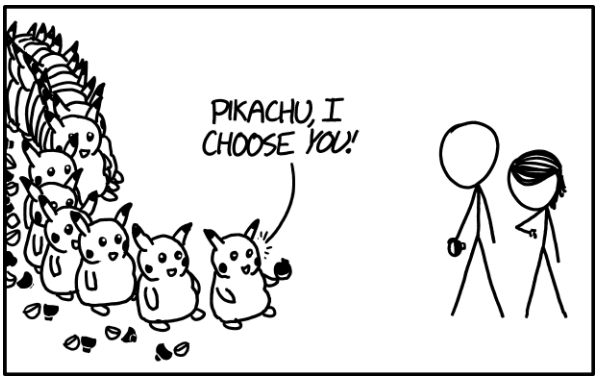
CS51 - Assignment 6
Mathematical Induction

Due: October 23, at 11:59pm



https://xkcd.com/1516/

For this assignment, we will gain more practice with:

- the principle of mathematical induction (both weak and strong), and
- writing math in LaTeX.

Background

You will find six problems that ask you to prove various claims (Problem 4 asks you to come up with the claim yourself). Unless specified otherwise, you should assume that you should prove the claim using regular/weak mathematical induction. You are encouraged to work on pen and paper or a board first before you proceed with typing your response in the provided `assignment6.tex`. As always, please keep track of the time you have spent on the assignment and submit your feedback.

Remember, the use of generative AI, such as ChatGPT, for the assignment completion is a violation of the academic honesty policy of this course.

Problem 1

Using mathematical induction, prove that for each integer $n \geq 0$, $\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

Problem 2

Using mathematical induction, prove that for each integer $n \geq 1$, $4^n - 1$ is evenly divisible by 3.

Problem 3

A sequence of numbers $a_1, a_2, a_3, ...$ is defined by letting $a_1 = 3$ and $a_k = 7a_{k-1}$ for all integers $k \geq 2$. Using mathematical induction, prove that $a_n = 3 \times 7^{(n-1)}$ for all integers $n \geq 1$.

Problem 4

Observe the following pattern:

$\frac{1}{1\times3} = \frac{1}{3}$

$\frac{1}{1\times3} + \frac{1}{3\times5} = \frac{2}{5}$

$\frac{1}{1\times3} + \frac{1}{3\times5} + \frac{1}{5\times7} = \frac{3}{7}$

$\frac{1}{1\times3} + \frac{1}{3\times5} + \frac{1}{5\times7} + \frac{1}{7\times9} = \frac{4}{9}$

Guess a general formula that describes this pattern and prove your claim by using mathematical induction.

Problem 5

Using **strong** induction, prove that for every integer $n \geq 4$, it is possible to make n dollars using only two- and five-dollar bills. That is, prove that any integer $n \geq 4$ can be written as $n = 2a + 5b$ for some integer $a \geq 0$ and some integer $b \geq 0$.

Problem 6

Using **strong** induction, prove that given any positive integer $n \geq 1$, there exists a binary representation of it in the form $n = c_0 + c_1 \times 2 + c_2 \times 2^2 + ... + c_{r-1}2^{r-1} + c_r 2^r$, where r is a non-negative integer, $c_j = 1$ or 0 for all $j = 0, 1, 2, ..., r - 1$.

Hint: It might help to prove it in terms of what happens when n is even and when it is odd.

Feedback

Create a file named `feedback6.txt` that answers the questions:

- How long did you spend on this assignment?
- Any comments or feedback? What things did you find interesting, challenging, or boring?

When you're done

Submit your `assignment6.tex`, resulting `assignment6.pdf`, and `feedback6.txt` files online using Gradescope; be sure you're uploading them to the right assignment. Note that you can resubmit the same assignment as many times as you would like up until the deadline.

Grading

Problem	points
Problem 1	1
Problem 2	2
Problem 3	2
Problem 4	2
Problem 5	2
Problem 6	2
Total	10