csci54 – discrete math & functional programming
counting, probability
Some probability questions

- A deck of cards consists of 52 cards, each with a rank (2-10, J, Q, K, A) and a suit (♧♢♡♤).

- If you draw a card from a perfectly-shuffled deck of cards, what is the probability that the card is a heart?

- If you draw a card from a perfectly-shuffled deck of cards, what is the probability that it is either the Queen of Hearts or the 9 of clubs?

- If you draw two cards (without replacement) from a perfectly-shuffled deck of cards, what is the probability that both cards are hearts?
A deck of cards consists of 52 cards, each with a rank (2-10, J, Q, K, A) and a suit (♧♢♡♤).

If you draw a card from a perfectly-shuffled deck of cards, what is the probability that the card is a heart?

One process:

- define a *sample space* $S$, which is a set containing all possible *outcomes*
- determine the subset of outcomes that defines the *event*
- calculate the probability of the outcomes in the event
- sum those probabilities
Probability – some definitions

**Definition 10.1: Outcomes and sample space.**
An outcome of a probabilistic process is the sequence of results for all randomly determined quantities. (An outcome can also be called a realization of the probabilistic process.) The sample space \( S \) is the set of all outcomes.

**Definition 10.2: Probability function.**
Let \( S \) be a sample space. A probability function \( \Pr : S \to \mathbb{R} \) describes, for each outcome \( s \in S \), the fraction of the time that \( s \) occurs. (We denote probabilities using square brackets, so the probability of \( s \in S \) is written \( \Pr [s] \).) We insist that the following two conditions hold of the probability function \( \Pr \):

\[
\sum_{s \in S} \Pr [s] = 1
\]

\( \Pr [s] \geq 0 \) for all \( s \in S \).

**Definition 10.3: Event.**
Let \( S \) be a sample space with probability function \( \Pr \). An event is a subset of \( S \). The probability of an event \( E \) is the sum of the probabilities of the outcomes in \( E \), and it is written \( \Pr [E] = \sum_{s \in E} \Pr [s] \).
In practice

A deck of cards consists of 52 cards, each with a rank (2-10, J, Q, K, A) and a suit (♧♢♡♤). For each of the following specify your sample space before answering the question.

If you draw a card from a perfectly-shuffled deck of cards, what is the probability that:

- the card is a heart?

If you draw two cards (without replacement) from a perfectly-shuffled deck of cards, what is the probability that

- both cards are hearts?
- the two cards have different suits?
- the two cards sum to 3 (i.e. you draw an Ace and a 2)
I randomly choose a number 1, 2, ..., 10. Consider the following 3 events. Are any pair of them independent?

- A: I choose an odd number
- B: I choose a prime number
- C: I choose a number (strictly) less than 5.
I randomly choose a number 1, 2, ..., 10. Consider the following two events. What are the conditional probabilities $\Pr[A|B]$ and $\Pr[B|A]$?

- A: I choose an odd number
- B: I choose a prime number
I have two coins in an opaque bag. The coins are visually indistinguishable, but one coin is fair (Prob H = 0.5); and the other coin is biased (Prob H = 0.75). I pull one of the two coins out at random. If I flip the coin and it comes up heads, what is the probability that I'm holding the biased coin?