In-Class Worksheet

Discrete Math & Functional Programming— CSCI 054— Fall 2024

Instructor: Osborn

Given an implication $p \Rightarrow q$, we can define the following related concepts:

- converse: $q \Rightarrow p$
- inverse: $\neg p \Rightarrow \neg q$
- contrapositive: $\neg q \Rightarrow \neg p$

Which, if any, are logically equivalent to the original implication?

If 2 is an even number, then 3 is an odd number

If x is an even number, then $x + 1$ is an odd number

Define the predicates:

- rested(n) = "n got at least 8 hours of sleep in the past 24 hours"
- bornMA(n) = "n was born in Massachusetts"

Which, if any, of the following propositions is True? Justify your answer.

- $\exists$ n in this room : rested(n)
- $\forall$ n in this room : (rested(n) $\Rightarrow$ bornMA(n))
- $\exists$ n currently enrolled at Pomona College : (rested(n) $\wedge$ bornMA(n))
- $\forall$ currently enrolled at Pomona College : (rested(n) $\vee$ bornMA(n))

Is the following a theorem? Explain.

$$[\forall x \in S : P(x)] \vee [\forall x \in S : \neg P(x)]$$