csci54 – discrete math & functional programming
relations, RSA

Relations recap

- A (binary) relation on a set A is a subset of AxA
- A relation can be any, or none, of the following:
  - reflexive
  - symmetric
  - transitive

- A relation that is reflexive, symmetric, and transitive is called an equivalence relation

- An equivalence relation on a set A partitions A into a set of equivalence classes

Practice questions

- For each of the following relations, indicate if it is reflexive, symmetric, and/or transitive. If it's all three and therefore an equivalence relation, describe the equivalence classes.

1. S = all juniors and seniors at Pomona. (x,y) in R_1 iff x and y share a major.
2. S = Z. (x,y) in R_2 iff x=y.
3. S = {1,2,3,4,5}. $R_3 = \{\langle 1, 5\rangle, \langle 2, 2\rangle, \langle 2, 4\rangle, \langle 4, 1\rangle, \langle 4, 2\rangle\}$.

- S = all students at Pomona. Define an equivalence relation on S that isn't one of the ones discussed in lecture last time.

Closures

Definition 8.11: Reflexive, symmetric, and transitive closures.

Let $R \subseteq A \times A$ be a relation. Then:

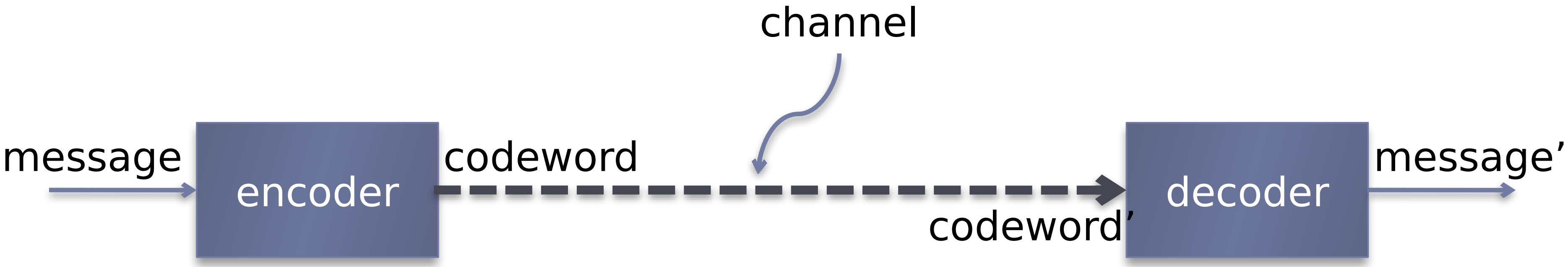
The *reflexive closure of* R is the smallest relation $R' \supseteq R$ such that R' is reflexive.

The *symmetric closure of* R is the smallest relation $R'' \supseteq R$ such that R'' is symmetric.

The *transitive closure of* R is the smallest relation $R^+ \supseteq R$ such that R^+ is transitive.

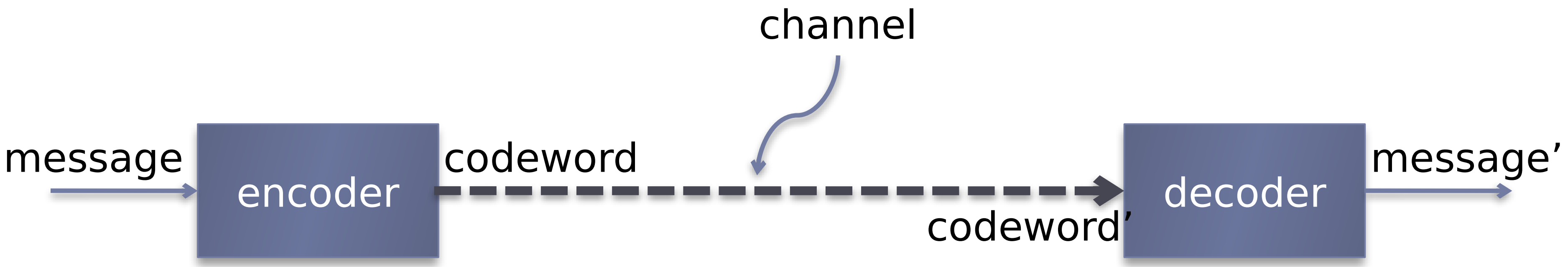
- Consider the relation R = {⟨1, 5⟩, ⟨2, 2⟩, ⟨2, 4⟩, ⟨4, 1⟩, ⟨4, 2⟩} on {1, 2, 3, 4, 5}
  - What is the reflexive closure?
  - What is the symmetric closure?
  - What is the transitive closure?
- S = all students at Pomona. (x,y) in R_1 if x and y share a major.

Transmitting information



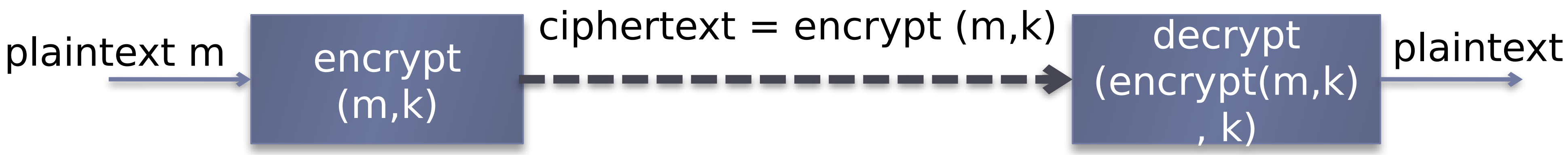
- cryptography
- error correction
- compression

Transmitting information - cryptography



- goal is to keep someone with access to the channel from finding out information about the message.
- assumptions (for now)
  - message = message'
  - codeword = codeword'
- why?
- how?

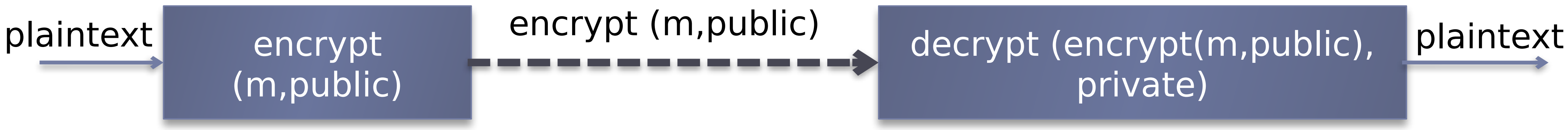
Private key cryptography



- Symmetric-key algorithms
- The communicating parties share a piece of secret information (the key k)

- Examples?
- Challenges?

Public key cryptography



- asymmetric-key algorithm

- Everyone publishes their public key.
- If you want to send a message to someone, you encrypt it with their public key.
- When you get a message you can decrypt it with your private key.

Public key encryption in practice

A

Richard Adams

Fingerprint: 3DAF 842A DAAF 190D AB13 D701 DCCE B6EA 2697 15E7

Email Address: richard.adams[@]theguardian.com

PUBLIC KEY

Esther Addley

Fingerprint: 081E D0F2 4742 3B23 2C53 B0DA FA39 808F D279 4676

Email Address: esther.addley[@]theguardian.com

PUBLIC KEY

Jonathan Alden

Fingerprint: 29C6 09D9 5CF7 1EEE 7549 66F6 CD1B 662A 4686 0F81

Email Address: jonathan.alden.freelance[@]theguardian.com

PUBLIC KEY

Lorena Allam

Fingerprint: 36D5 D5B6 0ADF FB83 0528 F6AF 5D00 EECE 797E 0CD7

Email Address: Lorena.Allam[@]theguardian.com

PUBLIC KEY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Encrypted Email

If you use PGP encryption, here is our fingerprint and link to our public key. If you use our public key with a mail encryption plugin, for example Mailvelope or Enigmail, this encrypts the contents of your message but not the subject line or the name of the sender.

Fingerprint:
EC6C 2905 F0F9 3C03 7394 6CA1 0642 427A 5FF7 80BE

Email: **lockbox@washpost.com**

The Post's public key

Copy

-----BEGIN PGP PUBLIC KEY BLOCK-----

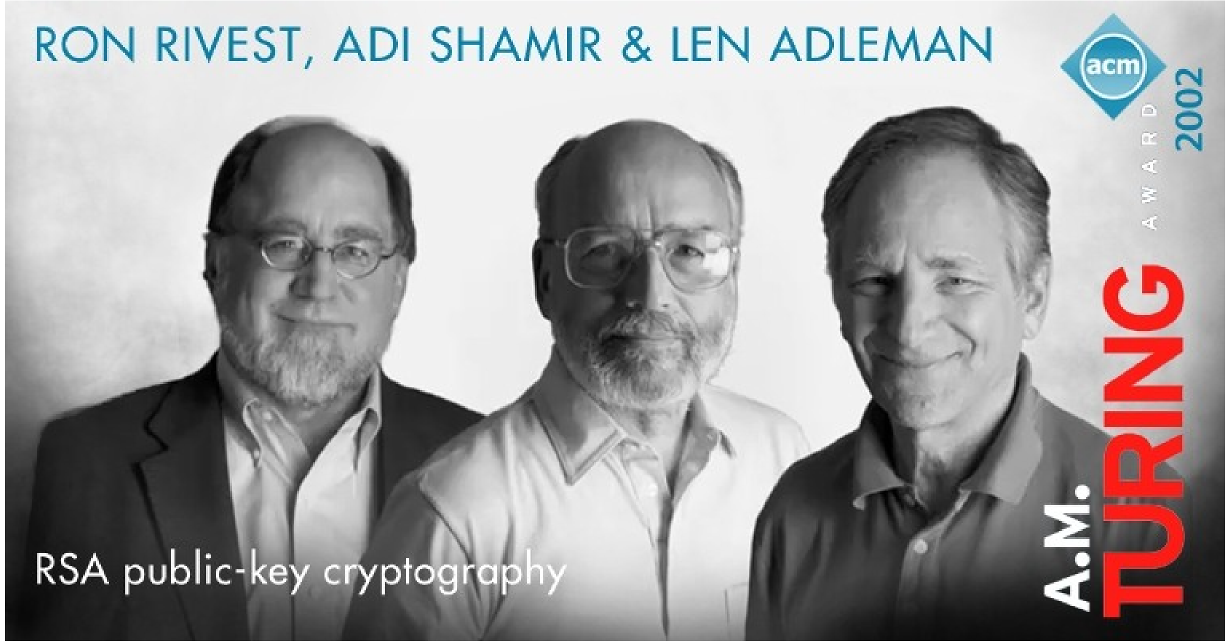
https://www.theguardian.com/pgp
https://www.washingtonpost.com/anonymous-news-tips/

RSA algorithm

- A very widely used public key encryption algorithm

- Three algorithmic components
  - key generation
  - encryption
  - decryption

- Our plan
  - What is the algorithm?
  - Why does it work?
  - How to implement it efficiently?



Modular arithmetic – definitions and properties

$a \equiv b \pmod{m}$ if and only if $a \bmod m = b \bmod m$

i.e., exists x, y, z in Z: $a = x * m + z \wedge b = y * m + z$

Some useful facts from Wikipedia:

(if $a1 \equiv a2 \pmod{m}$ and $b1 \equiv b2 \pmod{m}$):

- $a + k \equiv b + k \pmod{m}$ for any integer k (compatibility with translation)
- $k a \equiv k b \pmod{m}$ for any integer k (compatibility with scaling)
- $k a \equiv k b \pmod{k * m}$ for any integer k
- $a1 + a2 \equiv b1 + b2 \pmod{m}$ (compatibility with addition)
- $a1 - a2 \equiv b1 - b2 \pmod{m}$ (compatibility with subtraction)
- $a1 * a2 \equiv b1 * b2 \pmod{m}$ (compatibility with multiplication)
- $a^k \equiv b^k \pmod{m}$ for any non-negative integer k (compatibility with exponentiation)