In-Class Worksheet

CS 181 Advanced Algorithms — Spring 2026

Minimum-cost perfect-matchings: Consider the bipartite graph $G = (L \cup R, E)$ with perfect matching $M \subseteq E$ shown in blue.

What is the current cost of M? Show that M is not minimum cost by finding a net-negative alternating cycle. Toggle this cycle to obtain a new perfect matching M' of smaller cost. Is M' a minimum cost perfect matching?
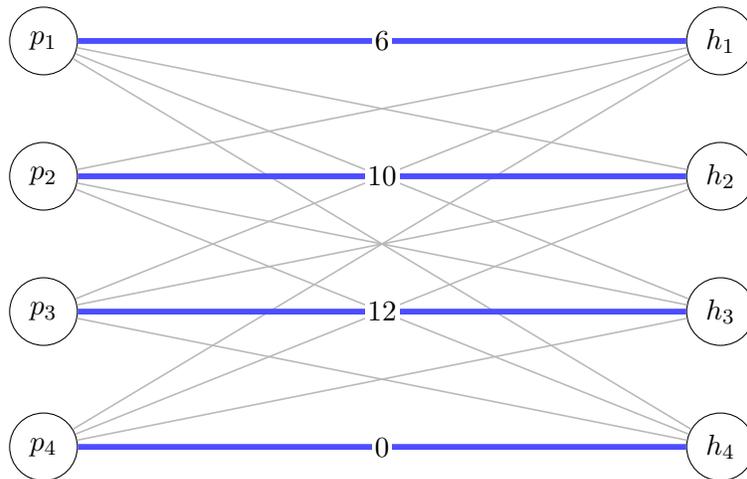

Figure 1: Unlabeled edges have cost 5.

Optimality Condition for Minimum Cost Bipartite Perfect Matching: Let $G = (L \cup R, E)$ be bipartite graph with costs on edges and let $M \subseteq E$ be a perfect matching in G. Then M is a minimum-cost perfect matching in G if and only if: _____.