LOGISTIC REGRESSION

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CS158 – Fall 2025

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Admin

Assignment 7

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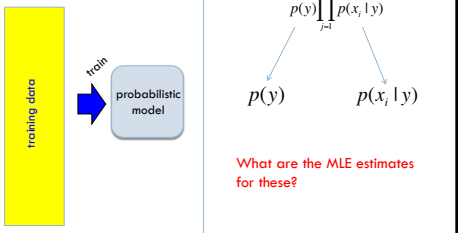
MLE estimation for NB



$$p(y)\prod_{i=1}^{m} p(x_i \mid y)$$

$p(y)$ $p(x_i \mid y)$

What are the MLE estimates for these?

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Maximum likelihood estimates

$$p(y) = \frac{count(y)}{n}$$

number of examples with label y / total number of examples

$$p(x_i \mid y) = \frac{count(x_i, y)}{count(y)}$$

number of examples with label y with feature $x_i = 1$ / number of examples with label

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Maximum likelihood estimates

$$p(y) = \frac{count(y)}{n}$$

x1	x2	label
1	1	1
1	0	1
1	1	1
0	1	-1
0	0	-1

p(1) = ?
p(-1) = ?

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Maximum likelihood estimates

$$p(y) = \frac{count(y)}{n}$$

x1	x2	label
1	1	1
1	0	1
1	1	1
0	1	-1
0	0	-1

p(1) = 3/5
p(-1) = 2/5

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Maximum likelihood estimates

$$p(x_i \mid y) = \frac{count(x_i, y)}{count(y)}$$

p(x1 = 1 | 1)	?
p(x1 = 0 | 1)	?
p(x2 = 1 | 1)	?
p(x2 = 0 | 1)	?

x1	x2	label
1	1	1
1	0	1
1	1	1
0	1	-1
0	0	-1

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Maximum likelihood estimates

$$p(x_i \mid y) = \frac{count(x_i, y)}{count(y)}$$

p(x1 = 1 | 1)	3/3
p(x1 = 0 | 1)	0/3
p(x2 = 1 | 1)	2/3
p(x2 = 0 | 1)	1/3

x1	x2	label
1	1	1
1	0	1
1	1	1
0	1	-1
0	0	-1

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Maximum likelihood estimates

$$p(x,y) = p(y)\prod_{j=1}^{m} p(x_i|y)$$

p(x₁ = 1 | 1)	3/3
p(x₁ = 0 | 1)	0/3
p(x₂ = 1 | 1)	2/3
p(x2 = 0| 1)	1/3

x1	x2	label
1	1	1
1	0	1
1	1	1
0	1	-1
0	0	-1

$p(x_1 = 1, x_2 = 1, y = 1) = ?$

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Maximum likelihood estimates

$$p(x,y) = p(y)\prod_{j=1}^{m} p(x_i|y)$$

p(x₁ = 1 | 1)	3/3
p(x₁ = 0 | 1)	0/3
p(x₂ = 1 | 1)	2/3
p(x2 = 0| 1)	1/3

x1	x2	label
1	1	1
1	0	1
1	1	1
0	1	-1
0	0	-1

$$p(x_1 = 1, x_2 = 1, y = 1) = p(1)p(x_1 = 1 \mid 1)p(x_2 = 1 \mid 1)$$
$$= \frac{3}{5} * 1 * \frac{2}{3}$$
$$= \frac{6}{15}$$

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Maximum likelihood estimates

$$p(x,y) = p(y)\prod_{j=1}^{m} p(x_i|y)$$

p(x₁ = 1 | 1)	3/3
p(x₁ = 0 | 1)	0/3
p(x₂ = 1 | 1)	2/3
p(x2 = 0| 1)	1/3

x1	x2	label
1	1	1
1	0	1
1	1	1
0	1	-1
0	0	-1

$p(x_1 = 0, x_2 = 1, y = 1) = ?$

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Maximum likelihood estimates

$$p(x,y) = p(y)\prod_{j=1}^{m} p(x_i|y)$$

p(x₁ = 1 | 1)	3/3
p(x₁ = 0 | 1)	0/3
p(x₂ = 1 | 1)	2/3
p(x2 = 0| 1)	1/3

x1	x2	label
1	1	1
1	0	1
1	1	1
0	1	-1
0	0	-1

$$p(x_1 = 0, x_2 = 1, y = 1) = p(1)p(x_1 = 0 \mid 1)p(x_2 = 1 \mid 1)$$
$$= \frac{3}{5} * 0 * \frac{2}{3}$$
$$= 0 \quad ☹$$

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Maximum likelihood estimates

Full model trained!

p(1) = 3/5
p(-1) = 2/5

x_1	x_2	label
1	1	1
1	0	1
1	1	1
0	1	-1
0	0	-1

$p(x_1 = 1 \mid 1)$	3/3
$p(x_2 = 1 \mid 1)$	2/3
$p(x_1 = 1 \mid -1)$	0/2
$p(x_2 = 1 \mid -1)$	1/2

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Basic steps for probabilistic modeling

Step 1: pick a model

Step 2: figure out how to estimate the probabilities for the model

Step 3 (optional): deal with overfitting

Probabilistic models

Which model do we use, i.e. how do we calculate p(*feature, label*)?

How do train the model, i.e. how to we we estimate the probabilities for the model?

How do we deal with overfitting?

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Likelihood distribution

The likelihood of the data, given the model parameters (Θ) is:

$p(data|\theta)$ likelihood of the data, given the current model

Maximum Likelihood Estimation (MLE), picks the parameter as:

$$\theta = argmax_\theta p(data|\theta)$$

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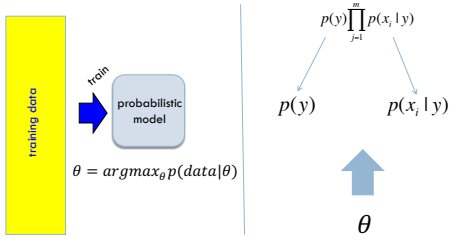
Training: MLE



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MLE problems

$p(x_1 = 1 \mid 1)$	3/3
$p(x_1 = 0 \mid 1)$	0/3
$p(x_2 = 1 \mid 1)$	2/3
$p(x2 = 0 \mid 1)$	1/3

$$\theta = argmax_\theta p(data|\theta)$$

The MLE is completely driven by the training data

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Priors

Coin1 data: 3 Heads and 1 Tail
Coin2 data: 30 Heads and 10 tails
Coin3 data: 2 Tails
Coin4 data: 497 Heads and 503 tails

If someone asked you what the probability of heads was for each of these coins, what would you say?

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Posterior distribution

We want to incorporate a prior belief of what the probabilities might be

$$p(\theta|data)$$

given the data, how likely are different model parameters

Maximum a posteriori principle, pick parameters that maximize the posterior distribution:

$$\theta = argmax_\theta p(\theta|data)$$

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MLE vs. Maximum posteriori

MLE	$\theta = argmax_\theta p(data\|\theta)$
Maximum posteriori	$\theta = argmax_\theta p(\theta\|data)$

Both are methods for picking model parameters

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Maximum posteriori

$p(\theta|data) = ?$ (Hint: Bayes' rule)

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Maximum posteriori

$$p(\theta|data) = \frac{p(data|\theta)p(\theta)}{p(data)}$$

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Maximum posteriori

What are each of these probabilities?

$$p(\theta|data) = \frac{p(data|\theta)p(\theta)}{p(data)}$$

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Maximum posteriori

likelihood of the data under the model

probability of different parameters, call the prior

$$p(\theta|data) = \frac{p(data|\theta)p(\theta)}{p(data)}$$

probability of seeing the data (regardless of model)

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Priors

$$\theta = \text{argmax}_\theta \frac{p(data \mid \theta)p(\theta)}{p(data)}$$

Does p(data) matter for the argmax?

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Priors

likelihood of the data under the model

probability of different parameters, call the prior

$$\theta = \text{argmax}_\theta \, p(data \mid \theta)p(\theta)$$

What does MLE assume for a prior on the model parameters?

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Priors

likelihood of the data under the model

probability of different parameters, call the prior

$$\theta = \text{argmax}_\theta \, p(data \mid \theta)p(\theta)$$

- Assumes a uniform prior, i.e. all Θ are equally likely!
- Relies solely on the likelihood

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A better approach

$$\theta = \text{argmax}_\theta \, p(data \mid \theta)p(\theta)$$

$$likelihood(data) = \prod_{i=1}^{n} p_\theta(x_i)$$

We can use any distribution we'd like.

This allows us to impart addition bias into the model

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Another view on the prior

Remember, the max is the same if we take the log:

$$\theta = \text{argmax}_\theta \log(p(data \mid \theta)) + \log(p(\theta))$$

$$\text{log-likelihood} = \sum_{i=1}^{n} \log(p(x_i))$$

We can use any distribution we'd like.

This allows us to impart addition bias into the model

Does this look like something we've seen before (Hint, think gradient descent)?

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Regularization vs prior

$$\theta = \text{argmax}_\theta \log(p(data \mid \theta)) + \log(p(\theta))$$

fit: likelihood based on the data / loss function based on the data

model bias: prior / regularizer

$$\text{argmin}_{w,b} \sum_{i=1}^{n} loss(yy') + \lambda regularizer(w)$$

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Prior for NB

$$\theta = \text{argmax}_\theta \log(p(data \mid \theta)) + \log(p(\theta))$$

Uniform prior

$$p(x_i \mid y) = \frac{count(x_i, y)}{count(y)}$$

Dirichlet prior

λ= 0 → increasing

$$p(x_i \mid y) = \frac{count(x_i, y) + \lambda}{count(y) + possible\_values\_of\_x_i * \lambda}$$

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Prior: another view

$$p(x_1, x_2, \ldots, x_m, y) = p(y) \prod_{j=1}^{m} p(x_i \mid y)$$

MLE: $p(x_i \mid y) = \frac{count(x_i, y)}{count(y)}$

What happens to our likelihood if, for one of the labels, we never saw a particular feature?

Goes to 0!

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Prior: another view

$$p(x_i \mid y) = \frac{count(x_i, y)}{count(y)}$$

$$p(x_i \mid y) = \frac{count(x_i, y) + \lambda}{count(y) + possible\_values\_of\_x_i * \lambda}$$

Adding a prior avoids this!

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Smoothing

training data

$$p(x_i \mid y) = \frac{count(x_i, y)}{count(y)}$$

$$p(x_i \mid y) = \frac{count(x_i, y) + \lambda}{count(y) + possible\_values\_of\_x_i * \lambda}$$

for each label, pretend like we've seen each feature value occur in λ additional examples

Sometimes this is also called smoothing because it is seen as smoothing or interpolating between the MLE and some other distribution

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Priors

Coin1 data: 3 Heads and 1 Tail

Coin2 data: 30 Heads and 10 tails

Coin3 data: 2 Tails

Coin4 data: 497 Heads and 503 tails

$$p(heads) = \frac{count(heads) + \lambda}{totalflips + 2\lambda}$$

Does this do the right thing in these cases?

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Priors

Coin1 data: 3 Heads and 1 Tail:

$p(heads) = \frac{4}{6} = 0.667$

Coin2 data: 30 Heads and 10 tails:

$p(heads) = \frac{31}{42} = 0.738$

Coin3 data: 2 Tails

$p(heads) = \frac{3}{4} = 0.75$

Coin4 data: 497 Heads and 503 tails

$p(heads) = \frac{498}{1002} = 0.497$

$$p(heads) = \frac{count(heads) + \lambda}{totalflips + 2\lambda}$$

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Maximum likelihood estimates

$$p(x_i \mid y) = \frac{count(x_i, y) + \lambda}{count(y) + possible\_values\_of\_x_i * \lambda}$$

λ = 1

x1	x2	label
1	1	1
1	0	1
1	1	1
0	1	-1
0	0	-1

p(x1 = 1 | 1)	?
p(x1 = 0 | 1)	?
p(x2 = 1 | 1)	?
p(x2 = 0 | 1)	?

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Maximum likelihood estimates

$$p(x_i \mid y) = \frac{count(x_i, y) + \lambda}{count(y) + possible\_values\_of\_x_i * \lambda}$$

λ = 1

x1	x2	label
1	1	1
1	0	1
1	1	1
0	1	-1
0	0	-1

p(x1 = 1 | 1)	4/5
p(x1 = 0 | 1)	1/5
p(x2 = 1 | 1)	3/5
p(x2 = 0 | 1)	2/5

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Avoids zero probability events!

MLE

p(x1 = 1 | 1)	3/3
p(x1 = 0 | 1)	0/3
p(x2 = 1 | 1)	2/3
p(x2 = 0 | 1)	1/3

smoothed/prior

p(x1 = 1 | 1)	4/5
p(x1 = 0 | 1)	1/5
p(x2 = 1 | 1)	3/5
p(x2 = 0 | 1)	2/5

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Basic steps for probabilistic modeling

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Probabilistic models

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How do train the model, i.e. how to we we estimate the probabilities for the model?

How do we deal with overfitting?

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Joint models vs conditional models

We've been trying to model the joint distribution (i.e. the data generating distribution):

$$p(x_1, x_2, \ldots, x_m, y)$$

trying to model the data generating distribution

However, if all we're interested in is classification, why not directly model the conditional distribution:

$$p(y \mid x_1, x_2, \ldots, x_m)$$

direct model for classification

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A first try: linear

$$p(y \mid x_1, x_2, \ldots, x_m) = x_1 w_1 + w_2 x_2 + \ldots + w_m x_m + b$$

Any problems with this?

- Nothing constrains it to be a probability
- Could still have combination of features and weight that exceeds 1 or is below 0

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The challenge

$x_1 w_1 + w_2 x_2 + \ldots + w_m x_m + b$

Linear model

$+\infty$

$-\infty$

$p(y \mid x_1, x_2, \ldots, x_m)$

probability

1

0

We like linear models!

Can we transform the probability into a function that ranges over all values?

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Odds ratio

Rather than predict the probability, we can predict the ratio of 1/0 (positive/negative)

Predict the **odds** that it is 1 (true): How much more likely is 1 than 0.

Does this help us?

$$\frac{P(1 \mid x_1, x_2, \ldots, x_m)}{P(0 \mid x_1, x_2, \ldots, x_m)} = \frac{P(1 \mid x_1, x_2, \ldots, x_m)}{1 - P(1 \mid x_1, x_2, \ldots, x_m)} = x_1 w_1 + w_2 x_2 + \ldots + w_m x_m + b$$

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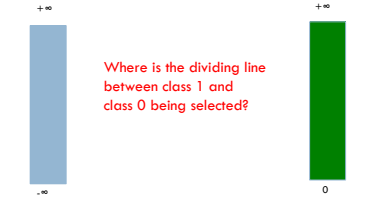
Odds ratio

$x_1w_1 + w_2x_2 + \ldots + w_mx_m + b$ $\quad \frac{P(1|x_1,x_2,\ldots,x_m)}{1-P(1|x_1,x_2,\ldots,x_m)}$

Linear model $\quad$ odds ratio



Where is the dividing line between class 1 and class 0 being selected?

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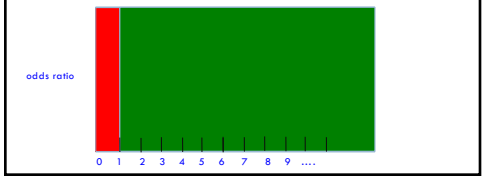
Odds ratio

$P(1|x_1,x_2,\ldots,x_m) > P(0|x_1,x_2,\ldots,x_m)$ $\quad \frac{P(1|x_1,x_2,\ldots,x_m)}{1-P(1|x_1,x_2,\ldots,x_m)}$

$P(1|x_1,x_2,\ldots,x_m) > 1 - P(1|x_1,x_2,\ldots,x_m)$

We're trying to find some transformation that transforms the odds ratio to a number that is -∞ to +∞

Does this suggest another transformation?



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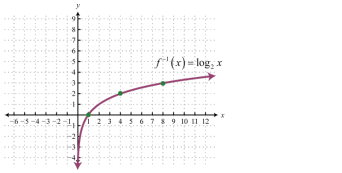

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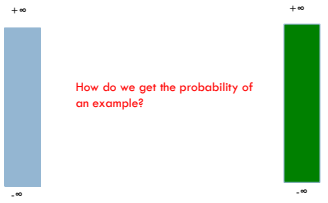
Log odds (logit function)

$x_1w_1 + w_2x_2 + \ldots + w_mx_m + b$ $\quad = \quad \log\frac{P(1|x_1,x_2,\ldots,x_m)}{1-P(1|x_1,x_2,\ldots,x_m)}$

Linear regression $\quad$ odds ratio



How do we get the probability of an example?

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Log odds (logit function)

$$\log \frac{P(1 \mid x_1, x_2, \ldots, x_m)}{1 - P(1 \mid x_1, x_2, \ldots, x_m)} = w_1 x_2 + w_2 x_2 + \ldots + w_m x_m + b$$

$$\frac{P(1 \mid x_1, x_2, \ldots, x_m)}{1 - P(1 \mid x_1, x_2, \ldots, x_m)} = e^{w_1 x_2 + w_2 x_2 + \ldots + w_m x_m + b}$$

$$P(1 \mid x_1, x_2, \ldots, x_m) = (1 - P(1 \mid x_1, x_2, \ldots, x_m)) e^{w_1 x_2 + w_2 x_2 + \ldots + w_m x_m + b}$$

…

$$P(1 \mid x_1, x_2, \ldots, x_m) = \frac{1}{1 + e^{-(w_1 x_2 + w_2 x_2 + \ldots + w_m x_m + b)}}$$

anyone recognize this?

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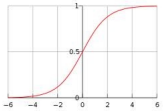
Logistic function

$$\text{logistic} = \frac{1}{1 + e^{-x}}$$



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Logistic regression

How would we classify examples once we had a trained model?

$$\log \frac{P(1 \mid x_1, x_2, \ldots, x_m)}{1 - P(1 \mid x_1, x_2, \ldots, x_m)} = w_1 x_2 + w_2 x_2 + \ldots + w_m x_m + b$$

If the sum > 0 then p(1)/p(0) > 1, so positive

if the sum < 0 then p(1)/p(0) < 1, so negative

Still a *linear* classifier (decision boundary is a line)

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Training logistic regression models

How should we learn the parameters for logistic regression (i.e. the w's and b)?

$$\log \frac{P(1 \mid x_1, x_2, \ldots, x_m)}{1 - P(1 \mid x_1, x_2, \ldots, x_m)} = w_1 x_2 + w_2 x_2 + \ldots + w_m x_m + b$$

parameters

$$P(1 \mid x_1, x_2, \ldots, x_m) = \frac{1}{1 + e^{-(w_1 x_2 + w_2 x_2 + \ldots + w_m x_m + b)}}$$

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MLE logistic regression

Find the parameters that maximize the likelihood (or log-likelihood) of the data:

$$\text{log-likelihood} = \sum_{i=1}^{n} \log(p(x_i))$$

$$= \sum_{i=1}^{n} \log\left(\frac{1}{1+e^{-y_i(w_1x_2+w_2x_2+\ldots+w_mx_m+b)}}\right)$$

assume labels 1, -1

$$= \sum_{i=1}^{n} -\log(1+e^{-y_i(w_1x_2+w_2x_2+\ldots+w_mx_m+b)})$$

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MLE logistic regression

$$\text{log-likelihood} = \sum_{i=1}^{n} -\log(1+e^{-y_i(w_1x_2+w_2x_2+\ldots+w_mx_m+b)})$$

We want to maximize, i.e.

$$MLE(data) = \text{argmax}_{w,b} \log\text{-likelihood}(data)$$

$$= \text{argmax}_{w,b} \sum_{i=1}^{n} -\log(1+e^{-y_i(w_1x_2+w_2x_2+\ldots+w_mx_m+b)})$$

$$= \arg\min_{w,b} \sum_{i=1}^{n} \log(1+e^{-y_i(w_1x_2+w_2x_2+\ldots+w_mx_m+b)})$$

Look familiar? Hint: anybody reading the book?

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MLE logistic regression

$$\text{argmin}_{w,b} \sum_{i=1}^{n} \log(1+e^{-y_i(w_1x_2+w_2x_2+\ldots+w_mx_m+b)})$$

Surrogate loss functions:

Zero/one:	$\ell^{(0/1)}(y,\hat{y}) = \mathbf{1}[y\hat{y} \le 0]$
Hinge:	$\ell^{(\text{hin})}(y,\hat{y}) = \max\{0, 1-y\hat{y}\}$
Logistic:	$\ell^{(\log)}(y,\hat{y}) = \frac{1}{\log 2}\log(1+\exp[-y\hat{y}])$
Exponential:	$\ell^{(\exp)}(y,\hat{y}) = \exp[-y\hat{y}]$
Squared:	$\ell^{(\text{sqr})}(y,\hat{y}) = (y-\hat{y})^2$

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logistic regression: three views

$$\log\frac{P(1\,|\,x_1,x_2,\ldots,x_m)}{1-P(1\,|\,x_1,x_2,\ldots,x_m)} = w_0 + w_1x_2 + w_2x_2 + \ldots + w_mx_m$$

linear classifier

$$P(1\,|\,x_1,x_2,\ldots,x_m) = \frac{1}{1+e^{-(w_0+w_1x_2+w_2x_2+\ldots+w_mx_m)}}$$

conditional model logistic

$$\text{argmin}_{w,b} \sum_{i=1}^{n} \log(1+e^{-y_i(w_1x_2+w_2x_2+\ldots+w_mx_m+b)})$$

linear model minimizing logistic loss

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Overfitting

$$\text{argmin}_{w,b} \sum_{i=1}^{n} \log(1 + e^{-y_i(w_1x_2 + w_2x_2 + \ldots + w_mx_m + b)})$$

If we minimize this loss function, in practice, the results aren't great and we tend to overfit

Solution?

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Regularization/prior

$$\text{argmin}_{w,b} \sum_{i=1}^{n} \log(1 + e^{-y_i(w_1x_2 + w_2x_2 + \ldots + w_mx_m + b)}) + \lambda regularizer(w,b)$$

or

$$\text{argmin}_{w,b} \sum_{i=1}^{n} \log(1 + e^{-y_i(w_1x_2 + w_2x_2 + \ldots + w_mx_m + b)}) - \log(p(w,b))$$

What are some of the regularizers we know?

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Regularization/prior

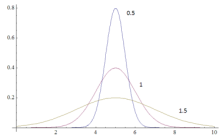
L2 regularization:

$$\text{argmin}_{w,b} \sum_{i=1}^{n} \log(1 + e^{-y_i(w_1x_2 + w_2x_2 + \ldots + w_mx_m + b)}) + \lambda \|w\|^2$$

Gaussian prior:

Gaussians are defined by a mean (μ) and a variance (σ^2)

p(w,b) ~



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Regularization/prior

L2 regularization:

$$\text{argmin}_{w,b} \sum_{i=1}^{n} \log(1 + e^{-y_i(w_1x_2 + w_2x_2 + \ldots + w_mx_m + b)}) + \lambda \|w\|^2$$

Gaussian prior:

$$\text{argmin}_{w,b} \sum_{i=1}^{n} \log(1 + e^{-y_i(w_1x_2 + w_2x_2 + \ldots + w_mx_m + b)}) + \frac{1}{2\sigma^2} \|w\|^2$$

Does the λ make sense?

$$\lambda = \frac{1}{2\sigma^2}$$

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Regularization/prior

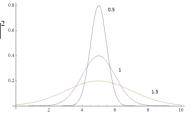
L2 regularization:

$$\text{argmin}_{w,b} \sum_{i=1}^{n} \log(1+e^{-y_i(w_1x_2+w_2x_2+\ldots+w_mx_m+b)}) + \lambda\|w\|^2$$

Gaussian prior:

$$\text{argmin}_{w,b} \sum_{i=1}^{n} \log(1+e^{-y_i(w_1x_2+w_2x_2+\ldots+w_mx_m+b)}) + \frac{1}{2\sigma^2}\|w\|^2$$

$$\lambda = \frac{1}{2\sigma^2}$$



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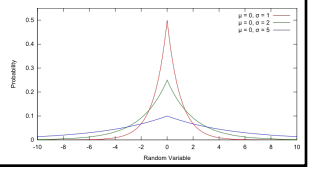
Regularization/prior

L1 regularization:

$$\text{argmin}_{w,b} \sum_{i=1}^{n} \log(1+e^{-y_i(w_1x_2+w_2x_2+\ldots+w_mx_m+b)}) + \lambda\|w\|$$

Laplacian prior:

p(w,b) ~



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Regularization/prior

L1 regularization:

$$\text{argmin}_{w,b} \sum_{i=1}^{n} \log(1+e^{-y_i(w_1x_2+w_2x_2+\ldots+w_mx_m+b)}) + \lambda\|w\|$$

Laplacian prior:

$$\text{argmin}_{w,b} \sum_{i=1}^{n} \log(1+e^{-y_i(w_1x_2+w_2x_2+\ldots+w_mx_m+b)}) + \frac{1}{\sigma}\|w\|$$

$$\lambda = \frac{1}{\sigma}$$

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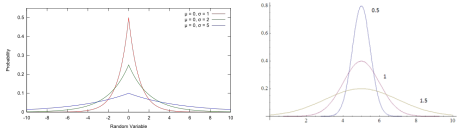
L1 vs. L2

L1 = Laplacian prior

L2 = Gaussian prior



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Logistic regression

Why is it called logistic regression?
It is a classifier??

$$\log \frac{P(1 \mid x_1, x_2, \ldots, x_m)}{1 - P(1 \mid x_1, x_2, \ldots, x_m)} = w_1 x_2 + w_2 x_2 + \ldots + w_m x_m + b$$

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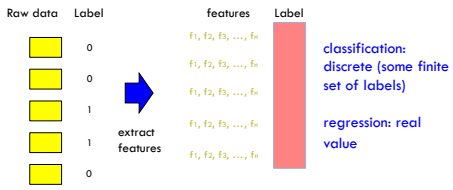
A digression: regression vs. classification



classification: discrete (some finite set of labels)

regression: real value

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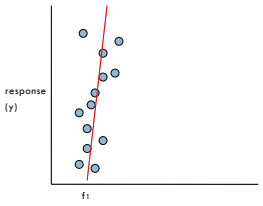
linear regression



Given some points, find the *line* that best fits/explains the data

Our model is a line, i.e. we're assuming a linear relationship between the feature and the label value

$$h(y) = w_1 x_1 + b$$

How can we find this line?

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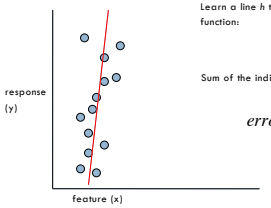
Linear regression



Learn a line *h* that minimizes some loss/error function:

$$error(h) = ?$$

Sum of the individual errors:

$$error(h) = \sum_{i=1}^{n} |y_i - h(f_i)|$$

0/1 loss!

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Error minimization

How do we find the minimum of an equation?

$$error(h) = \sum_{i=1}^{n} |y_i - h(f_i)|$$

Take the derivative, set to 0 and solve (going to be a min or a max)

Any problems here?

Ideas?

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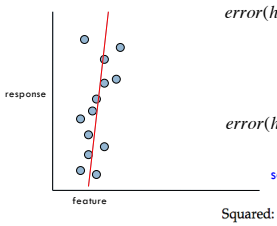
Linear regression



$$error(h) = \sum_{i=1}^{n} |y_i - h(f_i)|$$

$$error(h) = \sum_{i=1}^{n} (y_i - h(f_i))^2$$

squared error is convex!

Squared: $\ell^{(sqr)}(y, \hat{y}) = (y - \hat{y})^2$

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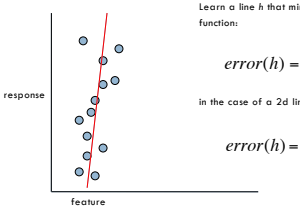
Linear regression



Learn a line h that minimizes an error function:

$$error(h) = \sum_{i=1}^{n} (y_i - h(f_i))^2$$

in the case of a 2d line:

$$error(h) = \sum_{i=1}^{n} (y_i - (w_1 x_1 + w_0))^2$$

function for a line

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Linear regression

We'd like to *minimize* the error

Find w_1 and w_0 such that the error is minimized

$$error(h) = \sum_{i=1}^{n} (y_i - (w_1 f_i + w_0))^2$$

We can solve this in closed form

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Multiple linear regression

If we have *m* features, then we have a line in *m* dimensions

$$h(\bar{f}) = w_0 + w_1 f_1 + w_2 f_2 + \ldots + w_m f_m$$

weights

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Multiple linear regression

We can still calculate the squared error like before

$$h(\bar{f}) = w_0 + w_1 f_1 + w_2 f_2 + \ldots + w_m f_m$$

$$error(h) = \sum_{i=1}^{n} (y_i - (w_0 + w_1 f_1 + w_2 f_2 + \ldots + w_m f_m))^2$$

Still can solve this exactly!

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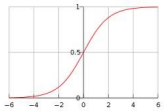
Logistic function

$$\text{logistic} = \frac{1}{1+e^{-x}}$$



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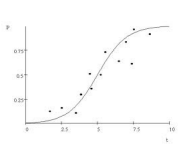
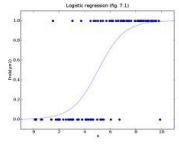
Logistic regression

Find the best fit of the data based on a logistic





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Basic steps for probabilistic modeling

	Probabilistic models
Step 1: pick a model	Which model do we use, i.e. how do we calculate p(*feature, label*)?
Step 2: figure out how to estimate the probabilities for the model	How do train the model, i.e. how to we we estimate the probabilities for the model?
Step 3 (optional): deal with overfitting	How do we deal with overfitting?

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Probabilistic models summarized

Two classification models:

- Naïve Bayes (models joint distribution)
- Logistic Regression (models conditional distribution)
  - In practice this tends to work better if all you want to do is classify

Priors/smoothing/regularization

- Important for both models
- In theory: allow us to impart some prior knowledge
- In practice: avoids overfitting and often tune on development data

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