

# WORD SIMILARITY

David Kauchak  
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1

## Admin

### Assignment 4

#### Quiz #2 Thursday

- ▣ 45 minutes
- ▣ Open book and notes
- ▣ Done with class after that

#### Assignment 5

- ▣ Two part assignment
- ▣ A due before spring break
- ▣ Have a proper spring break!
- ▣ B due a week after spring break

2

## Quiz #2

### Topics

- ▣ Linguistics 101
- ▣ Parsing
  - Grammars, CFGs, PCFGs
  - Top-down vs. bottom-up
  - CKY algorithm
  - Grammar learning
  - Evaluation
  - Improved models
- ▣ Text similarity (conceptual coverage)
  - Will also be covered on Quiz #3, though

3

## Text Similarity

A common question in NLP is how similar are texts

score:  $\text{sim}(\text{document}_1, \text{document}_2) = ?$

rank:  $\text{document}_1 \text{ ? } \text{document}_2$

4

## Bag of words representation

For now, let's ignore word order:

Obama said banana repeatedly last week on tv, "banana, banana, banana"

(4, 1, 1, 0, 0, 1, 0, 0, ...)	<p>"Bag of words representation": multi-dimensional vector, one dimension per word in our vocabulary</p>
-------------------------------	----------------------------------------------------------------------------------------------------------

banana  
obama  
said  
california  
across  
tv  
wrong  
capitol

Frequency of word occurrence

5

## Vector based word

**A**

a1: When	1
a2: the	2
a3: defendant	1
a4: and	1
a5: courthouse	0
...	

**B**

b1: When	1
b2: the	2
b3: defendant	1
b4: and	0
b5: courthouse	1
...	

Multi-dimensional vectors, one dimension per word in our vocabulary

6

## TF-IDF

One of the most common weighting schemes

**TF** = term frequency

**IDF** = inverse document frequency

$$a'_i = \underbrace{a_i}_{\text{TF}} \times \underbrace{\log N / df_i}_{\text{IDF (word importance weight)}}$$

We can then use this with any of our similarity measures!

7

## Normalized distance measures

**Cosine**

$$sim_{\cos}(A,B) = A \cdot B = \sum_{i=1}^n a_i b_i = \frac{\sum_{i=1}^n a_i b_i}{\sqrt{\sum_{i=1}^n a_i^2} \sqrt{\sum_{i=1}^n b_i^2}}$$

**L2**

$$dist_{L2}(A,B) = \sqrt{\sum_{i=1}^n (a_i - b_i)^2}$$

**L1**

$$dist_{L1}(A,B) = \sum_{i=1}^n |a_i - b_i|$$

a' and b' are length normalized versions of the vectors

8

## Our problems

Which of these have we addressed?

- word order
- length
- synonym
- spelling mistakes
- word importance
- word frequency

A model of word similarity!

9

## Word overlap problems

A: When the defendant and his lawyer walked into the court, some of the victim supporters turned their backs to him.

B: When the defendant walked into the courthouse with his attorney, the crowd truned their backs on him.

10

## Word similarity

How similar are two words?

score:  $\text{sim}(w_1, w_2) = ?$

rank:  $w \quad ? \quad \begin{matrix} w_1 \\ w_2 \\ w_3 \end{matrix}$  applications?

list:  $w_1$  and  $w_2$  are synonyms

11

## Word similarity applications

General text similarity

Thesaurus generation

Automatic evaluation

Text-to-text

- paraphrasing
- summarization
- machine translation

information retrieval (search)

12

## Word similarity

How similar are two words?

score:  $\text{sim}(w_1, w_2) = ?$

rank:  $w \ ?$

$w_1$  ideas? useful  
 $w_2$  resources?  
 $w_3$

list:  $w_1$  and  $w_2$  are synonyms

13

## Word similarity

Four categories of approaches (maybe more)

- Character-based
  - turned vs. truned
  - cognates (night, nacht, nicht, natt, nat, noc, noch)
- Semantic web-based (e.g. WordNet)
- Dictionary-based
- Distributional similarity-based
  - similar words occur in similar contexts

14

## Character-based similarity

$\text{sim}(\textit{turned}, \textit{truned}) = ?$

How might we do this using only the words (i.e. no outside resources?)

15

## Edit distance (Levenshtein distance)

The edit distance between  $w_1$  and  $w_2$  is the minimum number of operations to transform  $w_1$  into  $w_2$

Operations:

- insertion
- deletion
- substitution

$\text{EDIT}(\textit{turned}, \textit{truned}) = ?$   
 $\text{EDIT}(\textit{computer}, \textit{commuter}) = ?$   
 $\text{EDIT}(\textit{banana}, \textit{apple}) = ?$   
 $\text{EDIT}(\textit{wombat}, \textit{worcester}) = ?$

16

## Edit distance

EDIT(turned, truned) = 2

- delete u
- insert u

EDIT(computer, commuter) = 1

- replace p with m

EDIT(banana, apple) = 5

- delete b
- replace n with p
- replace a with p
- replace n with l
- replace a with e

EDIT(wombat, worcester) = 6

17

## Better edit distance

Are all operations equally likely?

- No

Improvement: give different weights to different operations

- replacing a for e is more likely than z for y

Ideas for weightings?

- Learn from actual data (known typos, known similar words)
- Intuitions: phonetics
- Intuitions: keyboard configuration

18

## Vector character-based word similarity

$\text{sim}(\textit{turned}, \textit{truned}) = ?$

Any way to leverage our vector-based similarity approaches from last time?

19

## Vector character-based word similarity

$\text{sim}(\textit{turned}, \textit{truned}) = ?$

a: 0  
b: 0  
c: 0  
d: 1  
e: 1  
f: 0  
g: 0  
...

a: 0  
b: 0  
c: 0  
d: 1  
e: 1  
f: 0  
g: 0  
...

Generate a feature vector based on the characters (or could also use the set based measures at the character level)

problems?

20

## Vector character-based word similarity

$$\text{sim}(\text{restful}, \text{fluster}) = ?$$

a:	0	a:	0	Character level loses a lot of information
b:	0	b:	0	
c:	0	c:	0	
d:	1	d:	1	
e:	1	e:	1	
f:	0	f:	0	
g:	0	g:	0	ideas?
...		...		

21

## Vector character-based word similarity

$$\text{sim}(\text{restful}, \text{fluster}) = ?$$

aa:	0	aa:	0	Use character bigrams or even trigrams
ab:	0	ab:	0	
ac:	0	ac:	0	
...		...		
es:	1	er:	1	
...		...		
fu:	1	fl:	1	
...		...		
re:	1	lu:	1	
...		...		

22

## Word similarity

### Four general categories

- Character-based
  - turned vs. truned
  - cognates (night, nacht, nicht, natt, nat, noc, noch)
- Semantic web-based (e.g. WordNet)
- Dictionary-based
- Distributional similarity-based
  - similar words occur in similar contexts

23

## WordNet

### Lexical database for English

- 155,287 words
- 206,941 word senses
- 117,659 synsets (synonym sets)
- ~400K relations between senses
- Parts of speech: nouns, verbs, adjectives, adverbs

Word graph, with word senses as nodes and edges as relationships

### Psycholinguistics

- WN attempts to model human lexical memory
- Design based on psychological testing

Created by researchers at Princeton

- <https://wordnet.princeton.edu/>

Lots of programmatic interfaces

24

WordNet relations

- synonym
- antonym
- hypernyms
- hyponyms
- holonym
- meronym
- troponym
- entailment
- (and a few others)

25

WordNet relations

- synonym – X and Y have similar meaning
- antonym – X and Y have opposite meanings
- hypernyms – subclass
  - beagle is a hypernym of dog
- hyponyms – superclass
  - dog is a hyponym of beagle
- holonym – contains part
  - car is a holonym of wheel
- meronym – part of
  - wheel is a meronym of car

26

WordNet relations

troponym – for verbs, a more specific way of doing an action

- run is a troponym of move
- dice is a troponym of cut

entailment – for verbs, one activity leads to the next

- sleep is entailed by snore

(and a few others)

27

WordNet



Graph, where nodes are words and edges are relationships

There is some hierarchical information, for example with hyp-er/o-nomy

28

## WordNet: run

**Noun**

- **run, tally** (a score in baseball made by a runner touching all four bases safely) "The Yankees scored 3 runs in the bottom of the 9th"; "their first tally came in the 3rd inning"
- **run, tally** (the act of testing something) "in the experiment he made the amount of carbon was measured separately"; "he called each slip of the test a new run"
- **run, race** (to run a race) "she broke the record for the half-mile run"
- **run, streak** (an unbroken series of events) "had a streak of bad luck"; "his team had a run of bad luck"
- **run, line** (the act of being something) "in the experimental made the amount of carbon was measured separately"; "he called each slip of the test a new run"
- **run, base** (to run a base) "he broke the record for the half-mile run"
- **run, base** (a regular step) "the ship made its run in record time"
- **run, base** (the act of running, running on foot at a fast pace) "he broke into a run"; "his daily run keeps him fit"
- **run** (the continuous period of time during which something is machine or a factory operates or continues in operation) "the assembly line was on a 24-hour run"
- **run** (unrestricted freedom to use) "he has the run of the house"
- **run** (the production withheld during a continuous period of operation) "the machine of factor etc.; a daily run of 100,000 gallons of fuel"
- **run** (to run, to run, to run, to run) "the play had a long run on Broadway"; "the team enjoyed a streak of victories"
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**Verb**

- **run** (to run, to run, to run) "the play had a long run on Broadway"; "the team enjoyed a streak of victories"
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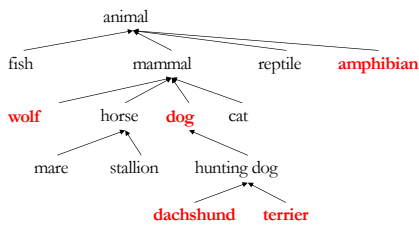
29

## WordNet: run

- **S: (n) run, tally** (a score in baseball made by a runner touching all four bases safely) "the Yankees scored 3 runs in the bottom of the 9th"; "their first tally came in the 3rd inning"
  - **direct hyponym / full hyponym**
    - **S: (n) earned run** (a run that was not scored as the result of an error by the other team)
    - **S: (n) unearned run** (a run that was scored as a result of an error by the other team)
    - **S: (n) run batted in, rbi** (a run that is the result of the batter's performance) "he had more than 100 rbi last season"
  - **direct hypernym / inherited hypernym / sister term**
  - **derivationally related form**

30

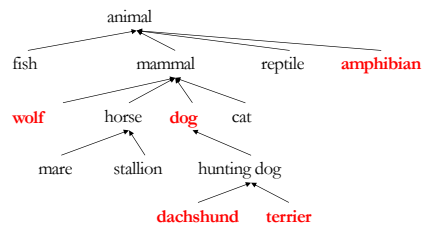
## WordNet-like Hierarchy



To utilize WordNet, we often want to think about some graph-based measure.

31

## WordNet-like Hierarchy



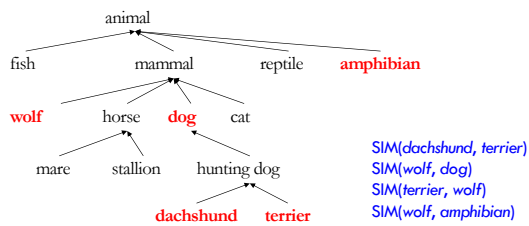
Rank the following based on similarity:

- SIM(wolf, dog)
- SIM(wolf, amphibian)
- SIM(terrier, wolf)
- SIM(dachshund, terrier)

32



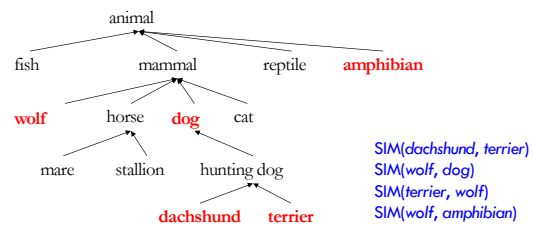
## WordNet-like Hierarchy



What information/heuristics did you use to rank these?

33

## WordNet-like Hierarchy



- path length is important (but not the only thing)
- words that share the same ancestor are related
- words lower down in the hierarchy are finer grained and therefore closer

34

## WordNet similarity measures

path length doesn't work very well

Some ideas:

- ▣ path length scaled by the depth (Leacock and Chodorow, 1998)

With a little cheating:

- ▣ Measure the "information content" of a word using a corpus: how specific is a word?
  - ▣ words higher up tend to have less information content
  - ▣ more frequent words (and ancestors of more frequent words) tend to have less information content

35

## WordNet similarity measures

Utilizing information content:

- ▣ information content of the lowest common parent (Resnik, 1995)
- ▣ information content of the words minus information content of the lowest common parent (Jiang and Conrath, 1997)
- ▣ information content of the lowest common parent divided by the information content of the words (Lin, 1998)

36

## Word similarity

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- Dictionary-based
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37

## Dictionary-based similarity

### Word

aardvark

beagle

dog

### Dictionary blurb

a large, nocturnal, burrowing mammal, *Orycteropus afer*, of central and southern Africa, feeding on ants and termites and having a long, extensile tongue, strong claws, and long ears.

One of a breed of small hounds having long ears, short legs, and a usually black, tan, and white coat.

Any carnivore of the family Canidae, having prominent canine teeth and, in the wild state, a long and slender muzzle, a deep-chested muscular body, a bushy tail, and large, erect ears. Compare canid.

38

## Dictionary-based similarity

### Utilize our text similarity measures

$\text{sim}(\text{dog}, \text{beagle}) =$

$\text{sim}(\text{One of a breed of small hounds having long ears, short legs, and a usually black, tan, and white coat.}, \text{Any carnivore of the family Canidae, having prominent canine teeth and, in the wild state, a long and slender muzzle, a deep-chested muscular body, a bushy tail, and large, erect ears. Compare canid.})$

39

## Dictionary-based similarity

- noun**
1. a domesticated canid, *Canis familiaris*, bred in many varieties.
  2. any carnivore of the dogfamily Canidae, having prominent canine teeth and, in the wild state, a long and slender muzzle; a deep-chested muscular body, a bushy tail, and large, erect ears. Compare canid.
  3. the male of such an animal.
  4. any of various animals resembling a dog.
  5. a despicable man or youth.
  6. Informal , a fellow in general; a lucky dog.
  7. dog; slang - fast.
  8. Slang
    - a. something worthless or of extremely poor quality: *That used car you bought is a dog.*
    - b. an utter failure; flop: *Critics say his new play is a dog.*
  9. Slang - an evil, boring, or crude person.
  10. Slang - hot dog.
  11. ( initial capital letter ) Astronomy , either of two constellations, Canis Major or Canis Minor.
  12. Machinery
    - a. any of various mechanical devices, as for gripping or holding something.
    - b. a projection on a moving part for moving steadily or for tripping another part with which it engages.
  13. Also called **grasper**, **nipper**. Metalworking - a device on a drawbench for drawing the work through the die.
  14. a cramp binding together two timbers.
  15. an iron bar driven into a stone or timber to provide a means of lifting it.
  16. an andiron; firedog.
  17. Meteorology - a sundog or fogdog.
  18. a word formerly used in communications to represent the letter D.

What about words that have multiple senses/parts of speech?

40

## Dictionary-based similarity

—noun

1. a domesticated canid, *Canis familiaris*, bred in many varieties.
2. any carnivore of the dogfamily Canidae, having prominent canine teeth and, in the wild state, a long and slender muzzle, a deep-chested muscular body, a bushy tail, and large, erect ears. Compare *canid*.
3. the male of such an animal.
4. any of various animals resembling a dog.
5. a despicable man or youth.
6. *Informal* : a fellow in general: a lucky dog.
7. dog; Stang - *feels*.
8. *Stang* :
  - a. something worthless or of extremely poor quality: *That used appliance bought is a dog.*
  - b. an utter failure; flop: *Critics say his new play is a dog.*
9. *Stang* : an ugly, boring, or crude person.
10. *Stang* - *but-dog*.
11. ( *initial capital letter* ) *Astronomy* : either of two constellations, *Canis Major* or *Canis Minor*.
12. *Machinery* :
  - a. any of various mechanical devices, as for gripping or holding something.
  - b. a projection on a moving part for moving steadily or for tripping another part with which it engages.
13. Also called *grigger*, *ripper*, *Metalworking* : a device on a drawbench for drawing the work through the die.
14. a cramp binding together two timbers.
15. an iron bar driven into a stone or timber to provide a means of lifting it.
16. an andiron; firedog.
17. *Meteorology* : a sundog or fogdog.
18. a word formerly used in communications to represent the letter D.

1. part of speech tagging
2. word sense disambiguation
3. most frequent sense
4. average similarity between all senses
5. max similarity between all senses
6. sum of similarity between all senses

41

## Dictionary + WordNet

WordNet also includes a “gloss” similar to a dictionary definition

Other variants include the overlap of the word senses as well as those word senses that are related (e.g. hypernym, hyponym, etc.)

- incorporates some of the path information as well
- Banerjee and Pedersen, 2003

42

## Word similarity

### Four general categories

- Character-based
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  - cognates (night, nacht, nicht, natt, nat, noc, noch)
- Semantic web-based (e.g. WordNet)
- Dictionary-based
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  - similar words occur in similar contexts

43

## Corpus-based approaches

Word

ANY blurb with the word

aardvark



beagle



Ideas?

dog



44

## Corpus-based

The **Beagle** is a breed of small to medium-sized dog. A member of the Hound Group, it is similar in appearance to the Foxhound but smaller, with shorter leg

**Beagles** are intelligent, and are popular as pets because of their size, even temper, and lack of inherited health problems.

Dogs of similar size and purpose to the modern **Beagle** can be traced in Ancient Greece[2] back to around the 5th century BC.

From medieval times, **beagle** was used as a generic description for the smaller hounds, though these dogs differed considerably from the modern breed.

In the 1840s, a standard **Beagle** type was beginning to develop: the distinction between the North Country Beagle and Southern

45

## Corpus-based: feature extraction

The **Beagle** is a breed of small to medium-sized dog. A member of the Hound Group, it is similar in appearance to the Foxhound but smaller, with shorter leg

We'd like to utilize our vector-based approach

How could we we create a vector from these occurrences?

- ❑ collect word counts from all documents with the word in it
- ❑ collect word counts from all sentences with the word in it
- ❑ collect all word counts from all words within *X* words of the word
- ❑ collect all words counts from words in specific relationship: subject-object, etc.

46

## Word-context co-occurrence vectors

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In the 1840s, a standard **Beagle** type was beginning to develop: the distinction between the North Country Beagle and Southern

47

## Word-context co-occurrence vectors

The <b>Beagle</b> is a breed	the:	2
	is:	1
<b>Beagles</b> are intelligent, and	a:	2
	breed:	1
to the modern <b>Beagle</b> can be traced	are:	1
	intelligent:	1
From medieval times, <b>beagle</b> was used as	and:	1
	to:	1
1840s, a standard <b>Beagle</b> type was beginning	modern:	1
	...	

Often do some preprocessing like lowercasing and removing stop words

48

## Corpus-based similarity

$$\text{sim}(\text{dog}, \text{beagle}) =$$

$$\text{sim}(\text{context\_vector}(\text{dog}), \text{context\_vector}(\text{beagle}))$$

the:	5	the:	2
is:	1	is:	1
a:	4	a:	2
breeds:	2	breed:	1
are:	1	are:	1
intelligent:	5	intelligent:	1
...		and:	1
		to:	1
		modern:	1
		...	

49

## Web-based similarity



beagle  Advanced search Language tools

Ideas?

50

## Web-based similarity

beagle



beagle  Advanced search Language tools

- Beagle** - Wikipedia, the free encyclopedia 

The **Beagle** is a breed of small to medium-sized dog. A member of the Hound Group, it is similar in appearance to the Foxhound but smaller, with shorter legs ...  
History · Description · Variations · Temperament  
en.wikipedia.org/wiki/Beagle · Cached · Similar
- Beagle (software)** - Wikipedia, the free encyclopedia 

**Beagle** is a search system for Linux and other such modern Unix-like systems ...  
en.wikipedia.org/wiki/Beagle\_(software) · Cached · Similar
- \* Show more results from wikipedia.org
- Beagle information and Pictures, Beagles**

All about the **Beagle**, info, pictures, breeders, rescues, care, temperament, health, puppy pictures and much more.  
www.dogandcatinfo.com/beagle.htm · Cached · Similar
- Beagles & Buddies: PET ADOPTION, BEAGLE SHELTER, DOG RESCUE**

Rescue shelter for **Beagles**, as well as other small dogs, from pounds, humane societies & off the street. We have a no-kill policy, our rescue facility keeps ...  
www.beagleandbuddies.com/ · Cached · Similar

51

## Web-based similarity

- Beagle** - Wikipedia, the free encyclopedia 

The **Beagle** is a breed of small to medium-sized dog. A member of the Hound Group, it is similar in appearance to the Foxhound but smaller, with shorter legs ...  
History · Description · Variations · Temperament  
en.wikipedia.org/wiki/Beagle · Cached · Similar
- Beagle (software)** - Wikipedia, the free encyclopedia 

**Beagle** is a search system for Linux and other such modern Unix-like systems ...  
en.wikipedia.org/wiki/Beagle\_(software) · Cached · Similar
- \* Show more results from wikipedia.org
- Beagle information and Pictures, Beagles**

All about the **Beagle**, info, pictures, breeders, rescues, care, temperament, health, puppy pictures and much more.  
www.dogandcatinfo.com/beagle.htm · Cached · Similar
- Beagles & Buddies: PET ADOPTION, BEAGLE SHELTER, DOG RESCUE**

Rescue shelter for **Beagles**, as well as other small dogs, from pounds, humane societies & off the street. We have a no-kill policy, our rescue facility keeps ...  
www.beagleandbuddies.com/ · Cached · Similar

Concatenate the snippets for the top N results

Concatenate the web page text for the top N results

52

## Another feature weighting

TF- IDF weighting takes into account the general importance of a feature

For distributional similarity, we have the feature ( $f_i$ ), but we also have the word itself ( $w$ ) that we can use for information

$\text{sim}(\text{context\_vector}(\text{dog}), \text{context\_vector}(\text{beagle}))$

the:	5	the:	2
is:	1	is:	1
a:	4	a:	2
breeds:	2	breed:	1
are:	1	are:	1
intelligent:	5	intelligent:	1
...		and:	1
		to:	1
		modern:	1
		...	

53

## Another feature weighting

Feature weighting ideas given this additional information?

$\text{sim}(\text{context\_vector}(\text{dog}), \text{context\_vector}(\text{beagle}))$

the:	5	the:	2
is:	1	is:	1
a:	4	a:	2
breeds:	2	breed:	1
are:	1	are:	1
intelligent:	5	intelligent:	1
...		and:	1
		to:	1
		modern:	1
		...	

54

## Another feature weighting

count *how likely* feature  $f_i$  and word  $w$  are to occur together

- incorporates co-occurrence
- but also incorporates how often  $w$  and  $f_i$  occur in other instances

$\text{sim}(\text{context\_vector}(\text{dog}), \text{context\_vector}(\text{beagle}))$

Does IDF capture this?

Not really. IDF only accounts for  $f_i$  regardless of  $w$

55

## Mutual information

A bit more probability ☺

$$I(X,Y) = \sum_x \sum_y p(x,y) \log \frac{p(x,y)}{p(x)p(y)}$$

When will this be high and when will this be low?

What happens if  $x$  and  $y$  are independent/dependent?

56

## Mutual information

A bit more probability ☺

$$I(X,Y) = \sum_x \sum_y p(x,y) \log \frac{p(x,y)}{p(x)p(y)}$$

if  $x$  and  $y$  are **independent** (i.e. one occurring doesn't impact the other occurring) then:

$$p(x,y) =$$

57

## Mutual information

A bit more probability ☺

$$I(X,Y) = \sum_x \sum_y p(x,y) \log \frac{p(x,y)}{p(x)p(y)}$$

if  $x$  and  $y$  are **independent** (i.e. one occurring doesn't impact the other occurring) then:

$$p(x,y) = p(x)p(y)$$

What does this do to the sum?

58

## Mutual information

A bit more probability ☺

$$I(X,Y) = \sum_x \sum_y p(x,y) \log \frac{p(x,y)}{p(x)p(y)}$$

if they are **dependent** then:

$$p(x,y) = p(x)p(y|x) = p(y)p(x|y)$$



$$I(X,Y) = \sum_x \sum_y p(x,y) \log \frac{p(y|x)}{p(y)}$$

59

## Mutual information

$$I(X,Y) = \sum_x \sum_y p(x,y) \log \frac{p(y|x)}{p(y)}$$

What is this asking?

When is this high?

How much more likely are we to see  $y$  given  $x$  has a particular value!

60

## Point-wise mutual information

### Mutual information

$$I(X,Y) = \sum_x \sum_y p(x,y) \log \frac{p(x,y)}{p(x)p(y)}$$

How related are two variables (i.e. over all possible values/events)

### Point-wise mutual information

$$PMI(x,y) = \log \frac{p(x,y)}{p(x)p(y)}$$

How related are two particular events/values

61

## PMI weighting

Mutual information is often used for feature selection in many problem areas

PMI weighting weights co-occurrences based on their correlation (i.e. high PMI)

### context\_vector(beagle)

the:	2	$\rightarrow \log \frac{p(\text{beagle, the})}{p(\text{beagle})p(\text{the})}$	How do we calculate these?
is:	1		
a:	2		
breed:	1	$\rightarrow \log \frac{p(\text{beagle, breed})}{p(\text{beagle})p(\text{breed})}$	
are:	1		
intelligent:	1		
and:	1		
to:	1		
modern:	1		
...			

62