Computer Systems

Numeral Systems

CS51 – Spring 2026

Administrative

Assignment 1

Assignment 2

Lab tomorrow

Practice problems at the end of slides

Slack/canvas

Booleans in Python

`bool` type in python: `True` or `False`

The comparison operators `==`, `!=`, `>`, `<`, `>=`, `<=` return `bool`s. For example,

- `10 < 0` evaluates to `False`
- `11 >= 11` evaluates to `True`
- `11 > 11.0` evaluates to `False`
- `10 == 10.0` evaluates to `True`
- `10 != 10.0` evaluates to `False`

Boolean Algebra in Python

We can combine bools and logical operators in Boolean expressions, for example, to evaluate them in **conditionals** (`if` statements) and **loops** (`for` and `while`).

Python supports three logical operators:

- `not` for negation (i.e., ¬)
- `and` for conjunction (i.e., ∧)
- `or` for disjunction (i.e., ∨)

Short-circuit evaluation in Python

Python supports **short-circuiting**: evaluates a Boolean expression from left to right and **stops evaluation** once the truth can be determined

When could this happen with a Boolean expression?

Short-circuit evaluation in Python

Python supports **short-circuiting**: evaluates a Boolean expression from left to right and **stops evaluation** once the truth can be determined

If any part of an `and` is `False`, the expression is `False`

`False and …`

If any part of an `or` is True, the expression is `True`

`True or …`

Short-circuit evaluation in Python

Python supports **short-circuiting**: evaluates a Boolean expression from left to right and **stops evaluation** once the truth can be determined

What is the benefit of this?

Short-circuit evaluation and optimization

```
if x == 0 or (x-1)/x > 0.5: …
```

How does short-circuiting help us?

Short-circuit evaluation and optimization

```
if x == 0 or (x-1)/x > 0.5: …
```

When x is equal to 0, evaluating would cause a divide-by-zero error. But the second disjunct isn’t evaluated when x is equal to 0 because the first operand was True!

Short-circuit evaluation and optimization

It also can be computationally faster:

```
if (simpleOrOftenFalse() and complexOrOftenTrue()):…
```

PRACTICE TIME – Boolean expressions

Assume the variable x currently stores the value 47. What will the following expressions evaluate to?

- True and not False
- not True or not False
- True and not 0
- False or not 1
- 2 < x and x < 50
- x > 2 and < 20
- (2 > x) or (x == 47)
- not (x == x)
- not (x != 3)
- x < 0 and (y/x) == 3
- x > 0 or (y/x) == 3

ANSWER – Boolean expressions

Assume the variable x currently stores the value 47. What will the following expressions evaluate to?

- True and not False - True
- not True or not False - True
- True and not 0 – True
- False or not 1 - False
- 2 < x and x < 50 - True
- x > 2 and < 20 – Invalid syntax
- (2 > x) or (x == 47) - True
- not (x == x) - False
- not (x != 3) – False
- x < 0 and (y/x) == 3 – False
- x > 0 or (y/x) == 3 - True

PRACTICE TIME – Conditionals

Rewrite the following code to make it cleaner and avoid the use of nested if statements:

```
def get_discount(member, purchase_amount):
    if member == True:
        if purchase_amount > 100:
            return "Discount applied"
        else:
            return "No discount"
    else:
        if purchase_amount > 200:
            return "Discount applied"
        else:
            return "No discount"
```

```
print(get_discount(True, 120)) # Expected: "Discount applied" print(get_discount(True, 80)) # Expected: "No discount"
print(get_discount(False, 220)) # Expected: "Discount applied" print(get_discount(False, 50)) # Expected: "No
discount"
```

ANSWER – Conditionals

- Rewrite the following code to make it cleaner and avoid the use of nested if statements:

```
def get_discount(member, purchase_amount):
    if (purchase_amount > 200) or (member and purchase_amount > 100):
        return "Discount applied"
    else:
        return "No discount"
```

```
print(get_discount(True, 120)) # Expected: "Discount applied" print(get_discount(True, 80)) # Expected: "No
discount" print(get_discount(False, 220)) # Expected: "Discount applied" print(get_discount(False, 50)) # Expected:
"No discount"
```

Numbers

What number is 9247?

Numbers

What number is 9247_{10} =

$= 9000 + 200 + 40 + 7$

$= 9 \times 10^3 + 2 \times 10^2 + 4 \times 10^1 + 7 \times 10^0$

Decimal numbers: base 10

For any base, the digits (individual numbers in the number) range from 0 to base - 1

Base 10 digits: $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$

Decimal numbers: base 10

n-digit number

$d_{n-1} \ldots d_2\, d_1 d_0$ represents the integer

$$d_{n-1}10^{n-1} + \ldots + d_2\, 10^2 + d_1 10^1 + d_0 10^0$$

$10^3 = 1000$	$10^2 = 100$	$10^1 = 10$	$10^0 = 1$
9	2	4	7

Decimal numbers: base 10

$d_{n-1} \dots d_2\, d_1 d_0$ represents the integer

$$d_{n-1} 10^{n-1} + \dots + d_2\, 10^2 + d_1 10^1 + d_0 10^0$$

$10^3 = 1000$	$10^2 = 100$	$10^1 = 10$	$10^0 = 1$
9	2	4	7

$$9247_{10}$$
$$= 9 \times 10^3 + 2 \times 10^2 + 4 \times 10^1 + 7 \times 10^0$$

Decimal numbers: base 10

$d_{n-1} \ldots d_2\, d_1 d_0$ represents the integer

$$d_{n-1} 10^{n-1} + \ldots + d_2\, 10^2 + d_1 10^1 + d_0 10^0$$

For a d digit decimal number, what are the range of values? E.g., three digits?

Decimal numbers: base 10

$d_{n-1} \ldots d_2\, d_1 d_0$ represents the integer

$d_{n-1}10^{n-1} + \ldots + d_2\, 10^2 + d_1 10^1 + d_0 10^0$

$0, 1, 2, 3, \ldots, 10^n - 1$

E.g., for 3 digit numbers: $0, 1, 2, 3, \ldots, 10^3 - 1 = 0 \ldots 999$

Numbers in other bases

What are valid digits for base 5 numbers?

Number in other bases

For any base, the digits (individual numbers in the number) range from 0 to base - 1

Base 5 digits: $0, 1, 2, 3, 4$

Numbers in other bases

What number is 243_5?

Numbers in other bases

What number is 243_5?

$5^2 = 25$	$5^1 = 5$	$5^0 = 1$
2	4	3

Numbers in other bases

What number is 243_5?

$5^2 = 25$	$5^1 = 5$	$5^0 = 1$
2	4	3

$$243_5 = 2 \times 5^2 + 4 \times 5^1 + 3 \times 5^0$$

$$= 2 \times 25 + 4 \times 5 + 3 = 73_{10}$$

Numbers in other bases

$d_{n-1} \ldots d_2\, d_1 d_0$ represents the integer

$$d_{n-1}5^{n-1} + \ldots + d_2\, 5^2 + d_1 5^1 + d_0 5^0$$

For a d digit base 5 number, what are the range of values? E.g., three digits?

Numbers in other bases

$d_{n-1} \ldots d_2\, d_1 d_0$ represents the integer

$d_{n-1} 5^{n-1} + \ldots + d_2\, 5^2 + d_1 5^1 + d_0 5^0$

$0, 1, 2, 3, \ldots, 5^n - 1$

E.g., for 3 digit numbers: $0, 1, 2, 3, \ldots, 5^3 - 1 = 0 \ldots 124$

Numbers in other bases

$d_{n-1} \ldots d_2\, d_1 d_0$ represents the integer

$d_{n-1} b^{n-1} + \ldots + d_2\, b^2 + d_1 b^1 + d_0 b^0$

For a d digit base b number, what are the range of values? E.g., three digits?

Numbers in other bases

$d_{n-1} \ldots d_2\, d_1 d_0$ represents the integer

$d_{n-1} b^{n-1} + \ldots + d_2\, b^2 + d_1 b^1 + d_0 b^0$

$0, 1, 2, 3, \ldots, b^n - 1$

E.g., for 3 digit numbers: $0, 1, 2, 3, \ldots, b^3 - 1$

Numbers in other bases

What are valid digits for base 2 numbers?

Number in other bases

For any base, the digits (individual numbers in the number) range from 0 to base - 1

Base 2 digits: $0, 1$

Numbers in other bases

What number is 1101_2?

Numbers in other bases

What number is 1101_2?

$2^3 = 8$	$2^2 = 4$	$2^1 = 10$	$2^0 = 1$
1	1	0	1

Numbers in other bases

What number is 1101_2?

$2^3 = 8$	$2^2 = 4$	$2^1 = 10$	$2^0 = 1$
1	1	0	1

$$1101_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 5^0$$

$$= 8 + 4 + 1 = 13_{10}$$

Binary numbers

$b_{n-1} \dots b_2\, b_1 b_0$ represents the integer $b_{n-1} 2^{n-1} + \dots + b_2\, 2^2 + b_1 2^1 + b_0 2^0$

For example, the binary number 10100111_2 can be written as:

$2^7 = 128$	$2^6 = 64$	$2^5 = 32$	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$
1	0	1	0	0	1	1	1

$$1 \times 2^7 + 0 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$

This is equal to the decimal representation $128 + 32 + 4 + 2 + 1 = 167_{10}$

binary numbers

For a `d` digit base `b` number, what are the range of values? E.g., three digits?

binary numbers

For a d digit binary number, what are the range of values? E.g., four digits?

$0, 1, 2, 3, \ldots, 2^n - 1$

E.g., for 4 digit numbers: $0, 1, 2, 3, \ldots, 2^4 - 1 = 0 \ldots 15$

Binary numbers and their decimal equivalents

1-bit binary	2-bit binary	3-bit binary	4-bit binary	Decimal equivalent
0	00	000	0000	0
1	01	001	0001	1
	10	010	0010	2
	11	011	0011	3
		100	0100	4
		101	0101	5
		110	0110	6
		111	0111	7
			1000	8
			1001	9
			1010	10
			1011	11
			1100	12
			1101	13
			1110	14
			1111	15

PRACTICE TIME

- Convert the following binary numbers to their decimal representation:
  - 1010_2
  - 1111_2
  - 10011011_2
  - 1100011_2
  - 100010_2

PRACTICE TIME - ANSWER

- Convert the following binary numbers to their decimal representation:
  - $1010_2 = 1 \times 2^3 + 1 \times 2^1 = 8 + 2 = 10_{10}$
  - $1111_2 = 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 8 + 4 + 2 + 1 = 15_{10}$
  - $10011011_2 = 1 \times 2^7 + 0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 128 + 16 + 8 + 2 + 1 = 155_{10}$
  - $1100011_2 = 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 99_{10}$
  - $100010_2 = 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 32 + 2 = 34_{10}$

Decimal to binary conversion

$2^7 = 128$	$2^6 = 64$	$2^5 = 32$	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$

What is 55_{10} in binary?

Decimal to binary conversion

$2^7 = 128$	$2^6 = 64$	$2^5 = 32$	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$

32 + 16 + 4 + 2 + 1 = 55

What is 55_{10} in binary?

Decimal to binary conversion

$2^7 =$ 128	$2^6 =$ 64	$2^5 =$ 32	$2^4 =$ 16	$2^3 =$ 8	$2^2 =$ 4	$2^1 =$ 2	$2^0 =$ 1
		1	1	0	1	1	1

$32 + 16 + 4 + 2 + 1 = 55 = 110111_2$

What is 55_{10} in binary?

Decimal to binary conversion

$2^7 = 128$	$2^6 = 64$	$2^5 = 32$	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$

What is 84_{10} in binary?

Decimal to binary conversion

$2^7 = 128$	$2^6 = 64$	$2^5 = 32$	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$

64 + 16 + 4 = 84

What is 84_{10} in binary?

Decimal to binary conversion

$2^7 =$ 128	$2^6 =$ 64	$2^5 =$ 32	$2^4 =$ 16	$2^3 =$ 8	$2^2 =$ 4	$2^1 =$ 2	$2^0 =$ 1
	1	0	1	0	1	0	0

$32 + 16 + 4 = 84 = 1010100_2$

What is 84_{10} in binary?

Decimal to binary: another approach

$$9247_{10} = 9 \times 10^3 + 2 \times 10^2 + 4 \times 10^1 + 7 \times 10^0$$

How can we get this digit?

Decimal to binary: another approach

$$9247_{10} = 9 \times 10^3 + 2 \times 10^2 + 4 \times 10^1 + 7 \times 10^0$$

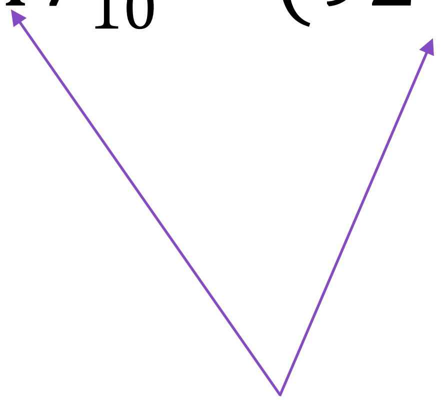
How can we get this digit?

Modulo (remainder) by the base (10)

Decimal to binary: another approach

$$9247_{10} = 9 \times 10^3 + 2 \times 10^2 + 4 \times 10^1 + 7 \times 10^0$$

$$9247_{10} = (924)*10 + 7$$



How can we get this digit?

Decimal to binary: another approach

$$9247_{10} = 9 \times 10^3 + 2 \times 10^2 + 4 \times 10^1 + 7 \times 10^0$$

$$9247_{10} = (924)^*10 + 7$$

Divide by 10 (base) then
Modulo (remainder) by the base (10)

How can we get this digit?

Decimal to binary: another approach

$$9247_{10} = 9 \times 10^3 + 2 \times 10^2 + 4 \times 10^1 + 7 \times 10^0$$

$$9247_{10} = (92) \times 10^2 + 4 \times 10 + 7$$

How can we get this digit?

Decimal to binary: another approach

$$9247_{10} = 9 \times 10^3 + 2 \times 10^2 + 4 \times 10^1 + 7 \times 10^0$$

$$9247_{10} = (92) \times 10^2 + 4 \times 10 + 7$$

Divide by 10^2 then
Modulo (remainder) by the base (10)

How can we get this digit?

Decimal to binary: another approach

$$9247_{10} = 9 \times 10^3 + 2 \times 10^2 + 4 \times 10^1 + 7 \times 10^0$$

$$= 9 \times 10^3 + 2 \times 10^2 + 4 \times 10^1 + 7$$

$$= (9 \times 10^2 + 2 \times 10^1 + 4) \times 10 + 7$$

$$= ((9 \times 10) + 2) \times 10 + 4) \times 10 + 7$$

Decimal to binary: another approach

$$9247_{10} = 9 \times 10^3 + 2 \times 10^2 + 4 \times 10^1 + 7 \times 10^0$$

$$= ((9 \times 10) + 2) \times 10 + 4) \times 10 + 7$$

Another way to build up a number:

- Remainder by base to get next (right-most) digit
- Divide by base
- Repeat

Decimal to binary: another approach

- Repeatedly divide the number by 2. The remainder goes to each column, from right to left.
- Example: Convert the decimal number 84_{10} to binary.

Divide by 2	Remainder	
84		LSB
		MSB

MSB LSB

Decimal to binary conversion

- Repeatedly divide the number by 2. The remainder goes to each column, from right to left.
- Example: Convert the decimal number 84_{10} to binary.
    - $\frac{84}{2} = 42$ with a remainder of 0, so 0 goes in the 1's column.

Divide by 2	**Remainder**
84	0
42	

LSB

MSB

						1
						0

MSB LSB

Decimal to binary conversion

- Repeatedly divide the number by 2. The remainder goes to each column, from right to left.
- Example: Convert the decimal number 84_{10} to binary.
  - $\frac{84}{2} = 42$ with a remainder of 0, so 0 goes in the 1's column.
  - $\frac{42}{2} = 21$ with a remainder of 0, so 0 goes in the 2's column.

Divide by 2	**Remainder**
84	0
42	0
21	

LSB

MSB

					2	1
					0	0

MSB LSB

Decimal to binary conversion

- Repeatedly divide the number by 2. The remainder goes to each column, from right to left.
- Example: Convert the decimal number 84_{10} to binary.
  - $\frac{84}{2} = 42$ with a remainder of 0, so 0 goes in the 1's column.
  - $\frac{42}{2} = 21$ with a remainder of 0, so 0 goes in the 2's column.
  - $\frac{21}{2} = 10$ with a remainder of 1, so there is a 1 going to the 4s column.

Divide by 2	**Remainder**
84	0
42	0
21	1

LSB ↑

MSB

				4	2	1
				1	0	0

MSB LSB

Decimal to binary conversion

- Repeatedly divide the number by 2. The remainder goes to each column, from right to left.
- Example: Convert the decimal number 84_{10} to binary.
  - $\frac{84}{2} = 42$ with a remainder of 0, so 0 goes in the 1’s column.
  - $\frac{42}{2} = 21$ with a remainder of 0, so 0 goes in the 2’s column.
  - $\frac{21}{2} = 10$ with a remainder of 1, so there is a 1 going to the 4s column.
  - $\frac{10}{2} = 5$ with a remainder of 0, so there is a 0 in the 8's column.

Divide by 2	**Remainder**
84	0
42	0
21	1
10	0
5	

LSB

MSB

			8	4	2	1
			0	1	0	0

MSB LSB

Decimal to binary conversion

- Repeatedly divide the number by 2. The remainder goes to each column, from right to left.
- Example: Convert the decimal number 84_{10} to binary.
  - $\frac{84}{2} = 42$ with a remainder of 0, so 0 goes in the 1’s column.
  - $\frac{42}{2} = 21$ with a remainder of 0, so 0 goes in the 2’s column.
  - $\frac{21}{2} = 10$ with a remainder of 1, so there is a 1 going to the 4s column.
  - $\frac{10}{2} = 5$ with a remainder of 0, so there is a 0 in the 8's column.
  - $\frac{5}{2} = 2$ with a remainder of 1, so there is a 1 going to the 16’s column.

Divide by 2	**Remainder**
84	0
42	0
21	1
10	0
5	1
2	

LSB

MSB

		16	8	4	2	1
		1	0	1	0	0

MSB LSB

Decimal to binary conversion

- Repeatedly divide the number by 2. The remainder goes to each column, from right to left.
- Example: Convert the decimal number 84_{10} to binary.
  - $\frac{84}{2} = 42$ with a remainder of 0, so 0 goes in the 1’s column.
  - $\frac{42}{2} = 21$ with a remainder of 0, so 0 goes in the 2’s column.
  - $\frac{21}{2} = 10$ with a remainder of 1, so there is a 1 going to the 4s column.
  - $\frac{10}{2} = 5$ with a remainder of 0, so there is a 0 in the 8's column.
  - $\frac{5}{2} = 2$ with a remainder of 1, so there is a 1 going to the 16’s column.
  - $\frac{2}{2} = 1$, with a remainder of 0, so 0 goes in the 32’s column.

Divide by 2	**Remainder**	
84	0	LSB
42	0	
21	1	
10	0	
5	1	
2	0	
1		MSB

	32	16	8	4	2	1
	0	1	0	1	0	0

MSB LSB

Decimal to binary conversion

- Repeatedly divide the number by 2. The remainder goes to each column, from right to left.
- Example: Convert the decimal number 84_{10} to binary.
  - $\frac{84}{2} = 42$ with a remainder of 0, so 0 goes in the 1’s column.
  - $\frac{42}{2} = 21$ with a remainder of 0, so 0 goes in the 2’s column.
  - $\frac{21}{2} = 10$ with a remainder of 1, so there is a 1 going to the 4s column.
  - $\frac{10}{2} = 5$ with a remainder of 0, so there is a 0 in the 8's column.
  - $\frac{5}{2} = 2$ with a remainder of 1, so there is a 1 going to the 16’s column.
  - $\frac{2}{2} = 1$, with a remainder of 0, so 0 goes in the 32’s column.
  - $\frac{1}{2} = 0$, with a remainder of 1, so 1 goes in the 64’s column.

Divide by 2	**Remainder**	
84	0	LSB
42	0	
21	1	
10	0	
5	1	
2	0	
1	1	MSB

64	32	16	8	4	2	1
1	0	1	0	1	0	0

MSB LSB

Decimal to binary conversion

- Repeatedly divide the number by 2. The remainder goes to each column, from right to left.
- Example: Convert the decimal number 84_{10} to binary.
  - $\frac{84}{2} = 42$ with a remainder of 0, so 0 goes in the 1’s column.
  - $\frac{42}{2} = 21$ with a remainder of 0, so 0 goes in the 2’s column.
  - $\frac{21}{2} = 10$ with a remainder of 1, so there is a 1 going to the 4s column.
  - $\frac{10}{2} = 5$ with a remainder of 0, so there is a 0 in the 8's column.
  - $\frac{5}{2} = 2$ with a remainder of 1, so there is a 1 going to the 16’s column.
  - $\frac{2}{2} = 1$, with a remainder of 0, so 0 goes in the 32’s column.
  - $\frac{1}{2} = 0$, with a remainder of 1, so 1 goes in the 64’s column.
- Putting this all together, $84_{10} = 1010100_2$.

Divide by 2	**Remainder**	
84	0	LSB
42	0	
21	1	
10	0	
5	1	
2	0	
1	1	MSB

64	32	16	8	4	2	1
1	0	1	0	1	0	0

MSB LSB

PRACTICE TIME

- Convert the following decimal numbers to their binary representation.
    - 125_{10}
    - 339_{10}
    - 75_{10}

$2^8 = 256$	$2^7 = 128$	$2^6 = 64$	$2^5 = 32$	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$

PRACTICE TIME

- Convert the following decimal numbers to their binary representation.
  - $125_{10} = 64 + 32 + 16 + 8 + 4 + 0 + 1 = 1111101_2$
  - $339_{10} = 256 + 0 + 64 + 0 + 16 + 0 \pm 0 + 2 + 1 = 101010011_2$
  - $75_{10} = 64 + 0 + 0 + 8 + 0 + 2 + 1 = 1001011_2$

$2^8 = 256$	$2^7 = 128$	$2^6 = 64$	$2^5 = 32$	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$

PRACTICE TIME - ANSWER

- Convert the following decimal numbers to their binary representation.
  - $125_{10} = 1111101_2$ $339_{10} = 101010011_2$ $75_{10} = 1001011_2$

Divide by 2	Remainder
125	1
62	0
31	1
15	1
7	1
3	1
1	1

LSB

MSB

Divide by 2	Remainder
339	1
169	1
84	0
42	0
21	1
10	0
5	1
2	0
1	1

LSB

MSB

Divide by 2	Remainder
75	1
37	1
18	0
9	1
4	0
2	0
1	1

LSB

MSB

Hexadecimal

What are valid digits for base 16 numbers (aka, hexadecimal numbers)?

Hexadecimal numbers

For any base, the digits (individual numbers in the number) range from 0 to base - 1

Base 16 digits: $0, 1, 2, 3, 4, \ldots, 15$

Hexadecimal numbers

For any base, the digits (individual numbers in the number) range from 0 to base - 1

Base 16 digits: $0, 1, 2, 3, 4, \ldots, 15$

Any problems with this?

Hexadecimal numbers

For any base, the digits (individual numbers in the number) range from 0 to base - 1

Base 16 digits: $0, 1, 2, \ldots, 9, \mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}, \mathrm{F}$

10, 11, 12, 13, 14, 15

Hexadecimal numbers

A, B, C, D, E, F
10, 11, 12, 13, 14, 15

What number is $2A7_{16}$?

Hexadecimal numbers

A, B, C, D, E, F
10, 11, 12, 13, 14, 15

What number is $2A7_{16}$?

Hexadecimal numbers

A, B, C, D, E, F

10, 11, 12, 13, 14, 15

What number is $2A7_{16}$?

$16^2 = 256$	$16^1 = 16$	$16^0 = 1$
2	A	7

Hexadecimal numbers

A, B, C, D, E, F

10, 11, 12, 13, 14, 15

What number is $2A7_{16}$?

$16^2 = 256$	$16^1 = 16$	$16^0 = 1$
2	A	7

$$2A7_{16} = 2 \times 16^2 + 10 \times 16^1 + 7 \times 16^0$$

$$= 2 \times 256 + 10 \times 16 + 7 = 679_{10}$$

Hexadecimal numbers

$d_{n-1} \ldots d_2\, d_1 d_0$ represents the integer

$d_{n-1} 16^{n-1} + \ldots + d_2\, 16^2 + d_1 16 + d_0 16^0$

For a d digit hexadecimal number, what are the range of values? E.g., three digits?

Hexadecimal numbers

$d_{n-1} \dots d_2\, d_1 d_0$ represents the integer

$d_{n-1} 16^{n-1} + \dots + d_2\, 16^2 + d_1 16 + d_0 16^0$

$0, 1, 2, 3, \dots, 16^n - 1$

E.g., for 3 digit numbers: $0, 1, 2, 3, \dots, 16^3 - 1 = 0 \dots 4095$

Hexadecimal numbers

Hex numbers are sometimes preceded by 0x to be distinguished from decimal numbers

0x15 is 15_{16} which is equal to 21_{10} and not 15_{10}.

Equivalence across number systems

Hexadecimal digit	Binary equivalent	Decimal equivalent
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
A	1010	10
B	1011	11
C	1100	12
D	1101	13
E	1110	14
F	1111	15

PRACTICE TIME

- Convert the following hexadecimal numbers to their decimal representation.
  - $32A_{16}$
  - $D1CE_{16}$
  - $1B7F_{16}$
  - 47_{16}

ANSWER

- Convert the following hexadecimal numbers to their decimal representation.
  - $32A_{16} = 3 \times 16^2 + 2 \times 16^1 + 10 \times 16^0 = 810_{10}$
  - $D1CE_{16} = 13 \times 16^3 + 1 \times 16^2 + 12 \times 16^1 + 14 \times 16^0 = 53710_{10}$
  - $1B7F_{16} = 1 \times 16^3 + 11 \times 16^2 + 7 \times 16^1 + 15 \times 16^0 = 7039_{10}$
  - $47_{16} = 4 \times 16^1 + 7 \times 16^0 = 71_{10}$

Decimal to hexadecimal conversion

- Repeatedly divide by 16 and leave remainder as LSB
- Example: Convert the decimal number 333_{10} to hexadecimal.

Divide by 16	Remainder
333	

LSB

MSB

Decimal to hexadecimal conversion

- Repeatedly divide by 16 and leave remainder as LSB
- Example: Convert the decimal number 333_{10} to hexadecimal.
  - $\frac{333}{16} = 20$ with a remainder of $13_{10} = D_{16}$ which goes in the 1's column.

Divide by 16	**Remainder**
333	$13_{10} = D_{16}$
20	

LSB

MSB

		1
		D

Decimal to hexadecimal conversion

- Repeatedly divide by 16 and leave remainder as LSB
- Example: Convert the decimal number 333_{10} to hexadecimal.
  - $\frac{333}{16} = 20$ with a remainder of $13_{10} = D_{16}$ which goes in the 1's column.
  - $\frac{20}{16} = 1$ with a remainder of 4, so 4 goes in the 16's column.

Divide by 16	**Remainder**
333	$13_{10} = D_{16}$
20	$4_{10} = 4_{16}$
1	

LSB

MSB

	16	1
	4	D

Decimal to hexadecimal conversion

- Repeatedly divide by 16 and leave remainder as LSB
- Example: Convert the decimal number 333_{10} to hexadecimal.
  - $\frac{333}{16} = 20$ with a remainder of $13_{10} = D_{16}$ which goes in the 1's column.
  - $\frac{20}{16} = 1$ with a remainder of 4, so 4 goes in the 16's column.
  - $\frac{1}{16} = 0$ with a remainder of 1, so there is a 1 going to the 256s column.

Divide by 16	**Remainder**
333	$13_{10} = D_{16}$
20	$4_{10} = 4_{16}$
1	$1_{10} = 1_{16}$

LSB

MSB

256	16	1
1	4	D

Decimal to hexadecimal conversion

- Repeatedly divide by 16 and leave remainder as LSB
- Example: Convert the decimal number 333_{10} to hexadecimal.
  - $\frac{333}{16} = 20$ with a remainder of $13_{10} = D_{16}$ which goes in the 1's column.
  - $\frac{20}{16} = 1$ with a remainder of 4, so 4 goes in the 16's column.
  - $\frac{1}{16} = 0$ with a remainder of 1, so there is a 1 going to the 256s column.
  - Putting this all together, $333_{10} = 14D_{16}$.

Divide by 16	**Remainder**	
333	$13_{10} = D_{16}$	LSB
20	$4_{10} = 4_{16}$	
1	$1_{10} = 1_{16}$	MSB

256	16	1
1	4	D

PRACTICE TIME

- Convert the following decimal numbers to their hexadecimal representation.
  - 2546_{10}
  - 1457_{10}
  - 269_{10}
  - 47_{10}

ANSWER

- Convert the following decimal numbers to their hexadecimal representation.
  - $2546_{10} = 9F2$ $1457_{10} = 5B1$ $269_{10} = 10D$ $47_{10} = 2F$

Divide by 16	Remainder
2546	$2_{10} = 2_{16}$
159	$15_{10} = F_{16}$
9	$9_{10} = 9_{16}$

Divide by 16	Remainder
1457	$1_{10} = 1_{16}$
91	$11_{10} = B_{16}$
5	$5_{10} = 5_{16}$

Divide by 16	Remainder
269	$13_{10} = D_{16}$
16	$0_{10} = 0_{16}$
1	$1_{10} = 1_{16}$

Divide by 16	Remainder
47	$15_{10} = F_{16}$
2	$2_{10} = 2_{16}$

LSB

MSB

Binary to Hexadecimal conversion

1100111000010000_2

What is the following binary number in hexadecimal?

4 bits is equal to one hex digit

Hexadecimal digit	Binary equivalent	Decimal equivalent
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
A	1010	10
B	1011	11
C	1100	12
D	1101	13
E	1110	14
F	1111	15

Binary to Hexadecimal conversion

110011100010000_2

$0110\ \ 0111\ \ 0001\ \ 0000_2$

break into 4 bit groups

Hexadecimal digit	Binary equivalent
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

Binary to Hexadecimal conversion

1100111000010000_2

break into 4 bit groups

$0110\ \ 0111\ \ 0001\ \ 0000_2$

convert each 4 bit group into a digit

Hexadecimal digit	Binary equivalent
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

Binary to Hexadecimal conversion

1100111000010000_2

break into 4 bit groups

$0110\ \ 0111\ \ 0001\ \ 0000_2$

convert each 4 bit group into a digit

$6\ \ 7\ \ 1\ \ 0_{16}$

Hexadecimal digit	Binary equivalent
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

PRACTICE TIME

- Convert the following binary numbers to their hexadecimal representation.
  - 10010011_2
  - 101100110101_2
  - 11011_2
  - 1111011101110010_2

Hexadecimal digit	**Binary equivalent**
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

ANSWER

- Convert the following binary numbers to their hexadecimal representation.
  - $1001\ 0011_2 = 93_{16}$
  - $1011\ 0011\ 0101_2 = B35_{16}$
  - $11011_2 = 0001\ 1011 = 1B_{16}$
  - $11111011101110010_2 = 0001\ 1111\ 0111\ 0111\ 0010 = 1\mathrm{F}772_{16}$

Hexadecimal digit	**Binary equivalent**
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

Hexadecimal to binary conversion

- We replace each hex digit with the four binary bits that correspond to its value.
- Example: Convert the hexadecimal number $CAFE_{16}$ to its binary representation.
  - $C_{16} = 1100_2$
  - $A_{16} = 1010_2$
  - $F_{16} = 1111_2$
  - $E_{16} \ = 1110_2$
  - Putting this all together, $CAFE_{16} \ = \ 1100101011111110_2$.

Hexadecimal digit	Binary equivalent
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

PRACTICE TIME

- Convert the following hexadecimal numbers to their binary representation.
  - $\mathrm{F00D}_{16}$
  - $2B3BAD_{16}$
  - $\mathrm{DEADC0DE}_{16}$

Hexadecimal digit	**Binary equivalent**
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

ANSWER

- Convert the following hexadecimal numbers to their binary representation.
  - $\mathrm{F00D}_{16} = 1111000000001101_2$
  - $2B3BAD_{16} = 001010110011101110101101_2$
  - $\mathrm{DEADC0DE}_{16} = 11011110101011011100000011011110_2$

Hexadecimal digit	Binary equivalent
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

Practice Problem

- For each row, fill in the missing entries by converting the given number from its representation to every other equivalent representation.

Binary	Decimal	Hexadecimal
	39_{10}	
100111010_2		
		$BADE_{16}$
	47_{10}	
101100111_2		
		$C0FFEE_{16}$

Practice Problem - ANSWER

- For each row, fill in the missing entries by converting the given number from its representation to every other equivalent representation.

Binary	Decimal	Hexadecimal
100111_2	39_{10}	27_{16}
100111010_2	314_{10}	$13A_{16}$
1011101011011110_2	47838_{10}	$BADE_{16}$
101111_2	47_{10}	$2F_{16}$
101100111_2	359_{10}	167_{16}
$110000001111111111101110_2$	12648430_{10}	$C0FFEE_{16}$