

In-Class Worksheet

CS 181 Advanced Algorithms — Spring 2026

Hall's theorem: Let $G = (L \cup R, E)$ be a bipartite graph with $|L| = |R|$. Then G has a perfect matching if and only if $\forall S \subseteq L$ we have $|N(S)| \geq |S|$.

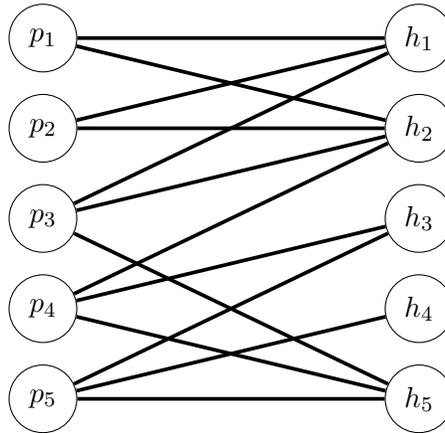


Figure 1: Determine the neighborhood $N(S)$ of $S = \{p_2, p_4\}$. Does this graph have a perfect matching? If yes, show it. If not, find a Hall-violating set.

Hackathon: There is a Hackathon attended by n Claremont students and n CMU students. Each Claremont student is friends with exactly k CMU students. Each CMU student is friends with exactly k Claremont students. Can we form pairs up all $2n$ students into n pairs of friends?

