

GRAMMARS

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some slides adapted from
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Admin

Assignment 3 out today: due next Monday

Quiz

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Context free grammar

$$S \rightarrow NP VP$$

left hand side right hand side
(single symbol) (one or more symbols)

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Formally...

$G = (NT, T, P, S)$

NT: finite set of nonterminal symbols

T: finite set of terminal symbols, NT and T are disjoint

P: finite set of productions of the form
 $A \rightarrow \alpha, A \in NT \text{ and } \alpha \in (T \cup NT)^*$

$S \in NT$: start symbol

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CFG: Example

Many possible CFGs for English, here is an example (fragment):

- S → NP VP
- VP → V NP
- NP → DetP N | DetP AdjP N
- AdjP → Adj | Adv AdjP
- N → kid | dog
- V → sees | likes
- Adj → big | small
- Adv → very
- DetP → a | the

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Derivations in a CFG

- S → NP VP
- VP → V NP
- NP → DetP N | DetP AdjP N
- AdjP → Adj | Adv AdjP
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S

What can we do?

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S

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NP VP

What can we do?

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Derivations in a CFG

$S \rightarrow NP VP$
 $VP \rightarrow V NP$
 $NP \rightarrow DetP N \mid DetP AdjP N$
 $AdjP \rightarrow Adj \mid Adv AdjP$
 $N \rightarrow kid \mid dog$
 $V \rightarrow sees \mid likes$
 $Adj \rightarrow big \mid small$
 $Adv \rightarrow very$
 $DetP \rightarrow a \mid the$

NP VP

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Derivations in a CFG

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 $VP \rightarrow V NP$
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DetP N VP

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Derivations in a CFG

$S \rightarrow NP VP$
 $VP \rightarrow V NP$
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 $N \rightarrow kid \mid dog$
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DetP N VP

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Derivations in a CFG

$S \rightarrow NP VP$
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 $NP \rightarrow DetP N \mid DetP AdjP N$
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 $N \rightarrow kid \mid dog$
 $V \rightarrow sees \mid likes$
 $Adj \rightarrow big \mid small$
 $Adv \rightarrow very$
 $DetP \rightarrow a \mid the$

the kid VP

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Derivations in a CFG

$S \rightarrow NP VP$
 $VP \rightarrow V NP$
 $NP \rightarrow \text{DetP } N \mid \text{DetP AdjP } N$
 $AdjP \rightarrow Adj \mid Adv AdjP$
 $N \rightarrow \text{kid} \mid \text{dog}$
 $V \rightarrow \text{sees} \mid \text{likes}$
 $Adj \rightarrow \text{big} \mid \text{small}$
 $Adv \rightarrow \text{very}$
 $\text{DetP} \rightarrow a \mid \text{the}$

the kid likes NP

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Derivations in a CFG

$S \rightarrow NP VP$
 $VP \rightarrow V NP$
 $NP \rightarrow \text{DetP } N \mid \text{DetP AdjP } N$
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 $Adj \rightarrow \text{big} \mid \text{small}$
 $Adv \rightarrow \text{very}$
 $\text{DetP} \rightarrow a \mid \text{the}$

the kid likes a dog

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Derivations in a CFG; Order of Derivation Irrelevant

$S \rightarrow NP VP$
 $VP \rightarrow V NP$
 $NP \rightarrow \text{DetP } N \mid \text{DetP AdjP } N$
 $AdjP \rightarrow Adj \mid Adv AdjP$
 $N \rightarrow \text{kid} \mid \text{dog}$
 $V \rightarrow \text{sees} \mid \text{likes}$
 $Adj \rightarrow \text{big} \mid \text{small}$
 $Adv \rightarrow \text{very}$
 $\text{DetP} \rightarrow a \mid \text{the}$

the kid likes a dog

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Derivations of CFGs

String rewriting system: we derive a string

Derivation history shows the constituent tree:

the kid likes a dog

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Parsing

Parsing is the field of NLP interested in automatically determining the syntactic structure of a sentence

Parsing can be thought of as determining what sentences are "valid" English sentences

As a byproduct, we often can get the structure

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Parsing

Given a CFG and a sentence, determine the possible parse tree(s)

```
S -> NP VP
NP -> N
NP -> PRP
NP -> N PP
VP -> V NP
VP -> V NP PP
PP -> IN N
PRP -> I
V -> eat
N -> sushi
N -> tuna
IN -> with
```

I eat sushi with tuna

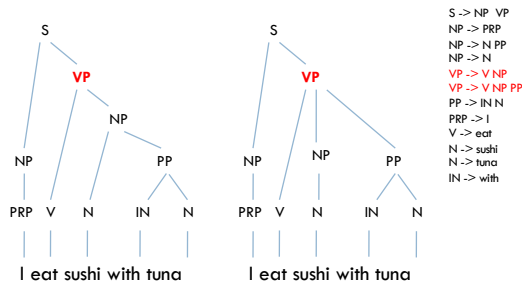
What parse trees are possible for this sentence?

How did you do it?

What if the grammar is much larger?

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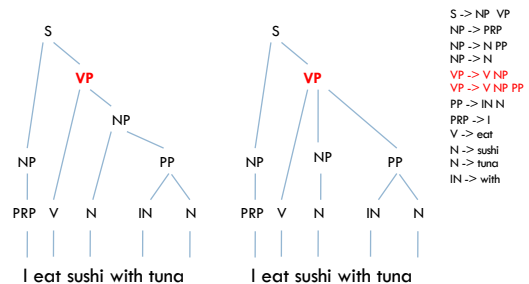
Parsing



What is the difference between these parses?

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Parsing ambiguity



How can we decide between these?

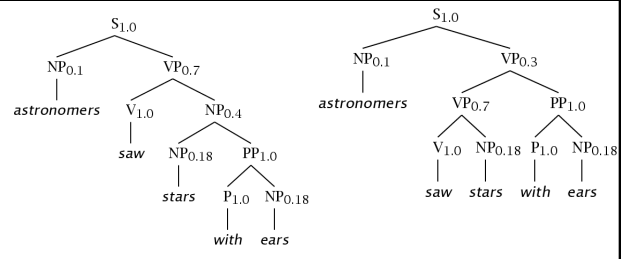
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A Simple PCFG

Probabilities!

S	→	NP VP	1.0	NP	→	NP PP	0.4
VP	→	V NP	0.7	NP	→	<i>astronomers</i>	0.1
VP	→	VP PP	0.3	NP	→	<i>ears</i>	0.18
PP	→	P NP	1.0	NP	→	<i>saw</i>	0.04
P	→	<i>with</i>	1.0	NP	→	<i>stars</i>	0.18
V	→	<i>saw</i>	1.0	NP	→	<i>telescope</i>	0.1

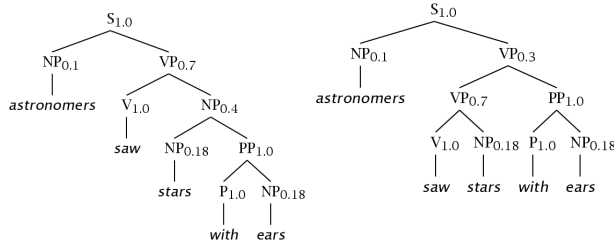
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Just like *n*-gram language modeling, PCFGs break the sentence generation process into smaller steps/probabilities

The probability of a parse is the product of the PCFG rules

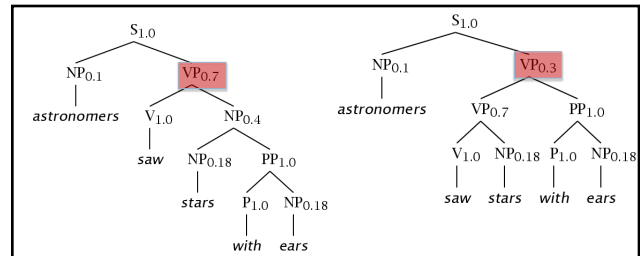
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What are the different interpretations here?

Which do you think is more likely?

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$$= 1.0 * 0.1 * 0.7 * 1.0 * 0.4 * 0.18$$

$$= 0.0009072$$

$$= 1.0 * 0.1 * 0.3 * 0.7 * 1.0 * 0.18$$

$$= 0.0006804$$

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Parsing problems

- Pick a model
 - e.g. CFG, PCFG, ...
- Train (or learn) a model
 - What CFG/PCFG rules should I use?
 - Parameters (e.g. PCFG probabilities)?
 - What kind of data do we have?
- Parsing
 - Determine the parse tree(s) given a sentence

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PCFG: Training

If we have example parsed sentences, how can we learn a set of PCFGs?

Tree Bank

Supervised PCFG Training

S → NP VP	0.9
S → VP	0.1
NP → Det AN	0.5
NP → NP PP	0.3
NP → PropN	0.2
A → ε	0.6
A → Adj A	0.4
PP → Prep NP	1.0
VP → V NP	0.7
VP → VPP	0.3

English

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Extracting the rules

I eat sushi with tuna

What CFG rules occur in this tree?

S	→	NP VP
NP	→	PRP
PRP	→	I
VP	→	V NP
V	→	eat
NP	→	N PP
N	→	sushi
PP	→	IN N
IN	→	with
N	→	tuna

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Estimating PCFG Probabilities

We can extract the rules from the trees

S	→	NP VP	1.0
NP	→	PRP	0.7
PRP	→	I	0.3
VP	→	V NP	1.0
V	→	eat	1.0
NP	→	N PP	1.0
N	→	sushi	1.0
PP	→	IN N	1.0
IN	→	with	1.0
N	→	tuna	1.0
...			

How do we go from the extracted CFG rules to PCFG rules?

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Estimating PCFG Probabilities

Extract the rules from the trees

Calculate the probabilities using MLE

$$\alpha \rightarrow \beta \quad \longrightarrow \quad p(\alpha \rightarrow \beta | \alpha)$$

$$P(\alpha \rightarrow \beta | \alpha) = \frac{\text{count}(\alpha \rightarrow \beta)}{\sum_{\gamma} \text{count}(\alpha \rightarrow \gamma)} = \frac{\text{count}(\alpha \rightarrow \beta)}{\text{count}(\alpha)}$$

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Estimating PCFG Probabilities

Occurrences

S	→ NP VP	10
S	→ V NP	3
S	→ VP PP	2
NP	→ N	7
NP	→ N PP	3
NP	→ DT N	6

$P(S \rightarrow V NP) = ?$

$$P(S \rightarrow V NP) = P(S \rightarrow V NP | S) = \frac{\text{count}(S \rightarrow V NP)}{\text{count}(S)} = \frac{3}{15}$$

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Grammar Equivalence

What does it mean for two grammars to be equal?

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Grammar Equivalence

Weak equivalence: grammars generate the same set of strings

- Grammar 1: NP → DetP N and DetP → a | the
- Grammar 2: NP → a N | the N

Strong equivalence: grammars have the same set of derivation trees

- With CFGs, possible only with useless rules
- Grammar 2: NP → a N | the N
- Grammar 3: NP → a N | the N, DetP → many

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Normal Forms

There are weakly equivalent **normal forms** (Chomsky Normal Form, Greibach Normal Form)

A CFG is in Chomsky Normal Form (CNF) if all productions are of one of two forms:

- $A \rightarrow BC$ with A, B, C nonterminals
- $A \rightarrow a$, with A a nonterminal and a a terminal

Every CFG has a weakly equivalent CFG in CNF

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CNF Grammar

S -> VP	S -> VP
VP -> VB NP	VP -> VB NP
VP -> VB NP PP	VP -> VP2 PP
NP -> DT NN	VP2 -> VB NP
NP -> NN	NP -> DT NN
NP -> NP PP	NP -> NN
PP -> IN NP	NP -> NP PP
DT -> the	PP -> IN NP
IN -> with	DT -> the
VB -> film	IN -> with
VB -> trust	VB -> film
NN -> man	VB -> trust
NN -> film	NN -> man
NN -> trust	NN -> film
	NN -> trust

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Probabilistic Grammar Conversion

Original Grammar		Chomsky Normal Form	
S → NP VP	0.8	S → NP VP	0.8
S → Aux NP VP	0.1	S → X1 VP	0.1
		X1 → Aux NP	1.0
S → VP	0.1	S → book include prefer	0.01 0.004 0.006
		S → Verb NP	0.05
		S → VP PP	0.03
NP → Pronoun	0.2	NP → I he she me	0.1 0.02 0.02 0.06
NP → Proper-Noun	0.2	NP → Houston NWA	0.16 0.04
NP → Det Nominal	0.6	NP → Det Nominal	0.6
Nominal → Noun	0.3	Nominal → book flight meal money	0.03 0.15 0.06 0.06
Nominal → Nominal Noun	0.2	Nominal → Nominal Noun	0.2
Nominal → Nominal PP	0.5	Nominal → Nominal PP	0.5
VP → Verb	0.2	VP → book include prefer	0.1 0.04 0.06
VP → Verb NP	0.5	VP → Verb NP	0.5
VP → VP PP	0.3	VP → VP PP	0.3
PP → Prep NP	1.0	PP → Prep NP	1.0

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Grammar questions

Can we determine if a sentence is grammatical?

Given a sentence, can we determine the syntactic structure?

Can we determine how likely a sentence is to be grammatical? to be an English sentence?

Can we generate candidate, grammatical sentences?

Next time: parsing

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