In-Class Worksheet

Discrete Math & Functional Programming— CSCI 054— Fall 2024
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Claim: $\forall n \in \mathbb{Z}_0^+ : \sum_{i=0}^{n} 2^i = 2^{n+1} - 1$

- We prove the claim using a proof by induction on:
- Base case:
- Inductive hypothesis (IHOP):
- Inductive step:
  - We want to show that:
  - Proof:
- Therefore by the principle of mathematical induction:

Identify the smallest integer p such that $\forall n \geq p : n! > 2^n$. Prove that your choice of p is correct.

Let's play with some definitions of evenness and oddness.

$$\texttt{even}(0) \wedge \forall n : \texttt{even}(n) \implies \texttt{even}(n+2)$$

$$\texttt{odd}(1) \wedge \forall n : \texttt{odd}(n) \implies \texttt{odd}(n+2)$$

Prove the following. If you're using induction, you probably don't want to go up one number at a time but 2 at a time, so we'll have n'+2 in the inductive case if we know n' is even. You can say "by induction on the evidence for n being even".

1. If a number is even, that number plus one is odd (using the $\exists k, x = \ldots$ definitions).
2. If a number is odd, that number plus one is even (using the $\exists k, x = \ldots$ definitions).
3. If a number is even, that number plus one is odd (using only the new definitions).
4. If a number is odd, that number plus one is even (using only the new definitions).
5. Prove that if a number isn't odd, it's even, and vice versa (using the definitions above)..
6. Adding two evens gives you an even, by the $\exists$ definition.
7. Adding two odds gives you an even, by the $\exists$ definition.
8. Adding an odd and an even gives you an odd, by the $\exists$ definition.
9. A number is even by the new definition if and only if it's also even by the exists definition (only using the facts above!).
10. Adding two evens gives you an even, by the new definition.
11. Adding two odds gives you an even, by the new definition.
12. Adding an odd and an even gives you an odd, by the new definition.