csci54 – discrete math & functional programming
basic data types: sets, function, relations
"Connecting Discrete Mathematics and Computer Science"
by David Liben-Nowell

https://cs.carleton.edu/faculty/dln/book/
Python has 4 built-in data types for storing collections of data. What are they? How are they different? What is each one good for?

- lists: ordered, indexed, mutable, allows duplicate values
- tuples: ordered, indexed, immutable, allows duplicate values
- dictionaries: unordered, mutable, cannot have duplicate keys
- sets: unordered, unindexed, cannot have duplicate keys, can add/remove elements but can't change existing elements
A set is an unordered collection of objects:

- Given a set $S$ and an object $o$, either $o \in S$ or $o \notin S$
- The cardinality of a set is written $|S|$ and is the number of elements in the set

Examples of sets we've seen:

- Int
- Integer
- Char
- Bool
- $\mathbb{Z}$: set of integers
- $\mathbb{Z}^+$: set of positive integers
- $\mathbb{N}$: set of non-negative integers
- $\mathbb{Q}$: set of rationals
- $\mathbb{R}$: set of reals
Defining a mathematical set

- exhaustive enumeration: list everything

  \[ S = \{1, 2, 17\} \]

- set abstraction: define a set using set operations or a “set builder notation” like our list comprehensions

  - \( Z \): set of integers
  - \( Z^+ \): set of positive integers
  - \( N \): set of non-negative integers
  - \( Q \): set of rationals
  - \( R \): set of reals

  The empty set, which contains no elements: \( \{\} \) or \( \emptyset \)

  The universal set, \( U \)
Set operations (what can you do with sets S and T?)

- Informally . . .

- S or $S^c$: set complement
  - set of elements that are in U (the universal set) but not in S

- $S \cup T$: set union
  - set of elements that are in S or in T

- $S \cap T$: set intersection
  - set of elements that are in S and in T

- S-T: set difference
  - set of elements that are in S and not in T
Set operations in set notation

- **S^c**: set complement
  - set of elements that are in U (the universal set) but not in S
  - \( S^c = \{x \in U : x \notin S\} \)

- **S∪T**: set union
  - set of elements that are in S or in T
  - \( ST \{x : x \in S \text{ or } x \in T\} \)

- **S∩T**: set intersection
  - set of elements that are in S and in T
  - \( ST \{x : x \in S \text{ and } x \in T\} \)

- **S-T**: set difference
  - set of elements that are in S and not in T
  - \( ST \{x : x \in S \text{ and } x \notin T\} \)

**Example**

- \( U = \mathbb{Z}^+ \)
- \( A = \{n : n \geq 6\} \)
- \( B = \{1, 2, 4, 5, 7, 8\} \)
- \( A^C \)
- \( A \cap B \)
- \( A \cup B \)
- \( |B| \)
What can you say about sets S and T?

- $\subseteq$: subset
  - S contains T

- $\subset$: proper subset
  - S contains T and S does not equal T

- $\supseteq$: superset
  - T contains S

- $\supset$: proper superset
  - T contains S and T does not equal S

Are either A or B a subset of the other?

Given an example of a proper superset of B.

$U = \mathbb{Z}^+$

$A = \{n : n \geq 6\}$

$B = \{1, 2, 4, 5, 7, 8\}$
Definition 2.46: Function.
Let $A$ and $B$ be sets. A function $f$ from $A$ to $B$, written $f : A \rightarrow B$, assigns to each input value $a \in A$ a unique output value $b \in B$; the unique value $b$ assigned to $a$ is denoted by $f(a)$. We sometimes say that $f$ maps $a$ to $f(a)$.

- Example(s) of function(s) from $\{1, 2, 3\}$ to $\{2, 4, 6\}$

- What is an example of a function
Defining functions

- symbolically

- exhaustively

- how would you define the function for "and"?
  - what does it map from/to?
Cartesian product

The Cartesian product of two sets is written \( A \times B \) and is defined as:

\[
A \times B = \{ (x,y) : x \in A \text{ and } y \in B \} 
\]

What is \( A \times B \) if \( A = \{1,2\} \) and \( B = \{\text{true, false}\} \)?

How would you define the function for "and"?

How would you define a function which takes two real numbers and returns their average?
Definitions related to functions

- **Given a function**
  - the domain is the set $A$
  - the co-domain is the set $B$
  - the range (or the image) is the subset of $B$ that are actually mapped to by an element in $A$.

- **Examples:**
  - `IsEven`
  - `Pow` (haskell `^`)
  - what's an example of a function whose domain, co-domain, and range are all the same?
classifying functions

- one-to-one: a function is one-to-one if, for every element of the co-domain, at most one element of the domain maps to it.

- onto: a function is onto if, for every element of the co-domain, there is an element of the domain that maps to it.
  - alternatively, a function is onto if the co-domain equals the range

- bijection: a function is a bijection if it is both one-to-one and onto
Hello world!

Let’s define a few sets

- Here’s a set with one element:

\[ S = \{ x \in \mathbb{Z} : x + 10 = 100 \} \]

- Here’s a set with an infinite number of elements: \( S^C \)

note: environments, math mode, packages

a quick intro:

https://www.overleaf.com/learn/latex/Learn_LaTeX_in_30_minutes