ADVANCED CLASSIFICATION TECHNIQUES

David Kauchak
CS 159 – Spring 2023

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Admin

Assignment 7

Next Monday: project proposal presentations
- informal
- 1 minute
- See the final project handout for details

Hack week QA session from OpenAI engineer (Friday @ 12:30pm)
- https://5chack.com/#hack-week

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Schedule for the rest of the semester

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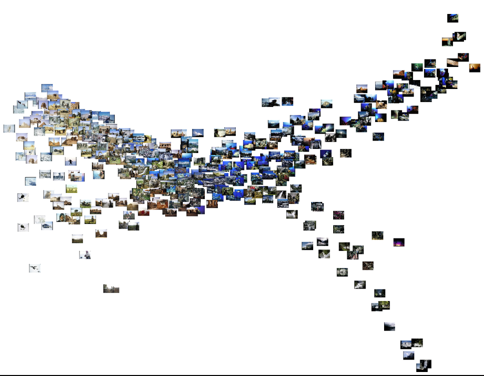
Machine Learning: A Geometric View



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Apples vs. Bananas

Weight	Color	Label
4	Red	Apple
5	Yellow	Apple
6	Yellow	Banana
3	Red	Apple
7	Yellow	Banana
8	Yellow	Banana
6	Yellow	Apple

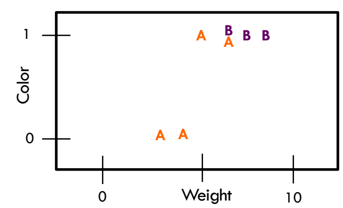
Can we visualize this data?

5

Apples vs. Bananas

Turn features into numerical values

Weight	Color	Label
4	0	Apple
5	1	Apple
6	1	Banana
3	0	Apple
7	1	Banana
8	1	Banana
6	1	Apple



We can view examples as points in an *n*-dimensional space where *n* is the number of features called the feature space

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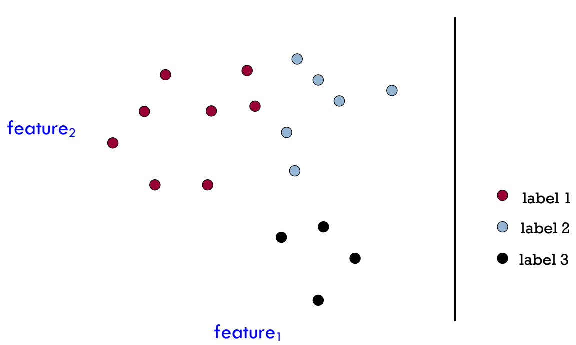
Examples in a feature space



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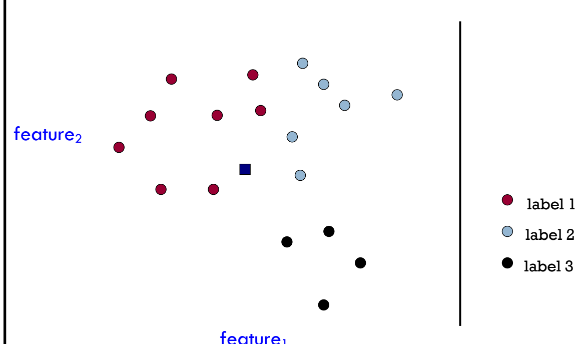
Test example: what class?



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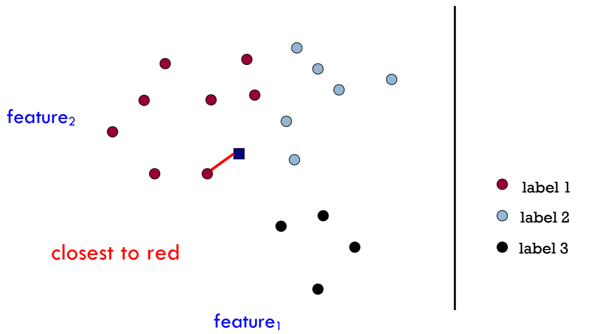
Test example: what class?



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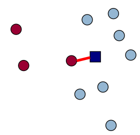
Another classification algorithm?

To classify an example ***d***:

Label ***d*** with the label of the closest example to ***d*** in the training set



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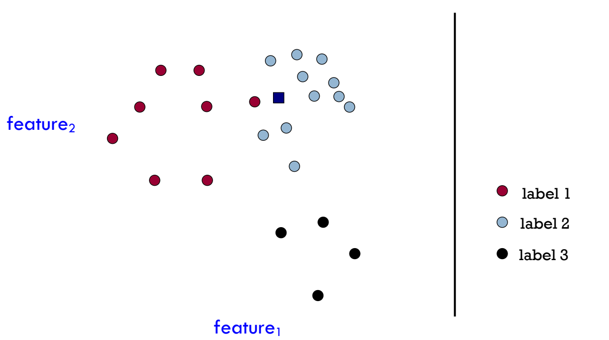
What about this example?



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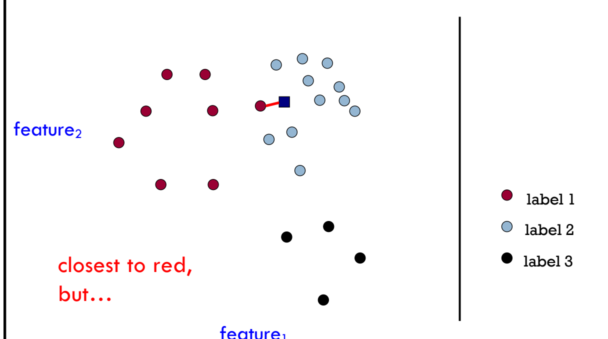
What about this example?



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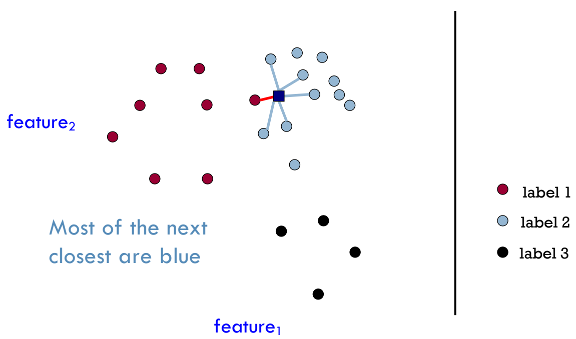
What about this example?



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k-Nearest Neighbor (k-NN)

To classify an example ***d***:

- Find ***k*** nearest neighbors of ***d***
- Choose as the label the majority label within the ***k*** nearest neighbors

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k-Nearest Neighbor (k-NN)

To classify an example ***d***:

- Find ***k*** *nearest* neighbors of ***d***
- Choose as the label the majority label within the ***k*** nearest neighbors

How do we measure "nearest"?

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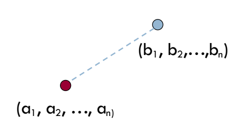
Euclidean distance

Euclidean distance! (or L1 or cosine or …)



$$D(a,b) = \sqrt{(a_1-b_1)^2 + (a_2-b_2)^2 + \ldots + (a_n-b_n)^2}$$

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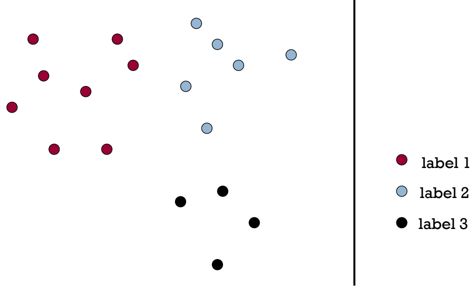
Decision boundaries

The **decision boundaries** are places in the features space where the classification of a point/example changes



Where are the decision boundaries for k-NN?

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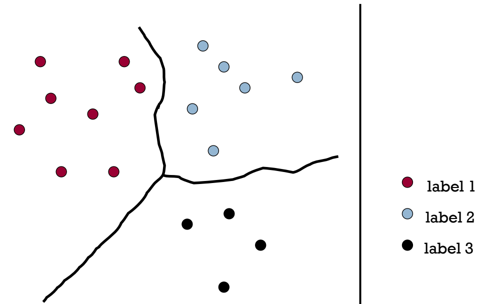
k-NN decision boundaries



k-NN gives locally defined decision boundaries between classes

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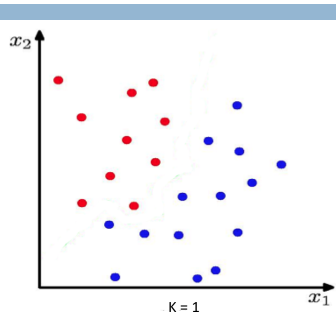
K Nearest Neighbour (k-NN) Classifier



What is the decision boundary for k-NN for this one?

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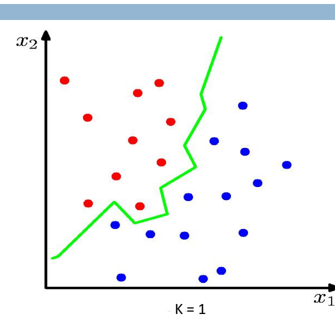
K Nearest Neighbour (*k*NN) Classifier



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Machine learning models

Some machine learning approaches make strong assumptions about the data

- If the assumptions are true this can often lead to better performance
- If the assumptions aren't true, they can fail miserably

Other approaches don't make many assumptions about the data

- This can allow us to learn from more varied data
- But, they are more prone to overfitting (biasing too much to the training data)
- and generally require more training data

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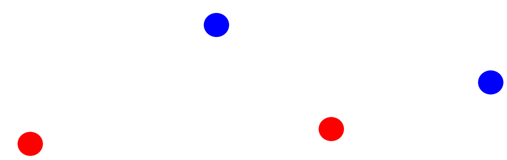
What is the data generating distribution?



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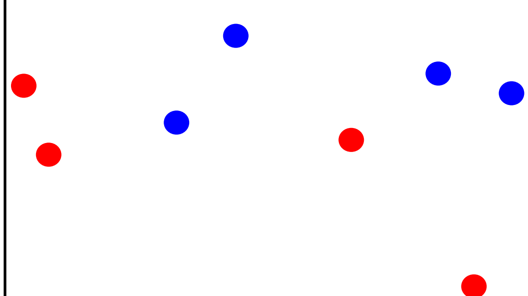

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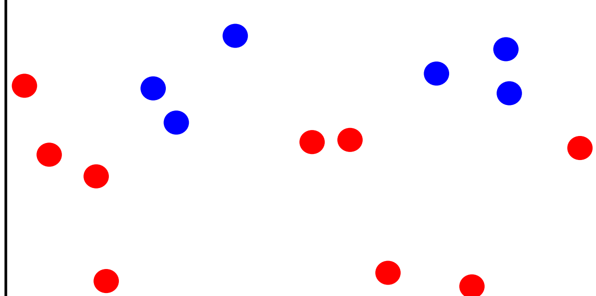

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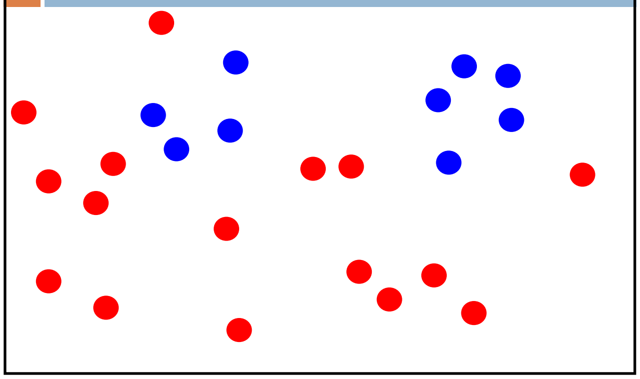

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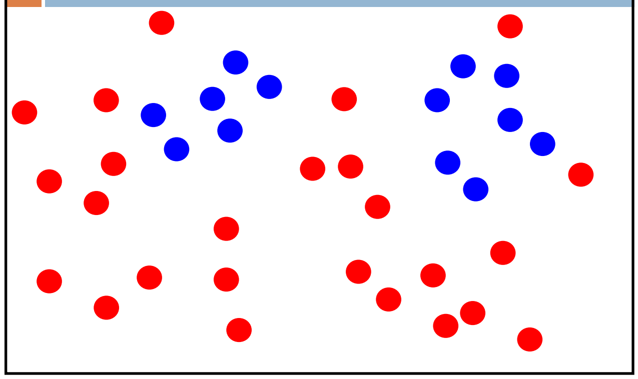
What is the data generating distribution?



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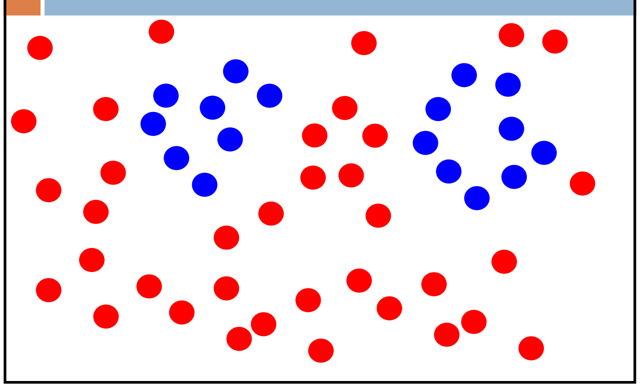

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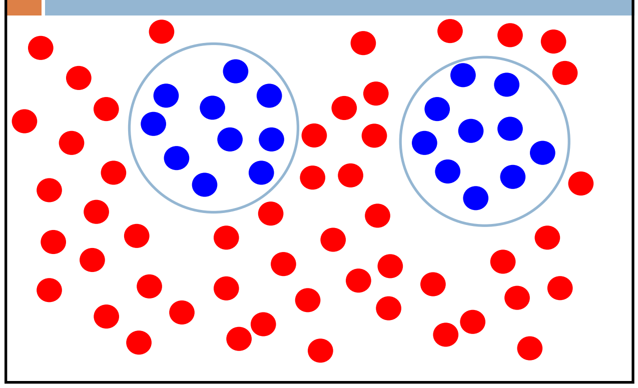
Actual model



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Model assumptions

If you don't have strong assumptions about the model, it can take you a longer to learn

Assume now that our model of the blue class is two circles

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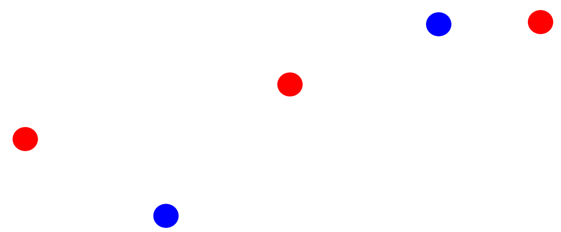
What is the data generating distribution?



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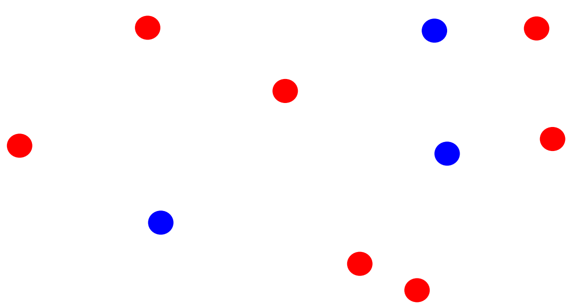
What is the data generating distribution?



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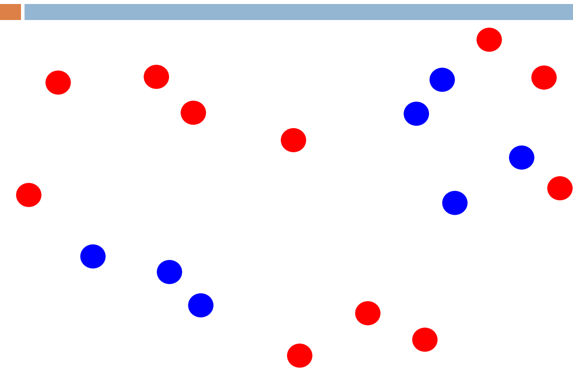

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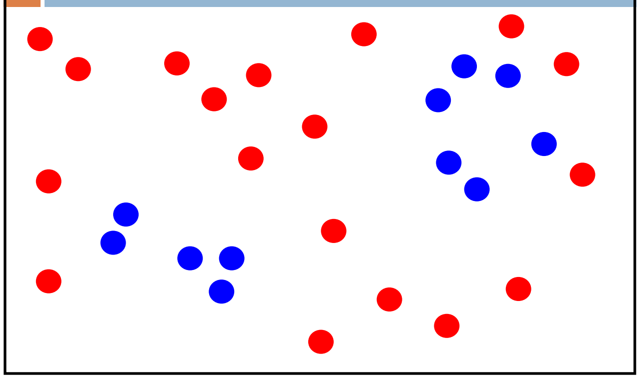

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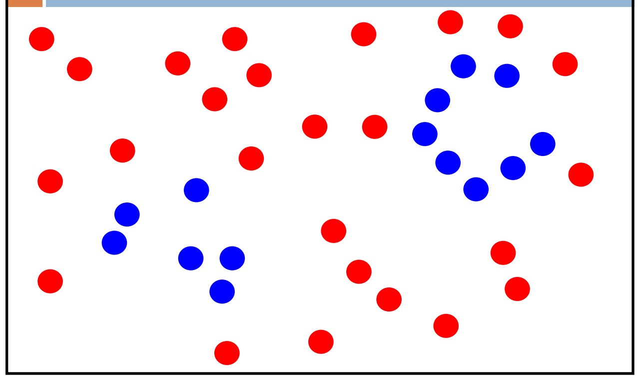

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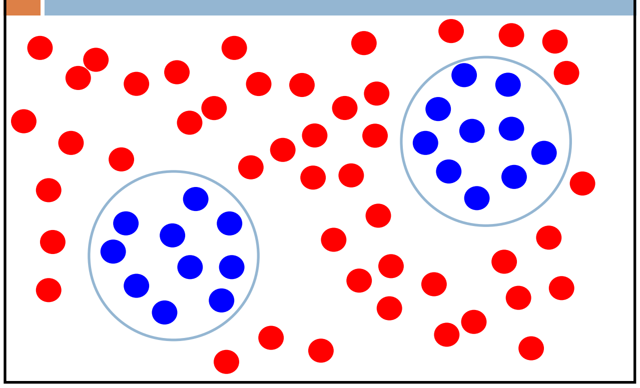
Actual model



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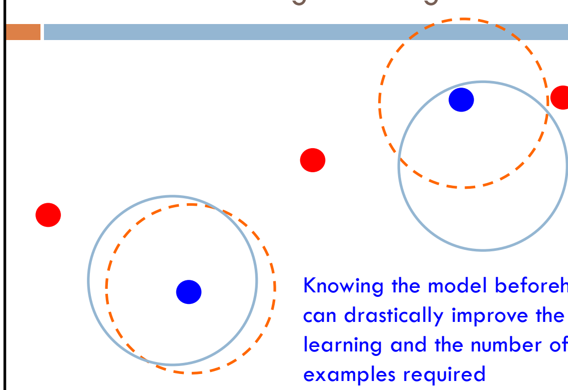

Knowing the model beforehand can drastically improve the learning and the number of examples required

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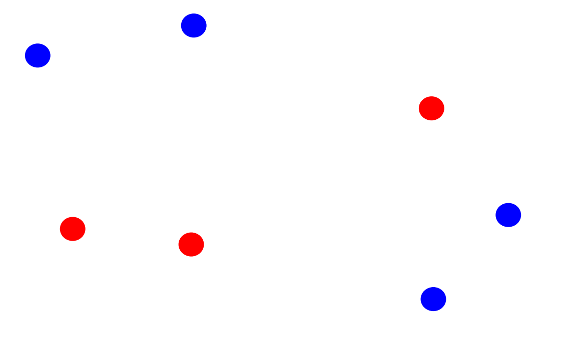
What is the data generating distribution?



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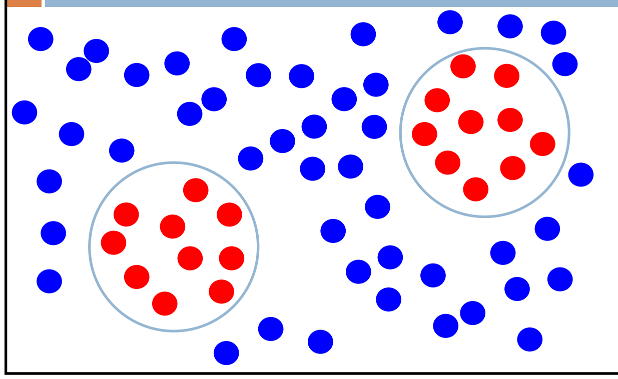
Make sure your assumption is correct, though!



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Machine learning models

What were the *model* assumptions (if any) that *k*-NN and NB made about the data?

Are there training data sets that could never be learned correctly by these algorithms?

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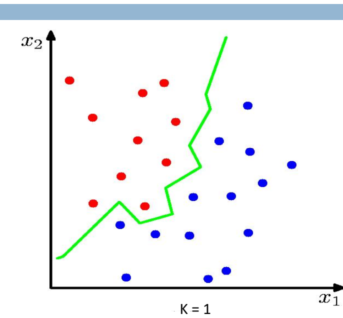
k-NN model



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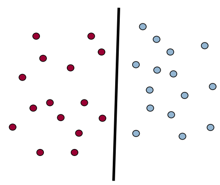
Linear models

A strong assumption is *linear separability*:

- in 2 dimensions, you can separate labels/classes by a line
- in higher dimensions, need hyperplanes

A *linear model* is a model that assumes the data is linearly separable



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Hyperplanes

A hyperplane is line/plane in a high dimensional space

What defines a line?
What defines a hyperplane?

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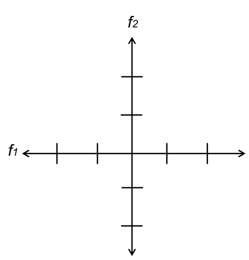
Defining a line

Any pair of values (w_1,w_2) defines a line through the origin:

$$0 = w_1 f_1 + w_2 f_2$$



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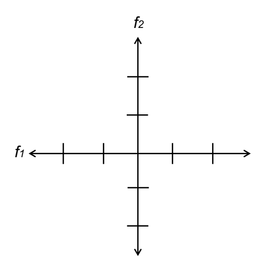
Defining a line

Any pair of values (w_1,w_2) defines a line through the origin:

$$0 = w_1 f_1 + w_2 f_2$$

$$0 = 1 f_1 + 2 f_2$$



What does this line look like?

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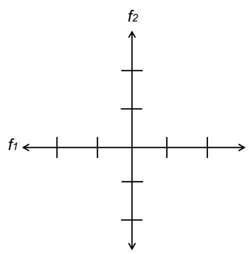
Defining a line

Any pair of values (w_1,w_2) defines a line through the origin:

$$0 = w_1 f_1 + w_2 f_2$$

$$0 = 1 f_1 + 2 f_2$$

-2	1
-1	0.5
0	0
1	-0.5
2	-1



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Defining a line

Any pair of values (w_1,w_2) defines a line through the origin:

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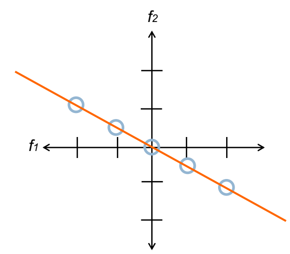

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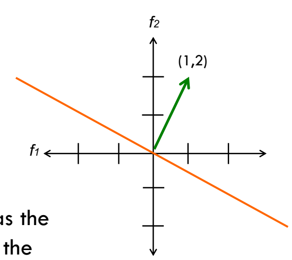
Defining a line

Any pair of values (w_1,w_2) defines a line through the origin:

$$0 = w_1 f_1 + w_2 f_2$$

$$0 = 1 f_1 + 2 f_2$$

w=(1,2)



We can also view it as the line perpendicular to the *weight vector*

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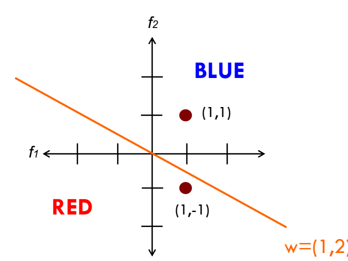
Classifying with a line

Mathematically, how can we classify points based on a line?

$$0 = 1 f_1 + 2 f_2$$



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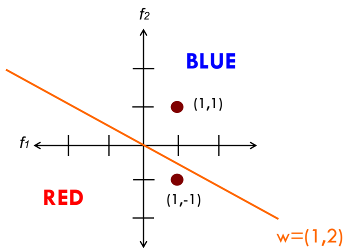
Classifying with a line

Mathematically, how can we classify points based on a line?

$$0 = 1f_1 + 2f_2$$

(1,1): $1*1+2*1=3$

(1,-1): $1*1+2*-1=-1$



The sign indicates which side of the line

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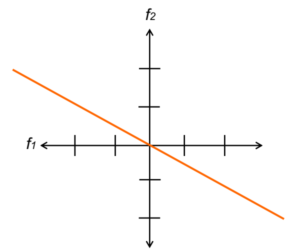
Defining a line

Any pair of values (w_1,w_2) defines a line through the origin:

$$0 = w_1 f_1 + w_2 f_2$$

$$0 = 1f_1 + 2f_2$$



How do we move the line off of the origin?

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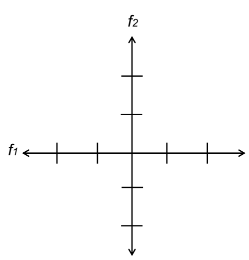
Defining a line

Any pair of values (w_1,w_2) defines a line through the origin:

$$a = w_1 f_1 + w_2 f_2$$

$$-1 = 1f_1 + 2f_2$$

-2
-1
0
1
2



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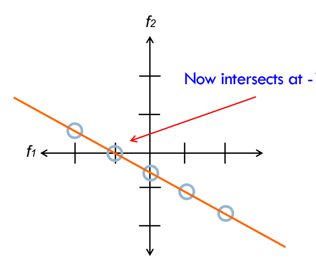
Defining a line

Any pair of values (w_1,w_2) defines a line through the origin:

$$a = w_1 f_1 + w_2 f_2$$

$$-1 = 1f_1 + 2f_2$$

-2	0.5
-1	0
0	-0.5
1	-1
2	-1.5



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Linear models

A linear model in *n*-dimensional space (i.e. *n* features) is define by *n*+1 weights:

In two dimensions, a line:

$$0 = w_1 f_1 + w_2 f_2 + b$$ (where b = -a)

In three dimensions, a plane:

$$0 = w_1 f_1 + w_2 f_2 + w_3 f_3 + b$$

In *n*-dimensions, a *hyperplane*

$$0 = b + \sum_{i=1}^{n} w_i f_i$$

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Classifying with a linear model

We can classify with a linear model by checking the sign:

$f_1, f_2, \ldots, f_m$ ➡ classifier

$b + \sum_{j=1}^{m} w_j f_j > 0$ Positive example

$b + \sum_{j=1}^{m} w_j f_j < 0$ Negative example

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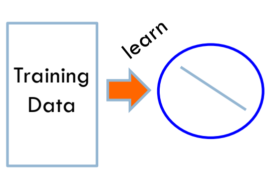
Learning a linear model

Geometrically, we know what a linear model represents

Given a linear model (i.e. a set of weights and b) we can classify examples



(data with labels)

How do we learn a linear model?

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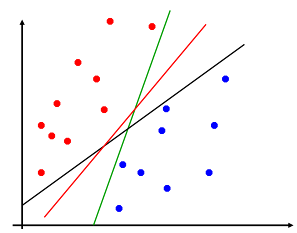
Which hyperplane would you choose?



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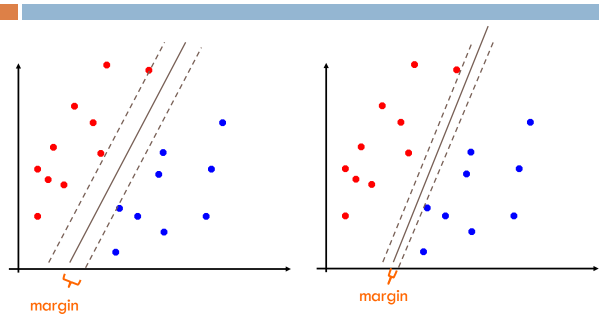
Large margin classifiers



Choose the line where the distance to the nearest point(s) is as large as possible

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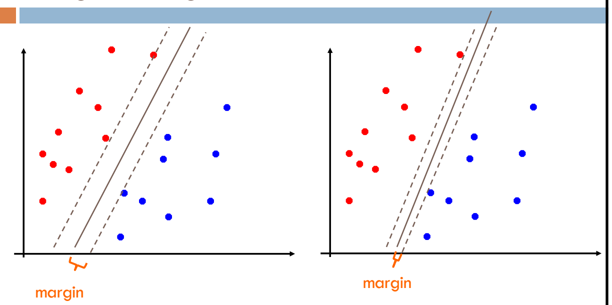
Large margin classifiers



The margin of a classifier is the distance to the closest points of either class

Large margin classifiers attempt to maximize this

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Large margin classifier setup

Select the hyperplane with the largest margin where the points are classified correctly!

Setup as a constrained optimization problem:

$$\max_{w,b} \ \text{margin}(w,b)$$

subject to:

$$y_i(w \cdot x_i + b) > 0 \quad \forall i$$

what does this say?

y_i: label for example i, either 1 (positive) or -1 (negative)
x_i: our feature vector for example i

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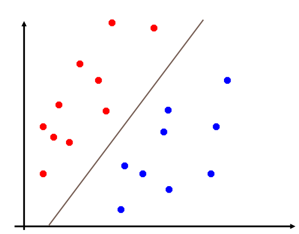
Measuring the margin

How do we calculate the margin?



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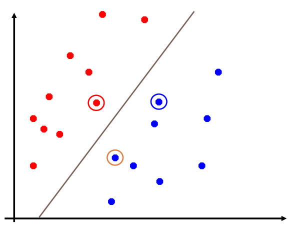
Support vectors

For any separating hyperplane, there exist some set of "closest points"

These are called the support vectors



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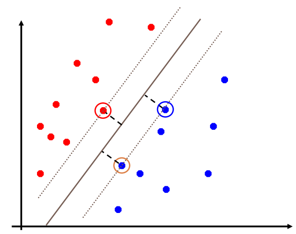
Measuring the margin

The margin is the distance to the support vectors, i.e. the "closest points", on either side of the hyperplane



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Support vector machine problem

Posed as a quadratic optimization problem

Maximize/minimize a quadratic function

Subject to a set of linear constraints

Many, many variants of solving this problem

One of the most successful classification approaches

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Support vector machines

One of the most successful (if not the most successful) classification approach:

decision tree	About 2,240,000 results (**0.32** sec)
Support vector machine	About 2,180,000 results (**0.36** sec)
k nearest neighbor	About 844,000 results (**0.33** sec)
Naïve Bayes	About 71,300 results (**0.32** sec)

Google scholar

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Other successful classifiers in NLP

Perceptron algorithm
- Linear classifier
- Trains "online"
- Fast and easy to implement
- Often used for tuning parameters (not necessarily for classifying)

Logistic regression classifier (aka Maximum entropy classifier)
- Probabilistic classifier
- Doesn't have the NB constraints
- Performs very well
- More computationally intensive to train than NB

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