# csci54 – discrete math & functional programming
# relations, RSA

# Relations recap

- A (binary) relation on a set A is a subset of AxA
- A relation can be any, or none, of the following:
  - reflexive
  - symmetric
  - transitive
- A relation that is reflexive, symmetric, and transitive is called an equivalence relation
- An equivalence relation on a set A partitions A into a set of equivalence classes

# Practice questions

- For each of the following relations, indicate if it is reflexive, symmetric, and/or transitive. If it's all three and therefore an equivalence relation, describe the equivalence classes.

1. S = all juniors and seniors at Pomona. (x,y) in $R_1$ iff x and y share a major.
2. S = Z. (x,y) in $R_2$ iff x=y.
3. S = {1,2,3,4,5}. $R_3$ = {⟨1, 5⟩, ⟨2, 2⟩, ⟨2, 4⟩, ⟨4, 1⟩, ⟨4, 2⟩}.

- S = all students at Pomona. Define an equivalence relation on S that isn't one of the ones discussed in lecture last time.

# Closures

**Definition 8.11: Reflexive, symmetric, and transitive closures.**

Let $R \subseteq A \times A$ be a relation. Then:

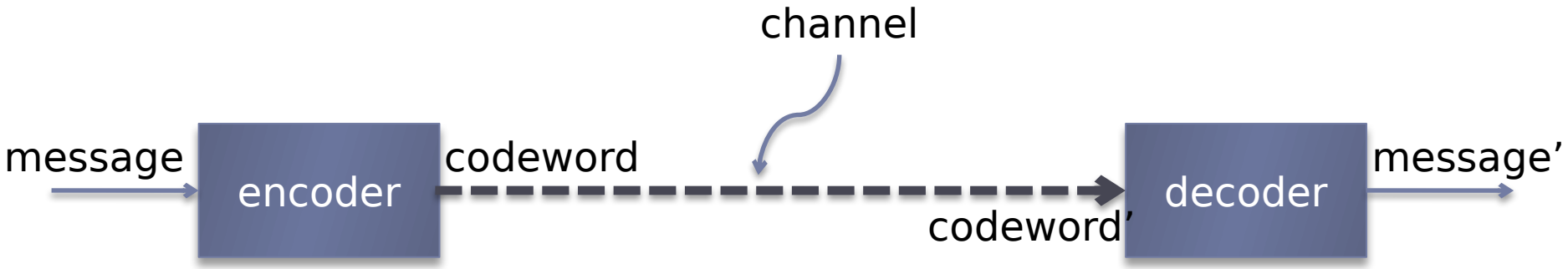
The *reflexive closure of* $R$ is the smallest relation $R' \supseteq R$ such that $R'$ is reflexive.

The *symmetric closure of* $R$ is the smallest relation $R'' \supseteq R$ such that $R''$ is symmetric.

The *transitive closure of* $R$ is the smallest relation $R^+ \supseteq R$ such that $R^+$ is transitive.

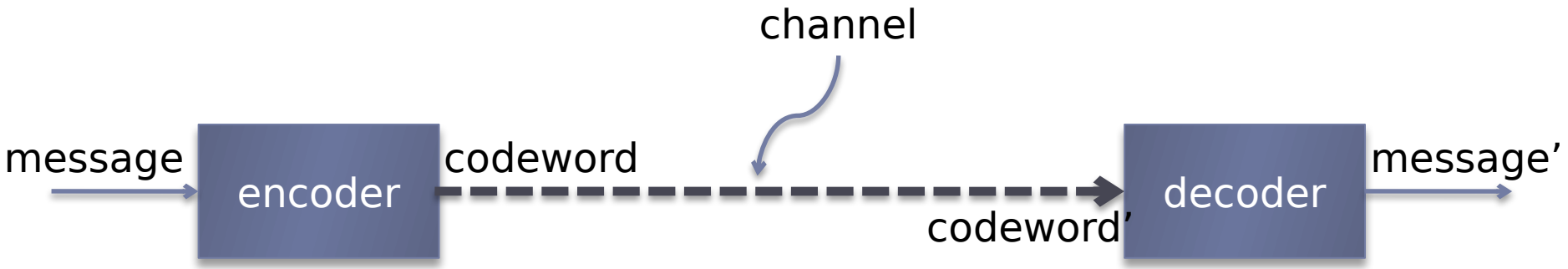
- Consider the relation R = {⟨1, 5⟩, ⟨2, 2⟩, ⟨2, 4⟩, ⟨4, 1⟩, ⟨4, 2⟩} on {1, 2, 3, 4, 5}
  - What is the reflexive closure?
  - What is the symmetric closure?
  - What is the transitive closure?
- S = all students at Pomona. (x,y) in $R_1$ if x and y share a major.

# Transmitting information



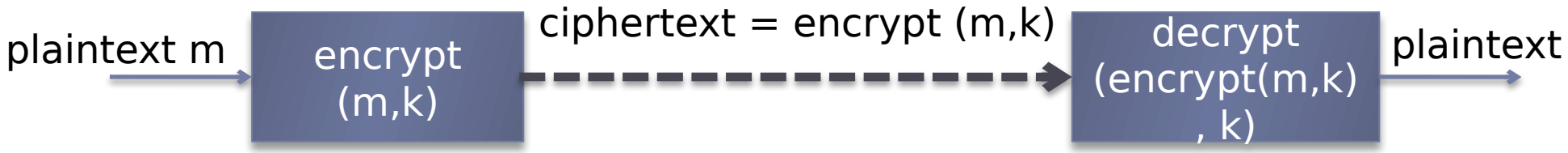

- cryptography
- error correction
- compression

# Transmitting information - cryptography




- goal is to keep someone with access to the channel from finding out information about the message.
- assumptions (for now)
  - message = message'
  - codeword = codeword'
- why?
- how?

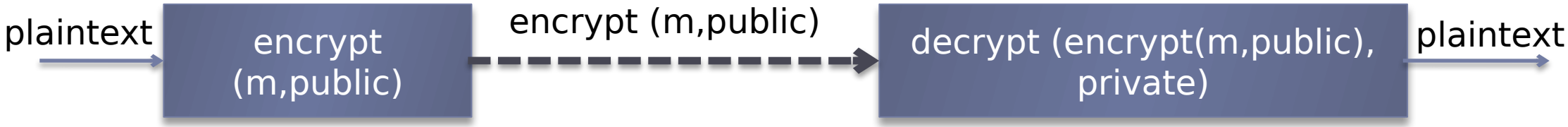
# Private key cryptography




- Symmetric-key algorithms
- The communicating parties share a piece of secret information (the key k)

- Examples?
- Challenges?

# Public key cryptography




- asymmetric-key algorithm

- Everyone publishes their public key.
- If you want to send a message to someone, you encrypt it with their public key.
- When you get a message you can decrypt it with your private key.

# Public key encryption in practice

A

**Richard Adams**

Fingerprint: `3DAF 842A DAAF 190D AB13 D701 DCCE B6EA 2697 15E7`

Email Address: `richard.adams[@]theguardian.com`

PUBLIC KEY

**Esther Addley**

Fingerprint: `081E D0F2 4742 3B23 2C53 B0DA FA39 808F D279 4676`

Email Address: `esther.addley[@]theguardian.com`

PUBLIC KEY

**Jonathan Alden**

Fingerprint: `29C6 09D9 5CF7 1EEE 7549 66F6 CD1B 662A 4686 0F81`

Email Address: `jonathan.alden.freelance[@]theguardian.com`

PUBLIC KEY

**Lorena Allam**

Fingerprint: `36D5 D5B6 0ADF FB83 0528 F6AF 5D00 EECE 797E 0CD7`

Email Address: `Lorena.Allam[@]theguardian.com`

PUBLIC KEY

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## Encrypted Email

If you use PGP encryption, here is our fingerprint and link to our public key. If you use our public key with a mail encryption plugin, for example Mailvelope or Enigmail, this encrypts the contents of your message but not the subject line or the name of the sender.

Fingerprint:
**EC6C 2905 F0F9 3C03 7394 6CA1 0642 427A 5FF7 80BE**

Email: **lockbox@washpost.com**

The Post's public key

Copy


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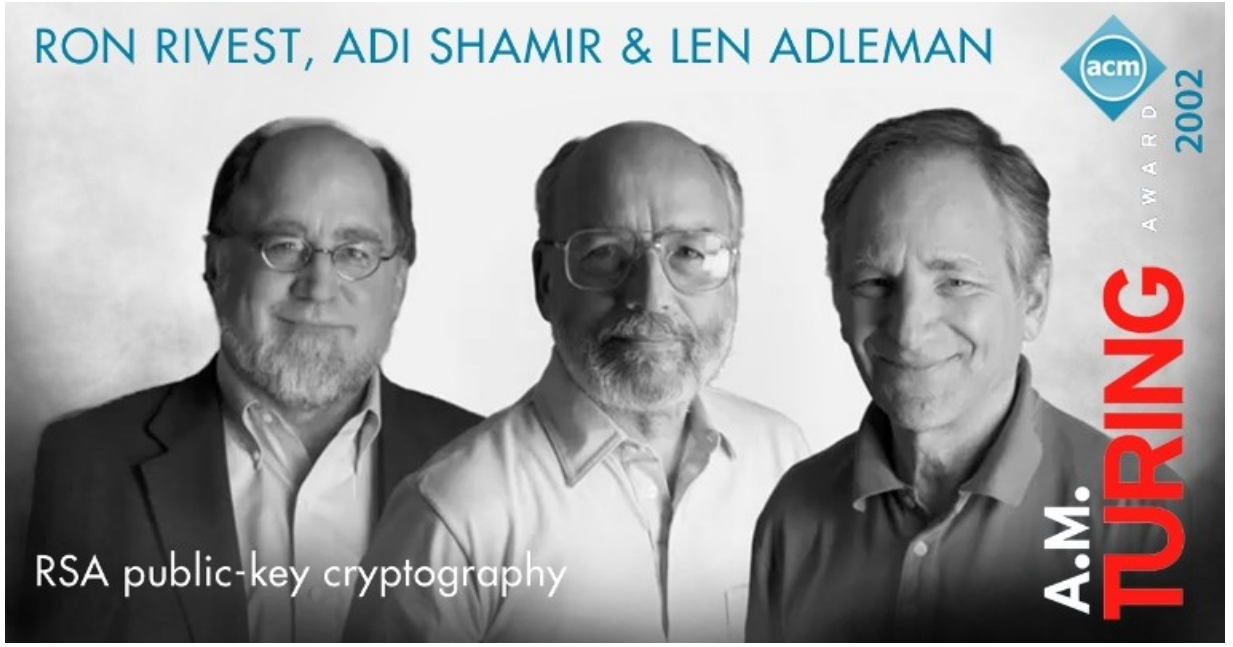
https://www.theguardian.com/pgp
https://www.washingtonpost.com/anonymous-news-tips/

# RSA algorithm

- A very widely used public key encryption algorithm
- Three algorithmic components
  - key generation
  - encryption
  - decryption
- Our plan
  - What is the algorithm?
  - Why does it work?
  - How to implement it efficiently?



# Modular arithmetic – definitions and properties

$a \equiv b \pmod m$ if and only if $a \bmod m = b \bmod m$

i.e., exists x, y, z in Z: $a = x * m + z \wedge b = y * m + z$

Some useful facts from Wikipedia:

(if $a1 \equiv a2 \pmod m$ and $b1 \equiv b2 \pmod m$):

- $a + k \equiv b + k \pmod m$ for any integer k (compatibility with translation)
- $k\,a \equiv k\,b \pmod m$ for any integer k (compatibility with scaling)
- $k\,a \equiv k\,b \pmod{k * m}$ for any integer k
- $a1 + a2 \equiv b1 + b2 \pmod m$ (compatibility with addition)
- $a1 - a2 \equiv b1 - b2 \pmod m$ (compatibility with subtraction)
- $a1 * a2 \equiv b1 * b2 \pmod m$ (compatibility with multiplication)
- $a^k \equiv b^k \pmod m$ for any non-negative integer k (compatibility with exponentiation)