

Computer Systems

Numeral Systems

CS51 – Fall 2025

Decimal numbers

- We are used to numbers that use ten digits: $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$.
- This arithmetic is called **base 10** (denoted as a subscript) and such numbers are called **decimal**.
- $d_n d_{n-1} \dots d_2\, d_1 d_0$ represents the integer $d_n 10^n + d_{n-1} 10^{n-1} + \dots + d_2\, 10^2 + d_1 10^1 + d_0 10^0$
- For example, the number 9247_{10} can be written as the sum of each of its digits multiplied by the weight of the corresponding column:

$10^3 = 1000$	$10^2 = 100$	$10^1 = 10$	$10^0 = 1$
9	2	4	7

  - $9247_{10} = 9 \times 10^3 + 2 \times 10^2 + 4 \times 10^1 + 7 \times 10^0$
- An n-digit decimal number is one of 10^n possibilities. For non-negative numbers, this provides a **range** of $0, 1, 2, 3, \dots, 10^n - 1$. For example, with 3 digits, we can represent $0, \dots, 999$.

Binary numbers

- We combine **bits**, that is binary digits that are either 0 or 1, to form **binary numbers**. This arithmetic is called base 2 and each column is twice as large as the one on its right.
- $b_n b_{n-1} \dots b_2\, b_1 b_0$ represents the integer $b_n 2^n + b_{n-1} 2^{n-1} + \dots + b_2\, 2^2 + b_1 2^1 + b_0 2^0$
- For example, the binary number 10100111_2 can be written as:

$2^7 = 128$	$2^6 = 64$	$2^5 = 32$	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$
1	0	1	0	0	1	1	1

  - $1 \times 2^7 + 0 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$
  - This is equal to the decimal representation $128 + 32 + 4 + 2 + 1 = 167_{10}$
- The left-most bit is the **most significant bit** (MSB). The right-most is the **least significant bit** (LSB).
- An n-digit binary number is one of 2^n possibilities: $0, 1, 2, 3, \dots, 2^n - 1$.
- We need at least $\log_2 n$ bits to represent information with n values.

Binary numbers and their decimal equivalents

1-bit binary	2-bit binary	3-bit binary	4-bit binary	Decimal equivalent
0	00	000	0000	0
1	01	001	0001	1
	10	010	0010	2
	11	011	0011	3
		100	0100	4
		101	0101	5
		110	0110	6
		111	0111	7
			1000	8
			1001	9
			1010	10
			1011	11
			1100	12
			1101	13
			1110	14
			1111	15

PRACTICE TIME

- Convert the following binary numbers to their decimal representation:
  - 10011011_2
  - 1100011_2
  - 100010_2

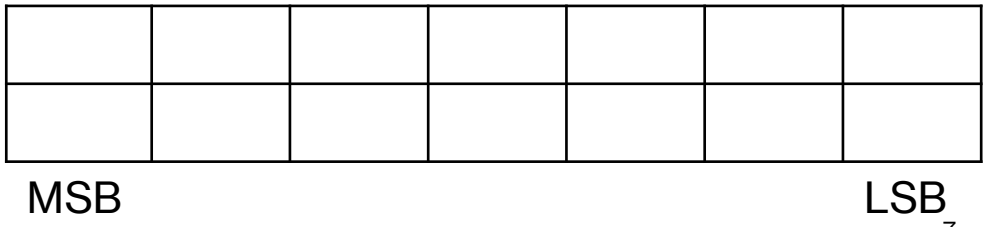
PRACTICE TIME - ANSWER

- Convert the following binary numbers to their decimal representation:
  - $10011011_2 = 1 \times 2^7 + 0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 128 + 16 + 8 + 2 + 1 = 155_{10}$
  - $1100011_2 = 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 99_{10}$
  - $100010_2 = 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 32 + 2 = 34_{10}$

Decimal to binary conversion

- Repeatedly divide the number by 2. The remainder goes to each column, from right to left.
- Example: Convert the decimal number 84_{10} to binary.

Divide by 2	Remainder	
84		LSB
		MSB



Decimal to binary conversion

- Repeatedly divide the number by 2. The remainder goes to each column, from right to left.
- Example: Convert the decimal number 84_{10} to binary.
  - $\frac{84}{2} = 42$ with a remainder of 0, so 0 goes in the 1's column.

Divide by 2	Remainder	
84	0	LSB
42		
		MSB



						1
						0

MSB LSB

Decimal to binary conversion

- Repeatedly divide the number by 2. The remainder goes to each column, from right to left.
- Example: Convert the decimal number 84_{10} to binary.
  - $\frac{84}{2} = 42$ with a remainder of 0, so 0 goes in the 1's column.
  - $\frac{42}{2} = 21$ with a remainder of 0, so 0 goes in the 2's column.

Divide by 2	Remainder	
84	0	LSB
42	0	
21		
		MSB



					2	1
					0	0
MSB						LSB

Decimal to binary conversion

- Repeatedly divide the number by 2. The remainder goes to each column, from right to left.
- Example: Convert the decimal number 84_{10} to binary.
  - $\frac{84}{2} = 42$ with a remainder of 0, so 0 goes in the 1's column.
  - $\frac{42}{2} = 21$ with a remainder of 0, so 0 goes in the 2's column.
  - $\frac{21}{2} = 10$ with a remainder of 1, so there is a 1 going to the 4s column.

Divide by 2	Remainder	
84	0	LSB
42	0	
21	1	
		MSB



				4	2	1
				1	0	0

MSB LSB

Decimal to binary conversion

- Repeatedly divide the number by 2. The remainder goes to each column, from right to left.
- Example: Convert the decimal number 84_{10} to binary.
  - $\frac{84}{2} = 42$ with a remainder of 0, so 0 goes in the 1's column.
  - $\frac{42}{2} = 21$ with a remainder of 0, so 0 goes in the 2's column.
  - $\frac{21}{2} = 10$ with a remainder of 1, so there is a 1 going to the 4s column.
  - $\frac{10}{2} = 5$ with a remainder of 0, so there is a 0 in the 8's column.

Divide by 2	**Remainder**	
84	0	LSB
42	0	
21	1	
10	0	
5		
		MSB



			8	4	2	1
			0	1	0	0

MSB LSB

Decimal to binary conversion

- Repeatedly divide the number by 2. The remainder goes to each column, from right to left.
- Example: Convert the decimal number 84_{10} to binary.
  - $\frac{84}{2} = 42$ with a remainder of 0, so 0 goes in the 1's column.
  - $\frac{42}{2} = 21$ with a remainder of 0, so 0 goes in the 2's column.
  - $\frac{21}{2} = 10$ with a remainder of 1, so there is a 1 going to the 4s column.
  - $\frac{10}{2} = 5$ with a remainder of 0, so there is a 0 in the 8's column.
  - $\frac{5}{2} = 2$ with a remainder of 1, so there is a 1 going to the 16's column.

Divide by 2	**Remainder**	
84	0	LSB
42	0	
21	1	
10	0	
5	1	
2		
		MSB



		16	8	4	2	1
		1	0	1	0	0

MSB LSB

Decimal to binary conversion

- Repeatedly divide the number by 2. The remainder goes to each column, from right to left.
- Example: Convert the decimal number 84_{10} to binary.
  - $\frac{84}{2} = 42$ with a remainder of 0, so 0 goes in the 1's column.
  - $\frac{42}{2} = 21$ with a remainder of 0, so 0 goes in the 2's column.
  - $\frac{21}{2} = 10$ with a remainder of 1, so there is a 1 going to the 4s column.
  - $\frac{10}{2} = 5$ with a remainder of 0, so there is a 0 in the 8's column.
  - $\frac{5}{2} = 2$ with a remainder of 1, so there is a 1 going to the 16's column.
  - $\frac{2}{2} = 1$, with a remainder of 0, so 0 goes in the 32's column.

Divide by 2	**Remainder**	
84	0	LSB
42	0	
21	1	
10	0	
5	1	
2	0	
1		MSB



	32	16	8	4	2	1
	0	1	0	1	0	0

MSB LSB

Decimal to binary conversion

- Repeatedly divide the number by 2. The remainder goes to each column, from right to left.
- Example: Convert the decimal number 84_{10} to binary.
  - $\frac{84}{2} = 42$ with a remainder of 0, so 0 goes in the 1's column.
  - $\frac{42}{2} = 21$ with a remainder of 0, so 0 goes in the 2's column.
  - $\frac{21}{2} = 10$ with a remainder of 1, so there is a 1 going to the 4s column.
  - $\frac{10}{2} = 5$ with a remainder of 0, so there is a 0 in the 8's column.
  - $\frac{5}{2} = 2$ with a remainder of 1, so there is a 1 going to the 16's column.
  - $\frac{2}{2} = 1$, with a remainder of 0, so 0 goes in the 32's column.
  - $\frac{1}{2} = 0$, with a remainder of 1, so 1 goes in the 64's column.

Divide by 2	**Remainder**	
84	0	LSB
42	0	
21	1	
10	0	
5	1	
2	0	
1	1	MSB



64	32	16	8	4	2	1
1	0	1	0	1	0	0

MSB LSB

Decimal to binary conversion

- Repeatedly divide the number by 2. The remainder goes to each column, from right to left.
- Example: Convert the decimal number 84_{10} to binary.
  - $\frac{84}{2} = 42$ with a remainder of 0, so 0 goes in the 1's column.
  - $\frac{42}{2} = 21$ with a remainder of 0, so 0 goes in the 2's column.
  - $\frac{21}{2} = 10$ with a remainder of 1, so there is a 1 going to the 4s column.
  - $\frac{10}{2} = 5$ with a remainder of 0, so there is a 0 in the 8's column.
  - $\frac{5}{2} = 2$ with a remainder of 1, so there is a 1 going to the 16's column.
  - $\frac{2}{2} = 1$, with a remainder of 0, so 0 goes in the 32's column.
  - $\frac{1}{2} = 0$, with a remainder of 1, so 1 goes in the 64's column.
- Putting this all together, $84_{10} = 1010100_2$.

Divide by 2	**Remainder**	
84	0	LSB
42	0	
21	1	
10	0	
5	1	
2	0	
1	1	MSB



64	32	16	8	4	2	1
1	0	1	0	1	0	0

MSB LSB

PRACTICE TIME

- Convert the following decimal numbers to their binary representation.
  - 125_{10}
  - 339_{10}
  - 75_{10}

PRACTICE TIME - ANSWER

- Convert the following decimal numbers to their binary representation.
  - $125_{10} = 1111101_2$
  - $339_{10} = 101010011_2$
  - $75_{10} = 1001011_2$

Divide by 2	Remainder	
125	1	LSB
62	0	
31	1	
15	1	
7	1	
3	1	
1	1	MSB

Divide by 2	Remainder	
339	1	LSB
169	1	
84	0	
42	0	
21	1	
10	0	
5	1	
2	0	
1	1	MSB

Divide by 2	Remainder	
75	1	LSB
37	1	
18	0	
9	1	
4	0	
2	0	
1	1	MSB

Hexadecimal numbers

- We also use a **hexadecimal** (or hex) notation where the arithmetic is base 16.
- Each digit is one of $0, 1, 2, 3, 4, 5, 6, 7, 8, 9, \mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}, \mathrm{F}$
  - (i.e. A for 10, B for 11, C for 12, D for 13, E for 14, and F for 15).
- $h_n h_{n-1} \dots h_2 \, h_1 h_0$ represents the integer $h_n 16^n + h_{n-1} 16^{n-1} + \dots + h_2 \, 16^2 + h_1 16^1 + h_0 16^0$
- For example, the hex number $FACE_{16}$ can be written as:

$16^3 = 4096$	$16^2 = 256$	$16^1 = 16$	$16^0 = 1$
15	10	12	14

  - $15 \times 16^3 + 10 \times 16^2 + 12 \times 16^1 + 14 \times 16^0$
  - This is equal to the decimal representation $61440 + 2560 + 192 + 14 = 64206_{10}$
- An n-digit hex number is one of 16^n possibilities: $0, 1, 2, 3, \dots, 16^n - 1$.
- Hex numbers are sometimes preceded by 0x to be distinguished from decimal numbers, e.g., 0x15 is 15_{16} which is equal to 21_{10} and not 15_{10}.

Equivalence across number systems

Hexadecimal digit	Binary equivalent	Decimal equivalent
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
A	1010	10
B	1011	11
C	1100	12
D	1101	13
E	1110	14
F	1111	15

PRACTICE TIME

- Convert the following hexadecimal numbers to their decimal representation.
  - $32A_{16}$
  - $D1CE_{16}$
  - $1B7F_{16}$
  - 47_{16}

ANSWER

- Convert the following hexadecimal numbers to their decimal representation.
  - $32A_{16} = 3 \times 16^2 + 2 \times 16^1 + 10 \times 16^0 = 810_{10}$
  - $D1CE_{16} = 13 \times 16^3 + 1 \times 16^2 + 12 \times 16^1 + 14 \times 16^0 = 53710_{10}$
  - $1B7F_{16} = 1 \times 16^3 + 11 \times 16^2 + 7 \times 16^1 + 15 \times 16^0 = 7039_{10}$
  - $47_{16} = 4 \times 16^1 + 7 \times 16^0 = 71_{10}$

Decimal to hexadecimal conversion

- Similarly to converting from decimal to binary, we will now repeatedly divide by 16 and the remainder is converted to hexadecimal and goes to the corresponding column from right to left.
- Example: Convert the decimal number 333_{10} to hexadecimal.

Divide by 16	Remainder
333	

LSB

MSB

Decimal to hexadecimal conversion

- Similarly to converting from decimal to binary, we will now repeatedly divide by 16 and the remainder is converted to hexadecimal and goes to the corresponding column from right to left.
- Example: Convert the decimal number 333_{10} to hexadecimal.
  - $\frac{333}{16} = 20$ with a remainder of $13_{10} = D_{16}$ which goes in the 1's column.

Divide by 16	**Remainder**
333	$13_{10} = D_{16}$
20	

LSB

MSB

		1
		D

Decimal to hexadecimal conversion

- Similarly to converting from decimal to binary, we will now repeatedly divide by 16 and the remainder is converted to hexadecimal and goes to the corresponding column from right to left.
- Example: Convert the decimal number 333_{10} to hexadecimal.
  - $\frac{333}{16} = 20$ with a remainder of $13_{10} = D_{16}$ which goes in the 1's column.
  - $\frac{20}{16} = 1$ with a remainder of 4, so 4 goes in the 16's column.

Divide by 16	Remainder	
333	$13_{10} = D_{16}$	LSB
20	$4_{10} = 4_{16}$	
1		MSB

	16	1
	4	D

Decimal to hexadecimal conversion

- Similarly to converting from decimal to binary, we will now repeatedly divide by 16 and the remainder is converted to hexadecimal and goes to the corresponding column from right to left.
- Example: Convert the decimal number 333_{10} to hexadecimal.
  - $\frac{333}{16} = 20$ with a remainder of $13_{10} = D_{16}$ which goes in the 1's column.
  - $\frac{20}{16} = 1$ with a remainder of 4, so 4 goes in the 16's column.
  - $\frac{1}{16} = 0$ with a remainder of 1, so there is a 1 going to the 256s column.

Divide by 16	**Remainder**	
333	$13_{10} = D_{16}$	LSB
20	$4_{10} = 4_{16}$	
1	$1_{10} = 1_{16}$	MSB

256	16	1
1	4	D

Decimal to hexadecimal conversion

- Similarly to converting from decimal to binary, we will now repeatedly divide by 16 and the remainder is converted to hexadecimal and goes to the corresponding column from right to left.
- Example: Convert the decimal number 333_{10} to hexadecimal.
  - $\frac{333}{16} = 20$ with a remainder of $13_{10} = D_{16}$ which goes in the 1's column.
  - $\frac{20}{16} = 1$ with a remainder of 4, so 4 goes in the 16's column.
  - $\frac{1}{16} = 0$ with a remainder of 1, so there is a 1 going to the 256s column.
  - Putting this all together, $333_{10} = 14D_{16}$.

Divide by 16	Remainder	
333	$13_{10} = D_{16}$	LSB
20	$4_{10} = 4_{16}$	
1	$1_{10} = 1_{16}$	MSB

256	16	1
1	4	D

PRACTICE TIME

- Convert the following decimal numbers to their hexadecimal representation.
  - 2546_{10}
  - 1457_{10}
  - 269_{10}
  - 47_{10}

ANSWER

- Convert the following decimal numbers to their hexadecimal representation.
  - $2546_{10} = 9F2$ $1457_{10} = 5B1$ $269_{10} = 10D$ $47_{10} = 2F$

Divide by 16	Remainder
2546	$2_{10} = 2_{16}$
159	$15_{10} = F_{16}$
9	$9_{10} = 9_{16}$

Divide by 16	Remainder
1457	$1_{10} = 1_{16}$
91	$11_{10} = B_{16}$
5	$5_{10} = 5_{16}$

Divide by 16	Remainder
269	$13_{10} = D_{16}$
16	$0_{10} = 0_{16}$
1	$1_{10} = 1_{16}$

Divide by 16	Remainder
47	$15_{10} = F_{16}$
2	$2_{10} = 2_{16}$

LSB

MSB

Binary to hexadecimal conversion

- We read the binary digits in groups of four and convert each group to its corresponding hexadecimal number. If needed, prepend leading 0s to make the number of bits a multiple of 4.
- Example: Convert the binary number 1100111000100000_2 to its hexadecimal representation.
  - Prepend a 0 to turn the number of digits to 16 (4 blocks of 4), i.e. we need to convert $0110\ 0111\ 0001\ 0000_2$
  - $0110_2 = 6_{16}$
  - $0111_2 = 7_{16}$
  - $0001_2 = 1_{16}$
  - $0000_2 = 0_{16}$
  - Putting this all together, $1100111000100000_2 = 6710_{16}$.

Hexadecimal digit	Binary equivalent
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

PRACTICE TIME

- Convert the following binary numbers to their hexadecimal representation.
  - 10010011_2
  - 101100110101_2
  - 1111101110110010_2

Hexadecimal digit	Binary equivalent
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

ANSWER

- Convert the following binary numbers to their hexadecimal representation.
  - $1001\,0011_2 = 93_{16}$
  - $1011\,0011\,0101_2 = B35_{16}$
  - $1111101110110010_2 = 0001\,1111\,0111\,0111\,0010 = 1\mathrm{F}772_{16}$

Hexadecimal digit	Binary equivalent
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

Hexadecimal to binary conversion

- We replace each hex digit with the four binary bits that correspond to its value.
- Example: Convert the hexadecimal number $CAFE_{16}$ to its binary representation.
  - $C_{16} = 1100_2$
  - $A_{16} = 1010_2$
  - $F_{16} = 1111_2$
  - $E_{16} = 1110_2$
  - Putting this all together, $CAFE_{16} = 1100101011111110_2$.

Hexadecimal digit	Binary equivalent
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

PRACTICE TIME

- Convert the following hexadecimal numbers to their binary representation.
  - $\mathrm{F00D}_{16}$
  - $2B3BAD_{16}$
  - $\mathrm{DEADC0DE}_{16}$

Hexadecimal digit	Binary equivalent
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

ANSWER

- Convert the following hexadecimal numbers to their binary representation.
  - $\mathrm{F00D}_{16} = 1111000000001101_2$
  - $2B3BAD_{16} = 00101011001110111010110 1_2$
  - $\mathrm{DEADC0DE}_{16} = 11011110101011011100000011011110_2$

Hexadecimal digit	Binary equivalent
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

Practice Problem

- For each row, fill in the missing entries by converting the given number from its representation to every other equivalent representation.

Binary	Decimal	Hexadecimal
	39_{10}	
100111010_2		
		$BADE_{16}$
	47_{10}	
101100111_2		
		$C0FFEE_{16}$

Practice Problem - ANSWER

- For each row, fill in the missing entries by converting the given number from its representation to every other equivalent representation.

Binary	Decimal	Hexadecimal
100111_2	39_{10}	27_{16}
100111010_2	314_{10}	$13A_{16}$
1011101011011110_2	47838_{10}	$BADE_{16}$
101111_2	47_{10}	$2F_{16}$
101100111_2	359_{10}	167_{16}
$110000001111111111101110_2$	12648430_{10}	$C0FFEE_{16}$