overline{B}]$.

Conditional probability

Theorem 10.14: The Law of Total Probability.

Let A and B be arbitrary events. Then $\Pr[A] = \Pr[A|B] \cdot \Pr[B] + \Pr[A|\overline{B}] \cdot \Pr[\overline{B}]$.

Theorem 10.15: Bayes' Rule.

For any two events A and B, we have
$$\Pr[A|B] = \frac{\Pr[B|A] \cdot \Pr[A]}{\Pr[B]}$$
.

I have two coins in an opaque bag. The coins are visually indistinguishable, but one coin is fair (Prob H = 0.5); and the other coin is biased (Prob H = 0.75). I pull one of the two coins out at random. If I flip the coin and it comes up heads, what is the probability that I'm holding the biased coin?