Computer Systems

Numeral Systems

CS51 – Fall 2026

Boolean Algebra in Python

Boolean variables in Python

- In Python, Boolean variables are represented through the `bool` data type. A bool variable can either be `True` or `False`. These values can also be seen as equal to 1 and 0, respectively.
- For example, `raining = False` declares a variable called `raining` and assigns to it the value `False`.
- The comparison operators `==`, `!=`, `>`, `<`, `>=`, `<=` return bools. For example,
  - `10 < 0` evaluates to `False`
  - `11 >= 11` evaluates to `True`
  - `11 > 11.0` evaluates to `False`
  - `10 == 10.0` evaluates to `True`
  - `10 != 10.0` evaluates to `False`

Boolean Algebra in Python

- We can combine bools and logical operators in Boolean expressions, for example, to evaluate them in **conditionals** (`if` statements) and **loops** (`for` and `while`).
- Python supports three logical operators:
  - `not` for negation (i.e., ¬)
  - `and` for conjunction (i.e., ∧)
  - `or` for disjunction (i.e., ∨)

Short-circuit evaluation in Python

- Python supports **short-circuiting**, a technique that evaluates a logical expression from left to right and *stops evaluation* once the truth can be determined.
- For example, if `x=3` and we have the expression:
  - `x>3 and x<24`
  - Once Python evaluates `x>3` being `False` and sees that and follows, then it can immediately evaluate the whole expression to `False`, without needing to evaluate whether `x<24`.

Think

- Assume `x=3` again.
- *Can you come up with an expression that includes an `or` operator and which would lead to short-circuiting?*

Think

- Assume `x=3` again.
- *Can you come up with an expression that includes an `or` operator and which would lead to short-circuiting?*
  - `x==3 or x>24`

Short-circuit evaluation and optimization

- Because of short-circuit evaluation, we can avoid deeply nested if statements to handle exceptional cases. For example,
  - if x == 0 or (x-1)/x > 0.5: …
  - When x is equal to 0, evaluating the second operand of the or statement would cause a divide-by-zero error. But the second disjunct isn't evaluated when x is equal to 0 because the first operand was True!

Short-circuit evaluation and optimization

- We can take advantage of short-circuit evaluation to make our code a bit faster. For example, when writing an if statement that contains the logical operator and, you should first write what is simpler to calculate or often false before something that is complex to calculate or often true:
  - if (simpleOrOftenFalse() and complexOrOftenTrue()):…

PRACTICE TIME – Boolean expressions

- Assume the variable x currently stores the value 47. What will the following expressions evaluate to?
  - True and not False
  - not True or not False
  - True and not 0
  - False or not 1
  - 2 < x and x < 50
  - x > 2 and < 20
  - (2 > x) or (x == 47)
  - not (x == x)
  - not (x != 3)
  - x < 0 and (y/x) == 3
  - x > 0 or (y/x) == 3

ANSWER – Boolean expressions

- Assume the variable x currently stores the value 47. What will the following expressions evaluate to?
  - True and not False - True
  - not True or not False - True
  - True and not 0 – True
  - False or not 1 - False
  - 2 < x and x < 50 - True
  - x > 2 and < 20 – Invalid syntax
  - (2 > x) or (x == 47) - True
  - not (x == x) - False
  - not (x != 3) – False
  - x < 0 and (y/x) == 3 – False
  - x > 0 or (y/x) == 3 - True

Numeral Systems

Decimal numbers

- We are used to numbers that use ten digits: $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$.
- This arithmetic is called **base 10** (denoted as a subscript) and such numbers are called **decimal**.
- To avoid confusion, we use subscripts to indicate a base of a number.
  - For example, instead 9247 we will write the number 9247_{10}.

Think

- *What number is* 9247_{10}*?*

Think

- *What number is 9247_{10}?*
  - $9247_{10} = 9000 + 200 + 40 + 7 =$
  - $= 9 \times 10^3 + 2 \times 10^2 + 4 \times 10^1 + 7 \times 10^0$

Decimal numbers

- The n digit number $d_{n-1} \ldots d_2\, d_1 d_0$ represents the integer

$$d_{n-1}10^{n-1} + \ldots + d_2\, 10^2 + d_1 10^1 + d_0 10^0$$

- For example, the number 9247_{10} can be written as the sum of each of its digits multiplied by the weight of the corresponding column:

$10^3 = 1000$	$10^2 = 100$	$10^1 = 10$	$10^0 = 1$
9	2	4	7

  - $9247_{10} = 9 \times 10^3 + 2 \times 10^2 + 4 \times 10^1 + 7 \times 10^0$

Think

- The n digit number $d_{n-1} \ldots d_2\, d_1 d_0$ represents the integer

$$d_{n-1}10^{n-1} + \ldots + d_2\, 10^2 + d_1 10^1 + d_0 10^0$$

- *If we had 3 digits, how many decimal numbers could we represent?*

Think

- The n digit number $d_{n-1} \ldots d_2\, d_1 d_0$ represents the integer

$$d_{n-1}10^{n-1} + \ldots + d_2\, 10^2 + d_1 10^1 + d_0 10^0$$

- *If we had 3 digits, how many decimal numbers could we represent?*
  - 1000 numbers: 0, 1, 2, 3,...,999

Think

- The n digit number $d_{n-1} \ldots d_2\, d_1 d_0$ represents the integer

$$d_{n-1}10^{n-1} + \ldots + d_2\, 10^2 + d_1 10^1 + d_0 10^0$$

- *If we had 3 digits, how many decimal numbers could we represent?*
  - 1000 numbers: 0, 1, 2, 3,...,999
- *If we had n digits, how many decimal numbers could we represent?*

Think

- The n digit number $d_{n-1} \ldots d_2\, d_1 d_0$ represents the integer

$$d_{n-1}10^{n-1} + \ldots + d_2\, 10^2 + d_1 10^1 + d_0 10^0$$

- *If we had 3 digits, how many decimal numbers could we represent?*
  - 1000 numbers: 0, 1, 2, 3,...,999
- *If we had n digits, how many decimal numbers could we represent?*
  - 10^n numbers: $0, 1, 2, 3, \ldots, 10^n - 1$

Think

- *What digits can we use if we are working on base 5?*

Think

- *What digits can we use if we are working on base 5?*
  - 0, 1, 2, 3, 4

Think

- *What number is* 243_5*?*

Think

- *What number is* 243_5*?*

$5^2 = 25$	$5^1 = 5$	$5^0 = 1$
2	4	3

- $243_5 = 2 \times 5^2 + 4 \times 5^1 + 3 \times 5^0$

Think

- *What number is* 243_5*?*

$5^2 = 25$	$5^1 = 5$	$5^0 = 1$
2	4	3

- $243_5 = 2 \times 5^2 + 4 \times 5^1 + 3 \times 5^0 = 2 \times 25 + 4 \times 5 + 3 = 73_{10}$

Base 5 numbers

- $d_{n-1} \ldots d_2\, d_1 d_0$ represents the integer $d_{n-1}5^{n-1} + \ldots + d_2\, 5^2 + d_1 5^1 + d_0 5^0$

Think

- $d_{n-1} \ldots d_2\, d_1 d_0$ represents the integer $d_{n-1}5^{n-1} + \ldots + d_2\, 5^2 + d_1 5^1 + d_0 5^0$
- *If we had 3 digits, how many base 5 numbers could we represent?*

Think

- $d_{n-1} \ldots d_2\, d_1 d_0$ represents the integer $d_{n-1}5^{n-1} + \ldots + d_2\, 5^2 + d_1 5^1 + d_0 5^0$
- *If we had 3 digits, how many base 5 numbers could we represent?*
  - 125 numbers: $0, 1, 2, 3, \ldots, 5^3 - 1 = 0, 1, 2, 3, \ldots, 124$

Think

- $d_{n-1} \ldots d_2\, d_1 d_0$ represents the integer $d_{n-1}5^{n-1} + \ldots + d_2\, 5^2 + d_1 5^1 + d_0 5^0$
- *If we had 3 digits, how many base 5 numbers could we represent?*
  - 125 numbers: $0, 1, 2, 3, \ldots, 5^3 - 1 = 0, 1, 2, 3, \ldots, 124$
- *If we had n digits, how many base 5 numbers could we represent?*

Think

- $d_{n-1} \ldots d_2\, d_1 d_0$ represents the integer $d_{n-1}5^{n-1} + \ldots + d_2\, 5^2 + d_1 5^1 + d_0 5^0$
- *If we had 3 digits, how many base 5 numbers could we represent?*
  - 125 numbers: $0, 1, 2, 3, \ldots, 5^3 - 1 = 0, 1, 2, 3, \ldots, 124$
- *If we had n digits, how many base 5 numbers could we represent?*
  - 5^n numbers: $0, 1, 2, 3, \ldots, 5^{\mathrm{n}} - 1$

Numbers in other bases

- $d_{n-1} \ldots d_2\, d_1 d_0$ represents the integer $d_{n-1} b^{n-1} + \ldots + d_2\, b^2 + d_1 b^1 + d_0 b^0$

Think

- $d_{n-1} \ldots d_2\, d_1 d_0$ represents the integer $d_{n-1}b^{n-1} + \ldots + d_2\, b^2 + d_1 b^1 + d_0 b^0$
- *If we had n digits, how many base b numbers could we represent?*

Think

- $d_{n-1} \ldots d_2\, d_1 d_0$ represents the integer $d_{n-1} b^{n-1} + \ldots + d_2\, b^2 + d_1 b^1 + d_0 b^0$
- *If we had n digits, how many base b numbers could we represent?*
  - b^n numbers: $0, 1, 2, 3, \ldots, \mathrm{b}^{\mathrm{n}} - 1$

Think

- *What are the valid digits for base 2 numbers?*

Think

- *What are the valid digits for base 2 numbers?*
  - *0, 1*

Think

- *What number is* 1101_2*?*

Think

- *What number is* 1101_2*?*

$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$
1	1	0	1

Think

- *What number is* 1101_2*?*

$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$
1	1	0	1

- $1101_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

Think

- *What number is* 1102_2*?*

$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$
1	1	0	1

- $1101_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 =$
- $= 8 + 4 + 0 + 1 = 13_{10}$

Least and most significant bits

- The left-most bit is the **most significant bit** (MSB).
- The right-most is the **least significant bit** (LSB).

Think

- $b_n b_{n-1} \ldots b_2\, b_1 b_0$ represents the integer $b_n 2^n + b_{n-1} 2^{n-1} + \ldots + b_2\, 2^2 + b_1 2^1 + b_0 2^0$
- *If we had n digits, how many binary numbers could we represent?*

Think

- $b_n b_{n-1} \ldots b_2\, b_1 b_0$ represents the integer $b_n 2^n + b_{n-1} 2^{n-1} + \ldots + b_2\, 2^2 + b_1 2^1 + b_0 2^0$
- *If we had n digits, how many binary numbers could we represent?*
  - 2^n numbers: $0, 1, 2, 3, \ldots, 2^n - 1$

Binary numbers and their decimal equivalents

1-bit binary	2-bit binary	3-bit binary	4-bit binary	Decimal equivalent
0	00	000	0000	0
1	01	001	0001	1
	10	010	0010	2
	11	011	0011	3
		100	0100	4
		101	0101	5
		110	0110	6
		111	0111	7
			1000	8
			1001	9
			1010	10
			1011	11
			1100	12
			1101	13
			1110	14
			1111	15

PRACTICE TIME

- Convert the following binary numbers to their decimal representation:
  - 1010_2
  - 1111_2
  - 10011011_2
  - 1100011_2
  - 100010_2

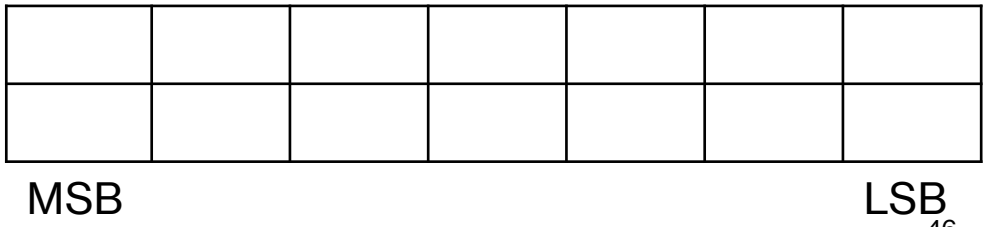
PRACTICE TIME - ANSWER

- Convert the following binary numbers to their decimal representation:
  - $1010_2 = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 8 + 2 = 10_{10}$
  - $1111_2 = 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 8 + 4 + 2 + 1 = 15_{10}$
  - $10011011_2 = 1 \times 2^7 + 0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 128 + 16 + 8 + 2 + 1 = 155_{10}$
  - $1100011_2 = 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 99_{10}$
  - $100010_2 = 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 32 + 2 = 34_{10}$

Decimal to binary conversion

- Repeatedly divide the number by 2. The remainder goes to each column, from right to left.
- Example: Convert the decimal number 84_{10} to binary.

Divide by 2	Remainder	
84		LSB
		MSB



Decimal to binary conversion

- Repeatedly divide the number by 2. The remainder goes to each column, from right to left.
- Example: Convert the decimal number 84_{10} to binary.
  - $\frac{84}{2} = 42$ with a remainder of 0, so 0 goes in the 1's column.

Divide by 2	Remainder	
84	0	LSB
42		
		MSB

						1
						0
MSB						LSB

Decimal to binary conversion

- Repeatedly divide the number by 2. The remainder goes to each column, from right to left.
- Example: Convert the decimal number 84_{10} to binary.
  - $\frac{84}{2} = 42$ with a remainder of 0, so 0 goes in the 1's column.
  - $\frac{42}{2} = 21$ with a remainder of 0, so 0 goes in the 2's column.

Divide by 2	Remainder	
84	0	LSB
42	0	
21		
		MSB

					2	1
					0	0

MSB LSB

Decimal to binary conversion

- Repeatedly divide the number by 2. The remainder goes to each column, from right to left.
- Example: Convert the decimal number 84_{10} to binary.
  - $\frac{84}{2} = 42$ with a remainder of 0, so 0 goes in the 1's column.
  - $\frac{42}{2} = 21$ with a remainder of 0, so 0 goes in the 2's column.
  - $\frac{21}{2} = 10$ with a remainder of 1, so there is a 1 going to the 4s.

Divide by 2	Remainder	
84	0	LSB
42	0	
21	1	
		MSB

				4	2	1
				1	0	0

MSB LSB

Decimal to binary conversion

- Repeatedly divide the number by 2. The remainder goes to each column, from right to left.
- Example: Convert the decimal number 84_{10} to binary.
  - $\frac{84}{2} = 42$ with a remainder of 0, so 0 goes in the 1's column.
  - $\frac{42}{2} = 21$ with a remainder of 0, so 0 goes in the 2's column.
  - $\frac{21}{2} = 10$ with a remainder of 1, so there is a 1 going to the 4s.
  - $\frac{10}{2} = 5$ with a remainder of 0, so there is a 0 in the 8's column.

Divide by 2	Remainder	
84	0	LSB
42	0	
21	1	
10	0	
5		
		MSB

			8	4	2	1
			0	1	0	0

MSB LSB

Decimal to binary conversion

- Repeatedly divide the number by 2. The remainder goes to each column, from right to left.
- Example: Convert the decimal number 84_{10} to binary.
  - $\frac{84}{2} = 42$ with a remainder of 0, so 0 goes in the 1's column.
  - $\frac{42}{2} = 21$ with a remainder of 0, so 0 goes in the 2's column.
  - $\frac{21}{2} = 10$ with a remainder of 1, so there is a 1 going to the 4s.
  - $\frac{10}{2} = 5$ with a remainder of 0, so there is a 0 in the 8's column.
  - $\frac{5}{2} = 2$ with a remainder of 1, so there is a 1 in 16's.

Divide by 2	Remainder	
84	0	LSB
42	0	
21	1	
10	0	
5	1	
2		
		MSB

		16	8	4	2	1
		1	0	1	0	0

MSB LSB

Decimal to binary conversion

- Repeatedly divide the number by 2. The remainder goes to each column, from right to left.
- Example: Convert the decimal number 84_{10} to binary.
  - $\frac{84}{2} = 42$ with a remainder of 0, so 0 goes in the 1's column.
  - $\frac{42}{2} = 21$ with a remainder of 0, so 0 goes in the 2's column.
  - $\frac{21}{2} = 10$ with a remainder of 1, so there is a 1 going to the 4s.
  - $\frac{10}{2} = 5$ with a remainder of 0, so there is a 0 in the 8's column.
  - $\frac{5}{2} = 2$ with a remainder of 1, so there is a 1 in 16's.
  - $\frac{2}{2} = 1$, with a remainder of 0, so 0 goes in the 32's.

Divide by 2	Remainder	
84	0	LSB
42	0	
21	1	
10	0	
5	1	
2	0	
1		MSB

	32	16	8	4	2	1
	0	1	0	1	0	0

MSB LSB

Decimal to binary conversion

- Repeatedly divide the number by 2. The remainder goes to each column, from right to left.
- Example: Convert the decimal number 84_{10} to binary.
  - $\frac{84}{2} = 42$ with a remainder of 0, so 0 goes in the 1's column.
  - $\frac{42}{2} = 21$ with a remainder of 0, so 0 goes in the 2's column.
  - $\frac{21}{2} = 10$ with a remainder of 1, so there is a 1 going to the 4s.
  - $\frac{10}{2} = 5$ with a remainder of 0, so there is a 0 in the 8's column.
  - $\frac{5}{2} = 2$ with a remainder of 1, so there is a 1 in 16's.
  - $\frac{2}{2} = 1$, with a remainder of 0, so 0 goes in the 32's.
  - $\frac{1}{2} = 0$, with a remainder of 1, so 1 goes in the 64's column.

Divide by 2	Remainder	
84	0	LSB
42	0	
21	1	
10	0	
5	1	
2	0	
1	1	MSB

64	32	16	8	4	2	1
1	0	1	0	1	0	0

MSB LSB

Decimal to binary conversion

- Repeatedly divide the number by 2. The remainder goes to each column, from right to left.
- Example: Convert the decimal number 84_{10} to binary.
  - $\frac{84}{2} = 42$ with a remainder of 0, so 0 goes in the 1's column.
  - $\frac{42}{2} = 21$ with a remainder of 0, so 0 goes in the 2's column.
  - $\frac{21}{2} = 10$ with a remainder of 1, so there is a 1 going to the 4s column.
  - $\frac{10}{2} = 5$ with a remainder of 0, so there is a 0 in the 8's column.
  - $\frac{5}{2} = 2$ with a remainder of 1, so there is a 1 going to the 16's column.
  - $\frac{2}{2} = 1$, with a remainder of 0, so 0 goes in the 32's column.
  - $\frac{1}{2} = 0$, with a remainder of 1, so 1 goes in the 64's column.
- Putting this all together, $84_{10} = 1010100_2$.

Divide by 2	**Remainder**	
84	0	LSB
42	0	
21	1	
10	0	
5	1	
2	0	
1	1	MSB

64	32	16	8	4	2	1
1	0	1	0	1	0	0

MSB LSB

PRACTICE TIME

- Convert the following decimal numbers to their binary representation.
  - 125_{10}
  - 339_{10}
  - 75_{10}

PRACTICE TIME - ANSWER

- Convert the following decimal numbers to their binary representation.
  - $125_{10} = 1111101_2$ $339_{10} = 101010011_2$ $75_{10} = 1001011_2$

Divide by 2	Remainder	
125	1	LSB
62	0	
31	1	
15	1	
7	1	
3	1	
1	1	MSB

Divide by 2	Remainder	
339	1	LSB
169	1	
84	0	
42	0	
21	1	
10	0	
5	1	
2	0	
1	1	MSB

Divide by 2	Remainder	
75	1	LSB
37	1	
18	0	
9	1	
4	0	
2	0	
1	1	MSB

Hexadecimal numbers

- We also use a **hexadecimal** (or hex) notation where the arithmetic is base 16.
- *What are the valid digits for hexadecimal numbers?*

Think

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- *What are the valid digits for hexadecimal numbers?*

Think

- We also use a **hexadecimal** (or hex) notation where the arithmetic is base 16.
- *What are the valid digits for hexadecimal numbers?*
  - One idea: 0, 1, 2, 3, ..., 15
  - *What's the problem here?*

Hexadecimal numbers

- We also use a **hexadecimal** (or hex) notation where the arithmetic is base 16.
- *What are the valid digits for hexadecimal numbers?*
  - 0, 1, 2, ..., 9, **A, B, C, D, E, F**
  - *Instead of **10, 11, 12, 13, 14, 15***

Think

A, B, C, D, E, F

10, 11, 12, 13, 14, 15

- *What number is* $2A7_{16}$?

Think

A, B, C, D, E, F

10, 11, 12, 13, 14, 15

- *What number is* $2A7_{16}$?

$16^2 = 256$	$16^1 = 16$	$16^0 = 1$
2	A	7

Think

A, B, C, D, E, F

10, 11, 12, 13, 14, 15

- *What number is* $2A7_{16}$?

$16^2 = 256$	$16^1 = 16$	$16^0 = 1$
2	A	7

- $2\text{A}7_{16} = 2 \times 16^2 + 10 \times 16^1 + 7 \times 16^0$

Think

A, B, C, D, E, F

10, 11, 12, 13, 14, 15

- *What number is* $2A7_{16}$?

$16^2 = 256$	$16^1 = 16$	$16^0 = 1$
2	A	7

- $2\text{A}7_{16} = 2 \times 16^2 + 10 \times 16^1 + 7 \times 16^0 =$
- $= 2 \times 256 + 10 \times 16 + 7 = 679_{10}$

Think

- $h_{n-1} \ldots h_2\, h_1 h_0$ represents the integer

$$h_{n-1}16^{n-1} + \cdots + h_2 16^2 + h_1 16^1 + h_0 16^0$$

- *If we had 3 digits, how many hex numbers could we represent?*

Think

- $h_{n-1} \ldots h_2\, h_1 h_0$ represents the integer

$$h_{n-1}16^{n-1} + \cdots + h_2 16^2 + h_1 16^1 + h_0 16^0$$

- *If we had 3 digits, how many hex numbers could we represent?*
  - 4096 numbers: $0, 1, 2, 3, \ldots, 16^3 - 1 = 0, 1, 2, 3, \ldots, 4095$

Think

- $h_{n-1} \ldots h_2\, h_1 h_0$ represents the integer

$$h_{n-1}16^{n-1} + \cdots + h_2 16^2 + h_1 16^1 + h_0 16^0$$

- *If we had 3 digits, how many hex numbers could we represent?*
  - 4096 numbers: $0, 1, 2, 3, \ldots, 16^3 - 1 = 0, 1, 2, 3, \ldots, 4095$
- *If we had n digits, how many hex numbers could we represent?*

Think

- $h_{n-1} \ldots h_2\, h_1 h_0$ represents the integer

$$h_{n-1}16^{n-1} + \cdots + h_2 16^2 + h_1 16^1 + h_0 16^0$$

- *If we had 3 digits, how many hex numbers could we represent?*
  - 4096 numbers: $0, 1, 2, 3, \ldots, 16^3 - 1 = 0, 1, 2, 3, \ldots, 4095$
- *If we had n digits, how many hex numbers could we represent?*
  - 16^n numbers: $0, 1, 2, 3, \ldots, 16^{\mathrm{n}} - 1$

A, B, C, D, E, F

10, 11, 12, 13, 14, 15

Hexadecimal numbers

- $h_n h_{n-1} \ldots h_2\, h_1 h_0$ represents the integer $h_n 16^n + h_{n-1} 16^{n-1} + \ldots + h_2\, 16^2 + h_1 16^1 + h_0 16^0$
- For example, the hex number $FACE_{16}$ can be written as:

$16^3 = 4096$	$16^2 = 256$	$16^1 = 16$	$16^0 = 1$
15	10	12	14

  - $15 \times 16^3 + 10 \times 16^2 + 12 \times 16^1 + 14 \times 16^0$
  - This is equal to the decimal representation $61440 + 2560 + 192 + 14 = 64206_{10}$
- Hex numbers are sometimes preceded by 0x to be distinguished from decimal numbers, e.g., 0x15 is 15_{16} which is equal to 21_{10} and not 15_{10}.

Equivalence across number systems

Hexadecimal digit	Binary equivalent	Decimal equivalent
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
A	1010	10
B	1011	11
C	1100	12
D	1101	13
E	1110	14
F	1111	15

A, B, C, D, E, F

10, 11, 12, 13, 14, 15

PRACTICE TIME

- Convert the following hexadecimal numbers to their decimal representation.
  - $32A_{16}$
  - $D1CE_{16}$
  - $1B7F_{16}$
  - 47_{16}

A, B, C, D, E, F

10, 11, 12, 13, 14, 15

ANSWER

- Convert the following hexadecimal numbers to their decimal representation.
  - $32A_{16} = 3 \times 16^2 + 2 \times 16^1 + 10 \times 16^0 = 810_{10}$
  - $D1CE_{16} = 13 \times 16^3 + 1 \times 16^2 + 12 \times 16^1 + 14 \times 16^0 = 53710_{10}$
  - $1B7F_{16} = 1 \times 16^3 + 11 \times 16^2 + 7 \times 16^1 + 15 \times 16^0 = 7039_{10}$
  - $47_{16} = 4 \times 16^1 + 7 \times 16^0 = 71_{10}$

Decimal to hexadecimal conversion

- Similarly to converting from decimal to binary, we will now repeatedly divide by 16 and the remainder is converted to hexadecimal and goes to the corresponding column from right to left.
- Example: Convert the decimal number 333_{10} to hexadecimal.

Divide by 16	**Remainder**
333	

LSB

MSB

Decimal to hexadecimal conversion

- Similarly to converting from decimal to binary, we will now repeatedly divide by 16 and the remainder is converted to hexadecimal and goes to the corresponding column from right to left.
- Example: Convert the decimal number 333_{10} to hexadecimal.
  - $\frac{333}{16} = 20$ with a remainder of $13_{10} = D_{16}$ which goes in the 1's column.

Divide by 16	**Remainder**
333	$13_{10} = D_{16}$
20	

LSB

MSB

		1
		D

Decimal to hexadecimal conversion

- Similarly to converting from decimal to binary, we will now repeatedly divide by 16 and the remainder is converted to hexadecimal and goes to the corresponding column from right to left.
- Example: Convert the decimal number 333_{10} to hexadecimal.
  - $\frac{333}{16} = 20$ with a remainder of $13_{10} = D_{16}$ which goes in the 1's column.
  - $\frac{20}{16} = 1$ with a remainder of 4, so 4 goes in the 16's column.

Divide by 16	**Remainder**
333	$13_{10} = D_{16}$
20	$4_{10} = 4_{16}$
1	

LSB

MSB

	16	1
	4	D

Decimal to hexadecimal conversion

- Similarly to converting from decimal to binary, we will now repeatedly divide by 16 and the remainder is converted to hexadecimal and goes to the corresponding column from right to left.
- Example: Convert the decimal number 333_{10} to hexadecimal.
  - $\frac{333}{16} = 20$ with a remainder of $13_{10} = D_{16}$ which goes in the 1's column.
  - $\frac{20}{16} = 1$ with a remainder of 4, so 4 goes in the 16's column.
  - $\frac{1}{16} = 0$ with a remainder of 1, so there is a 1 going to the 256s column.

Divide by 16	**Remainder**
333	$13_{10} = D_{16}$
20	$4_{10} = 4_{16}$
1	$1_{10} = 1_{16}$

LSB

MSB

256	16	1
1	4	D

Decimal to hexadecimal conversion

- Similarly to converting from decimal to binary, we will now repeatedly divide by 16 and the remainder is converted to hexadecimal and goes to the corresponding column from right to left.
- Example: Convert the decimal number 333_{10} to hexadecimal.
  - $\frac{333}{16} = 20$ with a remainder of $13_{10} = D_{16}$ which goes in the 1's column.
  - $\frac{20}{16} = 1$ with a remainder of 4, so 4 goes in the 16's column.
  - $\frac{1}{16} = 0$ with a remainder of 1, so there is a 1 going to the 256s column.
  - Putting this all together, $333_{10} = 14D_{16}$.

Divide by 16	**Remainder**
333	$13_{10} = D_{16}$
20	$4_{10} = 4_{16}$
1	$1_{10} = 1_{16}$

LSB

MSB

256	16	1
1	4	D

A, B, C, D, E, F

10, 11, 12, 13, 14, 15

PRACTICE TIME

- Convert the following decimal numbers to their hexadecimal representation.
  - 2546_{10}
  - 1457_{10}
  - 269_{10}
  - 47_{10}

A, B, C, D, E, F

10, 11, 12, 13, 14, 15

ANSWER

- Convert the following decimal numbers to their hexadecimal representation.
  - $2546_{10} = 9F2$ $1457_{10} = 5B1$ $269_{10} = 10D$ $47_{10} = 2F$

Divide by 16	Remainder
2546	$2_{10} = 2_{16}$
159	$15_{10} = F_{16}$
9	$9_{10} = 9_{16}$

Divide by 16	Remainder
1457	$1_{10} = 1_{16}$
91	$11_{10} = B_{16}$
5	$5_{10} = 5_{16}$

Divide by 16	Remainder
269	$13_{10} = D_{16}$
16	$0_{10} = 0_{16}$
1	$1_{10} = 1_{16}$

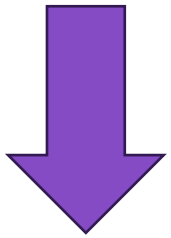
Divide by 16	Remainder
47	$15_{10} = F_{16}$
2	$2_{10} = 2_{16}$

LSB

MSB

Binary to hexadecimal conversion

$$1100111000010000_2$$



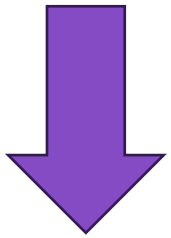
$$0110\ \ 0111\ \ 0001\ \ 0000_2$$

break into 4 bit groups

Hexadecimal digit	Binary equivalent
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

Binary to hexadecimal conversion

$$1100111000010000_2$$



$$0110\ \ 0111\ \ 0001\ \ 0000_2$$

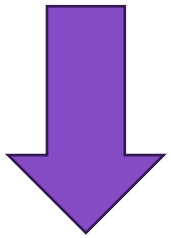
break into 4 bit groups

convert each 4 bit group into a digit

Hexadecimal digit	Binary equivalent
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

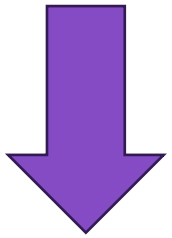
Binary to hexadecimal conversion

$$1100111000010000_2$$



break into 4 bit groups

$$0110\ \ 0111\ \ 0001\ \ 0000_2$$



convert each 4 bit group into a digit

$$6\ \ 7\ \ 1\ \ 0_{16}$$

Hexadecimal digit	Binary equivalent
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

PRACTICE TIME

- Convert the following binary numbers to their hexadecimal representation.
  - 10010011_2
  - 101100110101_2
  - 1111101110110010_2

Hexadecimal digit	Binary equivalent
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

ANSWER

- Convert the following binary numbers to their hexadecimal representation.
  - $1001\ 0011_2 = 93_{16}$
  - $1011\ 0011\ 0101_2 = B35_{16}$
  - $1111101110111 0010_2 = 0001\ 1111\ 0111\ 0111\ 0010 = 1\mathrm{F}772_{16}$

Hexadecimal digit	Binary equivalent
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

Hexadecimal to binary conversion

- We replace each hex digit with the four binary bits that correspond to its value.
- Example: Convert the hexadecimal number $CAFE_{16}$ to its binary representation.
  - $C_{16} = 1100_2$
  - $A_{16} = 1010_2$
  - $F_{16} = 1111_2$
  - $E_{16} = 1110_2$
  - Putting this all together, $CAFE_{16} = 1100101011111110_2$.

Hexadecimal digit	Binary equivalent
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

PRACTICE TIME

- Convert the following hexadecimal numbers to their binary representation.
  - $\mathrm{F00D}_{16}$
  - $2B3BAD_{16}$
  - $\mathrm{DEADC0DE}_{16}$

Hexadecimal digit	Binary equivalent
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

ANSWER

- Convert the following hexadecimal numbers to their binary representation.
  - $\mathrm{F00D}_{16} = 1111000000001101_2$
  - $2B3BAD_{16} = 00101011001110111010110 1_2$
  - $\mathrm{DEADC0DE}_{16} = 11011110101011011100000011011110_2$

Hexadecimal digit	Binary equivalent
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

Practice Problems

Practice Problems – Problem 1

Assuming x = 6, y = 2, z = 12. What would the following Python expressions evaluate to?

- x == 10 or x < 5
- x != 7 and x > 3
- not (x >= 5 and y <= 2)
- (x < 0) or (x >= 0)
- not (x != x)
- x > 1 and (z / x) != 2
- x <= 0 or (z / x) == 1
- x == 4 and y == 4

Practice Problems – Problem 2

- Rewrite the following code to make it cleaner and avoid the use of nested if statements:

```
def get_discount(member, purchase_amount):
        if member == True:
                if purchase_amount > 100:
                        return "Discount applied"
                else:
                        return "No discount"
        else:
                if purchase_amount > 200:
                        return "Discount applied"
                else:
                        return "No discount"
```

print(get_discount(True, 120)) # Expected: "Discount applied" print(get_discount(True, 80)) # Expected: "No discount" print(get_discount(False, 220)) # Expected: "Discount applied" print(get_discount(False, 50)) # Expected: "No discount"

Practice Problems – Problem 3

- For each row, fill in the missing entries by converting the given number from its representation to every other equivalent representation.

Binary	Decimal	Hexadecimal
	39_{10}	
100111010_2		
		$BADE_{16}$
	47_{10}	
101100111_2		
		$C0FFEE_{16}$

Practice Problem 1 – ANSWER

Assuming x = 6, y = 2, z = 12. What would the following Python expressions evaluate to?

- x == 10 or x < 5 → 6 == 10 or 6 < 5 → False or False → **False**
- x != 7 and x > 3 → 6 != 7 and 6 > 3 → True and True → **True**
- not (x >= 5 and y <= 2) → not (6 >= 5 and 2 <= 2) → not (True and True) → not True → **False**
- (x < 0) or (x >= 0) → 6 < 0 or 6 >= 0 → False or True → **True** (always true, no matter what x is)
- not (x != x) → not False → **True**
- x > 1 and (z / x) != 2 → 6 > 1 and (12 / 6 != 2) → True and False → **False**
- x <= 0 or (z / x) == 1 → 6 <= 0 or (12 / 6 == 1) → False or False → **False**
- x == 4 and y == 4 → 6 == 4 and 2 == 4 → False and False → **False**

Practice Problem 2 - ANSWER

- Rewrite the following code to make it cleaner and avoid the use of nested if statements:

```
def get_discount(member, purchase_amount):
        if (purchase_amount > 200) or (member and purchase_amount > 100):
                return "Discount applied"
        else:
                return "No discount"

print(get_discount(True, 120)) # Expected: "Discount applied" print(get_discount(True, 80)) # Expected: "No discount" print(get_discount(False, 220)) # Expected: "Discount applied" print(get_discount(False, 50)) # Expected: "No discount"
```

Practice Problem 3 - ANSWER

- For each row, fill in the missing entries by converting the given number from its representation to every other equivalent representation.

Binary	Decimal	Hexadecimal
100111_2	39_{10}	27_{16}
100111010_2	314_{10}	$13A_{16}$
1011101011011110_2	47838_{10}	$BADE_{16}$
101111_2	47_{10}	$2F_{16}$
101100111_2	359_{10}	167_{16}
$110000001111111111101110_2$	12648430_{10}	$C0FFEE_{16}$