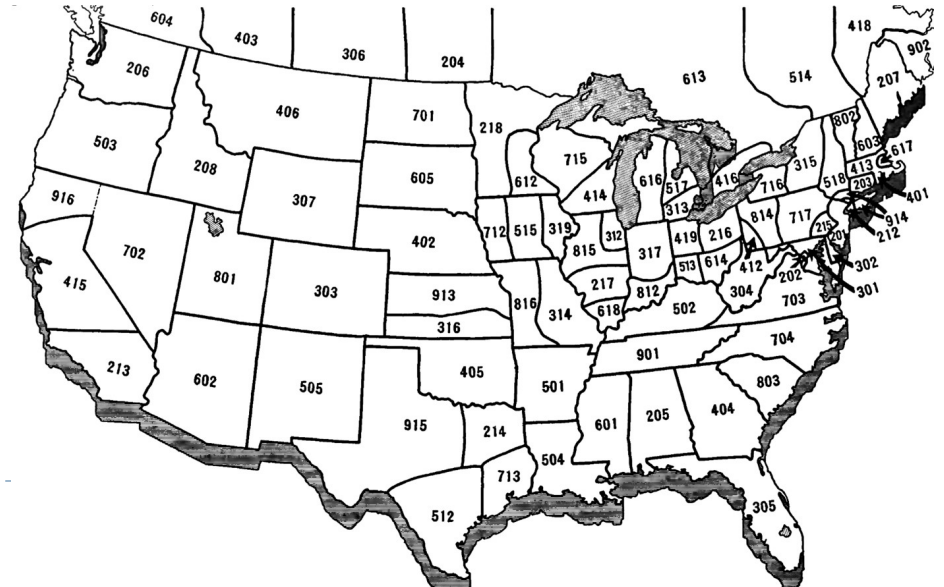
csci54 – discrete math & functional programming
counting

Counting questions

- If Steve has 4 pairs of pants and 5 soccer jerseys, how many different outfits (different combinations of pants and jersey) can he assemble?

- An area code is a 3-digit number whose middle digit (at least originally) had to be a 0 or a 1. Assuming no other restrictions, how many different area codes could there be?

- If there are 165 students, 119 of which are declared CS majors and 60 of whom are declared math majors, how many of the students are double majoring in CS and math?



Counting

- sum rule:
  - if A and B are disjoint, then $|A \cup B| = |A| + |B|$
- product rule:
  - the number of pairs (x,y) where $x \in A$ and $y \in B$ is $|A \times B| = |A| \cdot |B|$
- inclusion-exclusion:
  - $|A \cup B| = |A| + |B| - |A \cap B|$

More practice questions

- Different states have different standard license plate "numbers". How many different license plates are possible in each of the following states?
  - Connecticut: "digit-letter-letter-letter-letter-digit" (e.g. 9ABCD9)
  - Minnesota: "digit-digit-digit-letter-letter-letter" (e.g. 999ABC)

- If you wanted to replace CT license plate with k symbols (each could be either a digit or letter), how many symbols would you need to have at least as many possible distinct license plates?

- Same as above, but what if each symbol could appear at most once on the new CT license plates?

Practice problems - recap

- Let there be 165 students of whom 119 declared CS and 60 declared math. How many are double majoring in CS and math? — inclusion-exclusion
- How many distinct license plates are possible in each state? — product rule
- If you wanted to replace CT license plate with k symbols (each could be either a digit or letter), how many symbols would you need to have at least as many possible distinct license plates? — sum rule + product rule
- What if each symbol could appear at most once on the new license plates? — permutation

sum, product, inclusion-exclusion, permutations

Permutations

- A permutation of a set S is a sequence of elements from S that is of length |S| and contains no repetitions. In other words, a permutation of S is an ordering of the elements of S.

- Let S be any set and let n = |S|. The number of different permutations of S is n! (n factorial)

  where n! = n * (n-1)!, and 0! = 1

This is just the product rule in action! N ways to pick item 1, n-1 ways to pick item 2, ...

Practice questions

- What fraction of integers between 0 and 9,999,999,999 (all written as 10-digit numbers, including any leading zeros) have no repeated digits?

- How many integers between 0 and 999,999 (all written as 6-digit numbers, including any leading zeros) have at least one repeated digit?

- Suppose you have 17 tasks on your todo list, but only time to do 4 of them (and they all take the same amount of time, without dependencies). How many ways are there to select 4 of the possible tasks?

Permutations and combinations (choosing k from n)

- Permutation: choosing k items without replacement, order matters
  - Given a set of 12 runners, how many ways are there to choose a gold, silver, and bronze medalist?
    - 12x11x10
    - Alternatively, 12! / 9!
- Combination: choosing k without replacement, order doesn't matter
  - Given a set of 12 runners, how many ways are there to choose 3 of them to be in "group A"?
    - C(12,3) or $\binom{12}{3}$
    - read "12 choose 3"

Suppose you have 17 tasks on your todo list, but only time to do 4 of them. How many ways are there to select 4 of the possible tasks?

More practice questions

- Suppose two teams A and B play a best-of-three series of games. How many different sequences of outcomes are there in which A wins the overall series? What if they play a best-of-five?

- How many 10-bit strings have at most 2 ones?

- How many solutions are there to the equation a+b+c=8 where a, b, and c are all non-negative integers?

Choosing k of n elements

- Can you choose an element more than once? (is repetition allowed)
- Does the order matter?

- order matters, without repetition (n!/(n-k)!)

- order matters, with repetition (n^k)

- order doesn't matter, with repetition (n^k / k!)

- order doesn't matter, without repetition

Choosing with Repetition

- order doesn't matter, without repetition: (n+r-1)C(r)
- Example: we are ordering 3 dishes out of a possible 5 at a tapas restaurant.
- Doing something like 5^3 / 3! gives us 125/6 which is not an integer!
  - Because we don’t **know** that each of the 125 options has 6 equivalent orderings
- We could get there by sum rule: no repeated dishes + 1 repeated dish + all three the same
- But this is hard to scale up

Choosing with Repetition

- order doesn't matter, without repetition: (n+r-1)C(r)
- Example: We are ordering 3 tapas out of a possible 5
- We can shift the question slightly: imagine a finger going down the menu. At each step, we can either “move finger down” or “order one of these”
  - If we order 3 times, we need to “move down” until the end
  - If the finger is at the end, we need to order until done
- So we have a sequence of Move and Order which is 5 steps long (we need two moves to get to the end, and three orders)
- E.g. MOMOO, OMMOO, MMOOO, OOOMM
- How many ways can we put two M among 5 spots? (or, 3 O among 5 spots?) : (5+3-1) C (3)

More practice questions

- There are 141 CS majors. How many ways are there to choose a team of 3 CS majors?

- At the start of a programming contest, teams are given 10 questions to try to solve. At the start of the contest, each member of the team has to choose a problem to think about first. (More than one team member can think about the same problem.) How many ways are there for the 3 team members to choose a problem to think about first?

- Suppose that a team has calculated that they have time to code up and submit 20 different attempted answers to the 10 questions in the contest. How many different ways can they allocate their 20 submissions across the 10 problems? (The order of their submissions doesn't matter and they can submit more than once to each question.)