Adversarial Search

CS51A
David Kauchak
Fall 2025

Some material borrowed from:
Sara Owsley Sood and others

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Admin

Assignment 9

Assignment 10

Schedule for the rest of the semester

2

Midterm 2

Recursion (10/2) through adversarial search (11/11)

Can bring 2-pages of notes (one double-sided or two singled-sided)

Some sample problems posted

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A quick review of search

Problem solving via search:
To define the state space, define three things:
- is_goal
- next_states
- starting state

Uninformed search vs. informed search
- what's the difference?
- what are the techniques we've seen?
- pluses and minuses?

4

Why should we study games?

Clear success criteria

Important historically for AI

Fun ☺

Good application of search
- hard problems (chess 35^{100} states in search space, 10^{40} legal states)

Some real-world problems fit this model
- game theory (economics)
- multi-agent problems

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Types of games

What are some of the games you've played?

6

Types of games: game properties

single-player vs. 2-player vs. multiplayer

Fully observable (perfect information) vs. partially observable

Discrete vs. continuous

real-time vs. turn-based

deterministic vs. non-deterministic (chance)

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Strategic thinking $\stackrel{?}{=}$ intelligence

Two-player games have been a focus of AI since its inception…

Important question: Is strategic thinking the same as intelligence?

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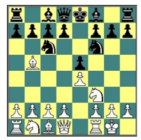
Strategic thinking $\stackrel{?}{=}$ intelligence

Humans and computers have different relative strengths in these games:

humans

?



computers

?

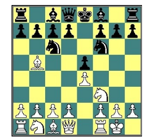
9

Strategic thinking $\stackrel{?}{=}$ intelligence

Humans and computers have different relative strengths in these games:

humans

good at evaluating the strength of a board for a player



computers

good at looking ahead in the game to find winning combinations of moves

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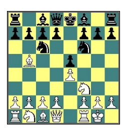
How humans play games…

Positions were shown to novice and expert players…



- experts could reconstruct these perfectly
- novice players did far worse…

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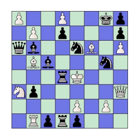
How humans play games…

Random chess positions (not legal ones) were then shown to the two groups



experts and novices did just as badly at reconstructing them!

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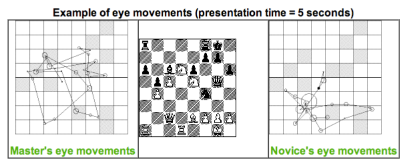
People are still working on this problem…



http://people.brunel.ac.uk/~hsstffg/frg-research/chess_expertise/

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Tic Tac Toe as search

If we want to write a program to play tic tac toe, what question are we trying to answer?

Given a state (i.e. board configuration), what move should we make!

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Tic Tac Toe as search

X		

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Tic Tac Toe as search

X	X	O
	O	O
X	O	X

X	X	O
X	O	O
X	O	X

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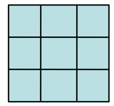
Tic Tac Toe as search



How can we pose this as a search problem?

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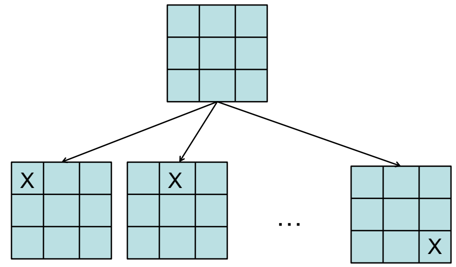
Tic Tac Toe as search



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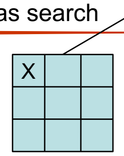
Tic Tac Toe as search



Now what?

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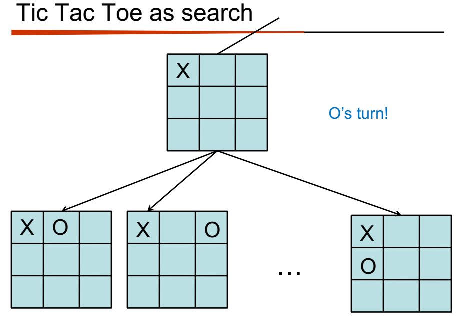
Tic Tac Toe as search



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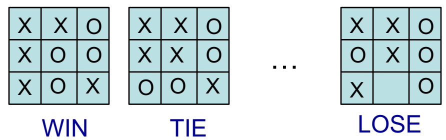
Tic Tac Toe as search

Eventually, we'll get to an ending state



How does this help us?

Try and make moves that move us towards a win, i.e. where there are leaves with a WIN.

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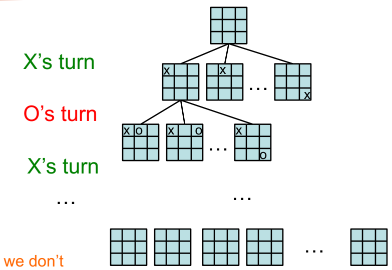
Tic Tac Toe



Problem: we don't know what O will do

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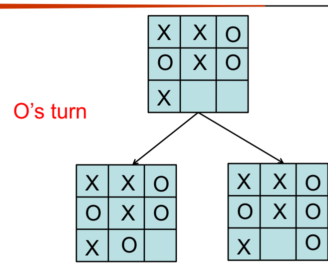
I'm X, what will 'O' do?



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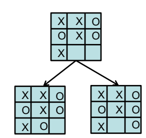
Minimizing risk

The computer doesn't know what move O (the opponent) will make

It can *assume* that it will try and make the best move possible

Even if O actually makes a different move, we're no worse off. Why?



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Optimal Strategy

An Optimal Strategy is one that is at least as good as any other, no matter what the opponent does

– If there's a way to force the win, it will

– Will only lose if there's no other option

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Defining a scoring function

X	X	O
X	O	O
X	O	X

WIN
+1

X	X	O
X	X	O
O	O	X

TIE
0

…

X	X	O
O	X	O
X		O

LOSE
-1

Idea:
- define a function that gives us a "score" for how good each state is
- higher scores mean better

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Defining a scoring function

Our (X) turn

X	X	O
	O	O
X	O	X

What should be the score of this state?

+1: we can get to a win

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Defining a scoring function

Opponent's (O) turn

X	X	O
O	X	O
X		

What should be the score of this state?

-1: opponent can get to a win

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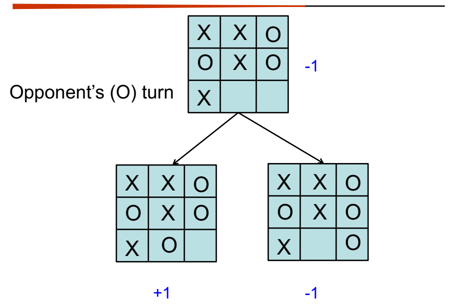
Defining a scoring function



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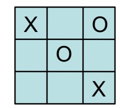
Defining a scoring function

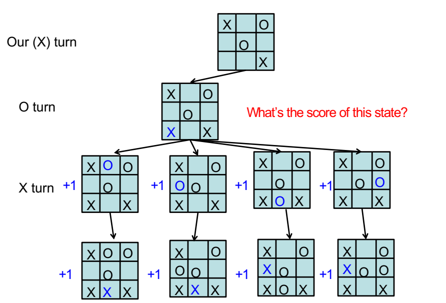
Our (X) turn



What should be the score of this state?

31

Defining a scoring function



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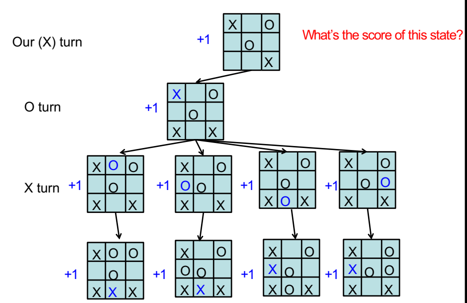
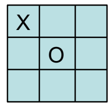
Defining a scoring function



33

Defining a scoring function

Our (X) turn



What should be the score of this state?

0: If we play perfectly and so does O, the best we can do is a tie (could do better if O makes a mistake)

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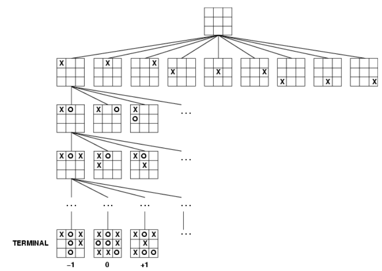
How can X play optimally?



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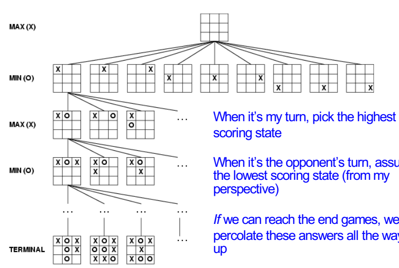
How can X play optimally?



When it's my turn, pick the highest scoring state

When it's the opponent's turn, assume the lowest scoring state (from my perspective)

If we can reach the end games, we can percolate these answers all the way back up

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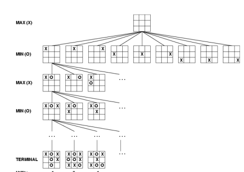
How can X play optimally?

Start from the bottom and propagate the score up:

- if X's turn, pick the move that maximizes the utility
- if O's turn, pick the move that minimizes the utility

Is this optimal?



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Minimax Algorithm: An Optimal Strategy

```
minimax(state) =
    if state is a terminal state
        score(state)
    else if MY turn
        over all next states, s: return the maximum of minimax(s)
    else if OPPONENTS turn
        over all next states, s: return the minimum  of minimax(s)
```

Uses recursion to compute the “value” of each state

Searches down to the leaves, then the values are “backed up” through the tree as the recursion finishes

What type of search is this?

What does this assume about how MIN will play? What if this isn’t true?

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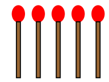
Baby Nim

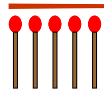


Take 1 or 2 at each turn
Goal: take the last match

What move should I take?

39

Baby Nim



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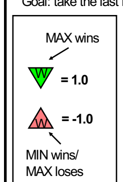
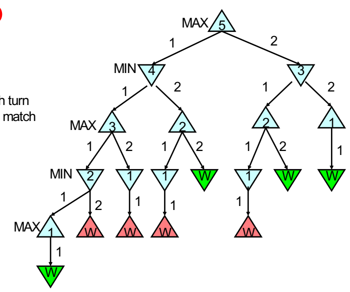
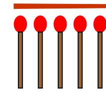




40

Baby Nim



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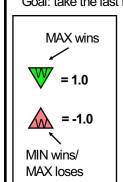

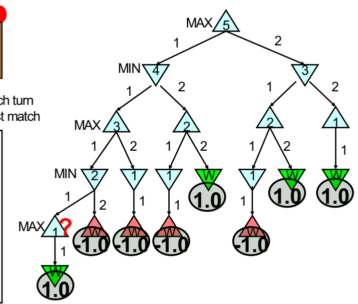

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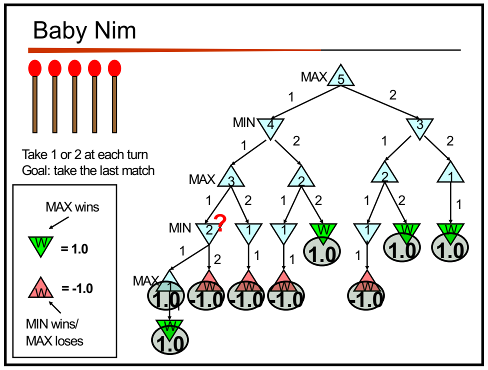
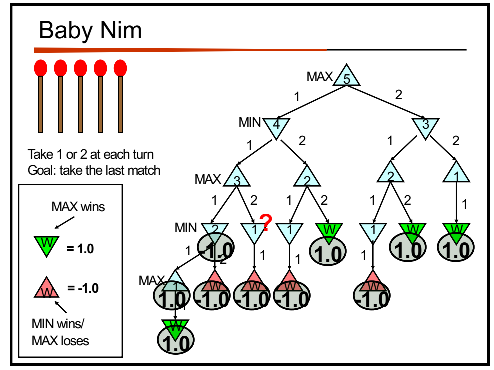

Baby Nim
Take 1 or 2 at each turn
Goal: take the last match
MAX wins
= 1.0
= -1.0
MIN wins/
MAX loses
42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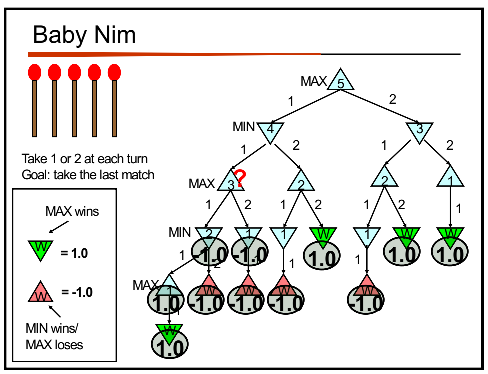
43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Goal: take the last match
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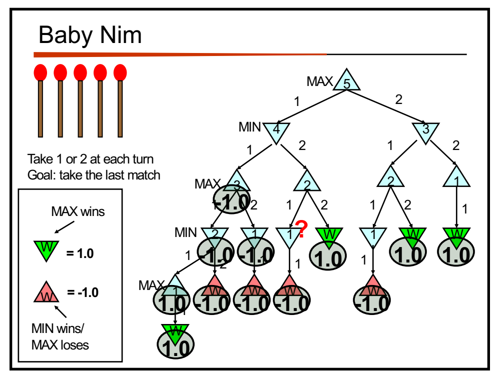
44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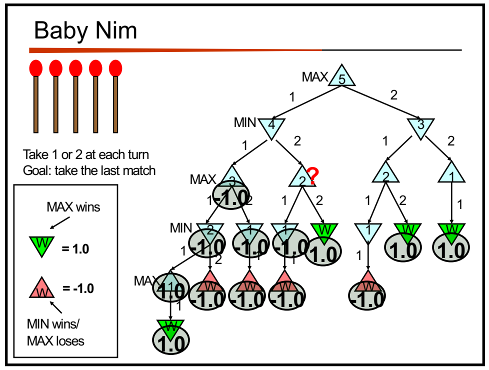
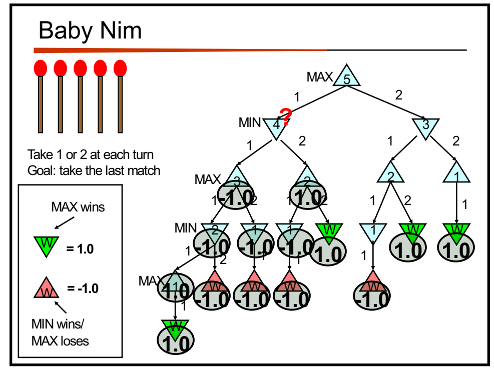
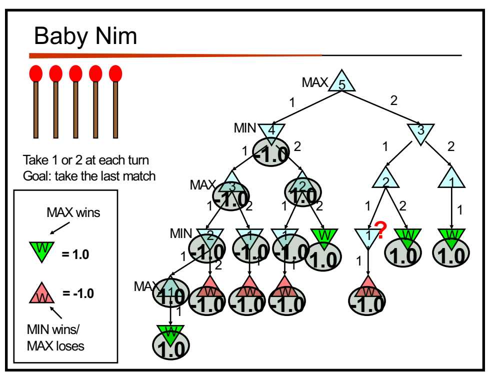
MAX wins
= 1.0
= -1.0
MIN wins/
MAX loses
45


Baby Nim
Take 1 or 2 at each turn
Goal: take the last match
MAX wins
= 1.0
= -1.0
MIN wins/
MAX loses

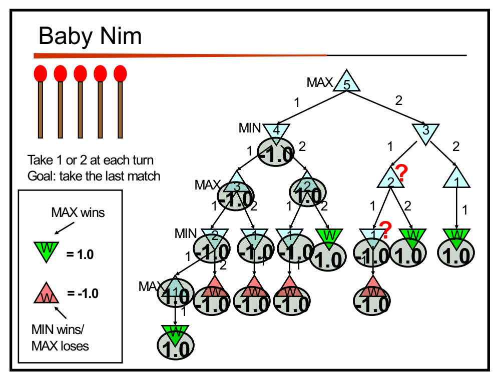
46


Baby Nim
Take 1 or 2 at each turn
Goal: take the last match
MAX wins
= 1.0
= -1.0
MIN wins/
MAX loses

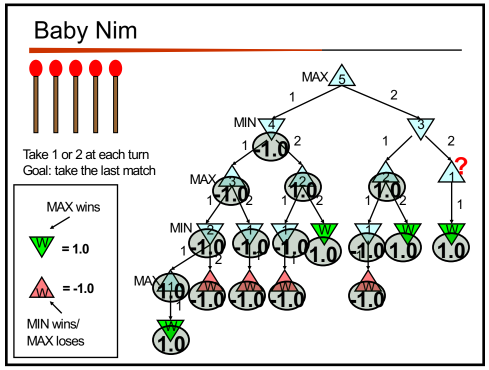
47


Baby Nim
Take 1 or 2 at each turn
Goal: take the last match
MAX wins
= 1.0
= -1.0
MIN wins/
MAX loses

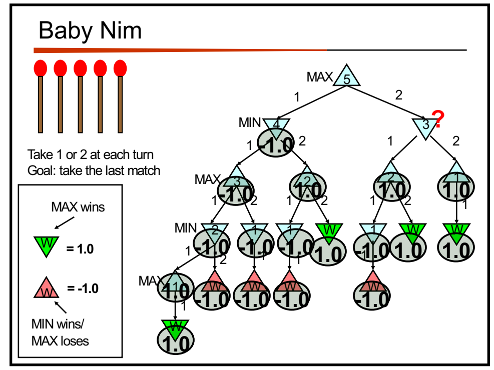
48


Baby Nim
Take 1 or 2 at each turn
Goal: take the last match
MAX wins
= 1.0
= -1.0
MIN wins/
MAX loses

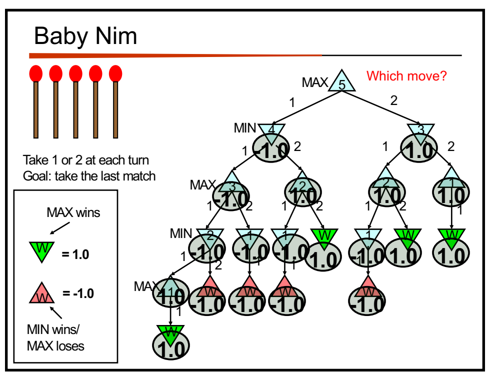
49


Baby Nim
Take 1 or 2 at each turn
Goal: take the last match
MAX wins
= 1.0
= -1.0
MIN wins/
MAX loses

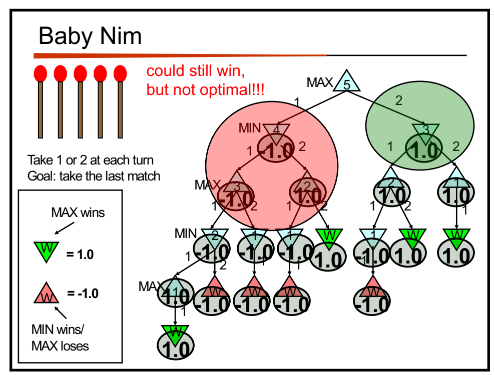
50


Baby Nim
Take 1 or 2 at each turn
Goal: take the last match
MAX wins
= 1.0
= -1.0
MIN wins/
MAX loses

51


Baby Nim
Which move?
Take 1 or 2 at each turn
Goal: take the last match
MAX wins
= 1.0
= -1.0
MIN wins/
MAX loses

52


Baby Nim
could still win,
but not optimal!!!
Take 1 or 2 at each turn
Goal: take the last match
MAX wins
= 1.0
= -1.0
MIN wins/
MAX loses

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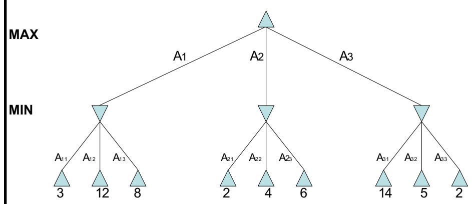
Minimax example 2



Which move should be made: A_1, A_2 or A_3?

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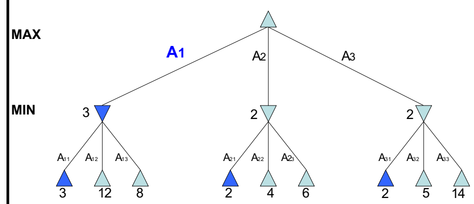
Minimax example 2



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Properties of minimax

Minimax is optimal!

Are we done?

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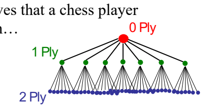
Games State Space Sizes

On average, there are ~35 possible moves that a chess player can make from any board configuration…

18 Ply!!

17005

Hydra at home in the United Arab Emirates…



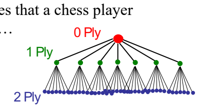
Branching Factor Estimates for different two-player games

Tic-tac-toe	4
Connect Four	7
Checkers	10
Othello	30
Chess	35
Go	300

57

Games State Space Sizes

On average, there are ~35 possible moves that a chess player can make from any board configuration…



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Boundaries for qualitatively different games…

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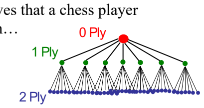
Games State Space Sizes

On average, there are ~35 possible moves that a chess player can make from any board configuration…



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Can search entire space

"solved" games

CHINOOK (2007) →

computer-dominated

Can't ☹

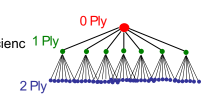
Is this true? human-dominated

59

Games State Space Sizes

AlphaGo (created by Google), in April 2016 beat one of the best Go players:

http://www.nytimes.com/2016/04/05/science/google-alphago-artificial-intelligence.html



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Can search entire space

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Can't ☹

What do we do?

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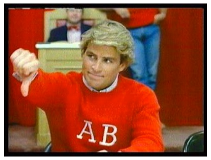
Alpha-Beta pruning

An optimal pruning strategy

- only prunes paths that are suboptimal (i.e. wouldn't be chosen by an optimal playing player)
- returns the *same* result as minimax, but faster



Name the movie ☺

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Alpha-Beta pruning

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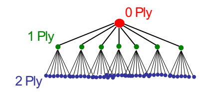
Games State Space Sizes

Pruning helps get a bit deeper

For many games, still can't search the entire tree

Now what?



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computer-dominated

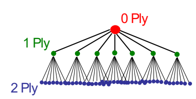
63

Games State Space Sizes

Pruning helps get a bit deeper

For many games, still can't search the entire tree

Go as deep as you can:
- *estimate* the score/quality of the state (called an evaluation function)
- use that instead of the real score



Branching Factor Estimates for different two-player games

Tic-tac-toe	4
Connect Four	7
Checkers	10
Othello	30
Chess	35
Go	300

computer-dominated

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Tic Tac Toe evaluation functions

O	X	X
	O	

Ideas?

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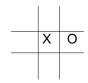
Example Tic Tac Toe EVAL

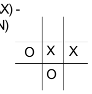
Tic Tac Toe
Assume MAX is using "X"

EVAL(*state*) =

if state is win for MAX:
+ ∞
if state is win for MIN:
- ∞
else:
(number of rows, columns and diagonals available to MAX) -
(number of rows, columns and diagonals available to MIN)



= 6 - 4 = 2



= 4 - 3 = 1

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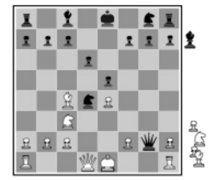
Chess evaluation functions



Ideas?

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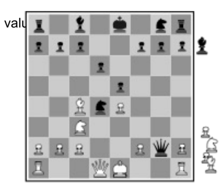
Chess EVAL

Assume each piece has the following valu

pawn	= 1;	
knight	= 3;	
bishop	= 3;	
rook	= 5;	
queen	= 9;	

EVAL(*state*) =
sum of the value of white pieces –
sum of the value of black pieces



= 31 - 36 = -5

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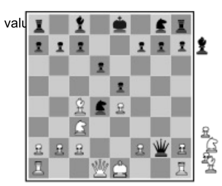
Chess EVAL

Assume each piece has the following valu

pawn	= 1;
knight	= 3;
bishop	= 3;
rook	= 5;
queen	= 9;

EVAL(*state*) =
sum of the value of white pieces –
sum of the value of black pieces



Any problems with this?

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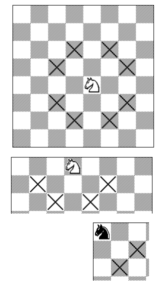
Chess EVAL

Ignores actual positions!

Actual heuristic functions are often a weighted combination of features

$$EVAL(s) = w_1 f_1(s) + w_2 f_2(s) + w_3 f_3(s) + \ldots$$



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Chess EVAL

$$EVAL(s) = w_1 f_1(s) + w_2 f_2(s) + w_3 f_3(s) + \ldots$$

number of pawns

number of attacked knights

1 if king has knighted, 0 otherwise

A feature can be any numerical information about the board

- as general as the number of pawns
- to specific board configurations

Deep Blue: 8000 features!

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history/end-game tables

History

- keep track of the quality of moves from previous games
- use these instead of search

end-game tables

- do a reverse search of certain game configurations, for example all board configurations with king, rook and king
- tells you what to do in ***any*** configuration meeting this criterion
- if you ever see one of these during search, you lookup exactly what to do

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end-game tables

Devastatingly good

Allows much deeper branching

- for example, if the end-game table encodes a 20-move finish and we can search up to 14
- can search up to depth 34

Stiller (1996) explored all end-games with 5 pieces

- one case check-mate required 262 moves!

Knoval (2006) explored all end-games with 6 pieces

- one case check-mate required 517 moves!

Traditional rules of chess require a capture or pawn move within 50 or it's a stalemate

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Opening moves

At the very beginning, we're the farthest possible from any goal state

People are good with opening moves

Tons of books, etc. on opening moves

Most chess programs use a database of opening moves rather than search

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Nim

K piles of coins

On your turn you must take one or more coins from one pile

Player that takes the last coin wins

Example:
https://www.goobix.com/games/nim/

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