NEURAL NETWORKS

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CS158 – Fall 2023

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Admin

Assignment 7

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Perceptron learning algorithm

repeat until convergence (or for some # of iterations):
  for each training example (f_1, f_2, …, f_n, label):
    $prediction = b + \sum_{i=1}^{n} w_i f_i$

    if *prediction* * *label* ≤ 0: // they don't agree
      for each w_i:
        $w_i = w_i + f_i$*label
      $b = b$ + label

Why is it called the "perceptron" learning algorithm if what it learns is a line? Why not "line learning" algorithm?

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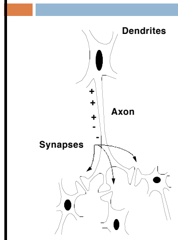
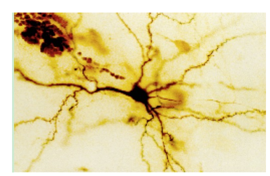
Our Nervous System





Neuron

What do you know?

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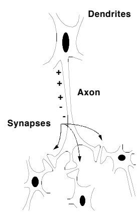
Our nervous system: the computer science view



the human brain is a large collection of interconnected neurons

a NEURON is a brain cell

- they collect, process, and disseminate electrical signals
- they are connected via synapses
- they FIRE depending on the conditions of the neighboring neurons

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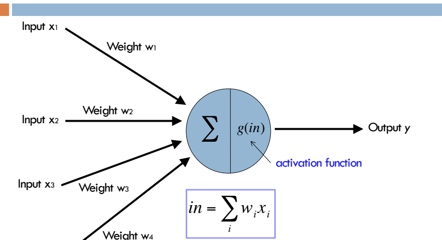
A neuron/perceptron



How is this a linear classifier (i.e. perceptron)?

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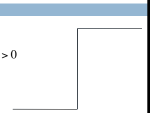
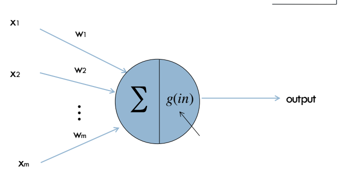
Hard threshold = linear classifier

hard threshold:

$$g(in) = \begin{cases} 1 & \text{if } in > -b \\ 0 & otherwise \end{cases} \qquad output = \begin{cases} 1 & \text{if } \sum_i w_i x_i + b > 0 \\ 0 & otherwise \end{cases}$$





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Neural Networks

Neural Networks try to mimic the structure and function of our nervous system

People like biologically motivated approaches

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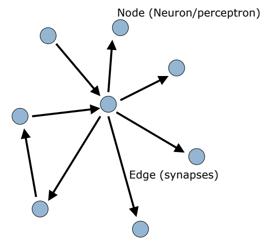
Artificial Neural Networks



our approximation

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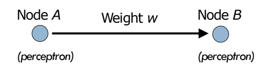

$$output = \begin{cases} 1 & \text{if } \sum_i w_i x_i + b > 0 \\ 0 & otherwise \end{cases}$$

W is the strength of signal sent between A and B.

If *A* fires and *w* is **positive**, then *A* **stimulates** *B*.

If *A fires* and *w* is **negative**, then *A* **inhibits** *B*.

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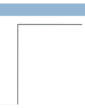
Other activation functions

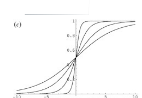
hard threshold:

$$g(in) = \begin{cases} 1 & \text{if } in > -b \\ 0 & otherwise \end{cases}$$

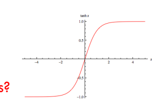


sigmoid

$$g(x) = \frac{1}{1 + e^{-ax}}$$



tanh x



why other threshold functions?

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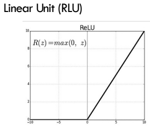
Many other activation functions

Rectified Linear Unit (RLU)



Softmax (for probabilities)

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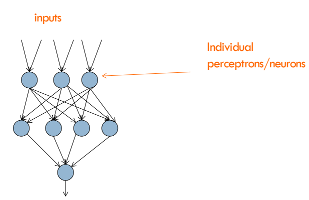
Neural network



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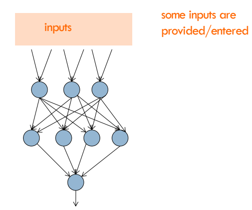
Neural network



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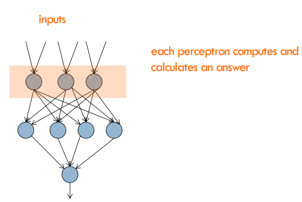
Neural network



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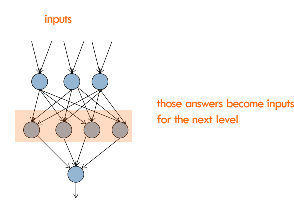
Neural network



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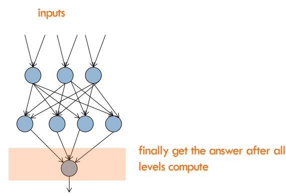
Neural network



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Activation spread

http://www.youtube.com/watch?v=Ya7d4ROvZ6I

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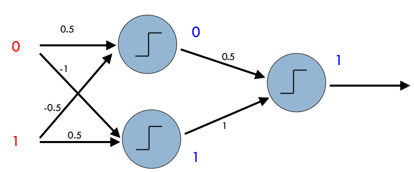
Computation (assume 0 bias)



$$g(in) = \begin{cases} 1 & \text{if } in > -b \\ 0 & otherwise \end{cases}$$

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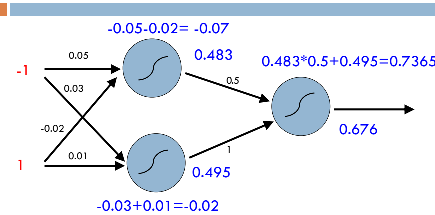
Computation



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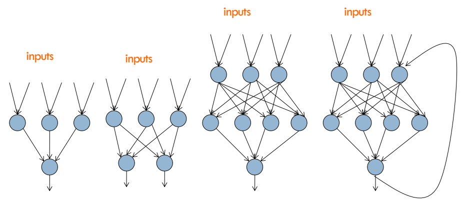
Neural networks

Different kinds/characteristics of networks



How are these different?

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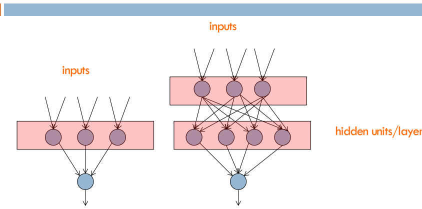
Hidden units/layers



Feed forward networks

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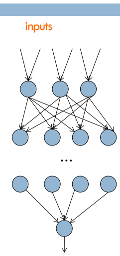
Hidden units/layers



Can have many layers of hidden units of differing sizes

To count the number of layers, you count all but the inputs

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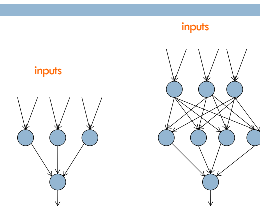
Hidden units/layers



2-layer network 3-layer network

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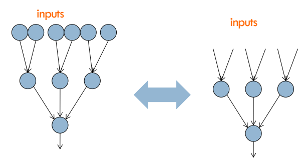
Alternate ways of visualizing

Sometimes the input layer will be drawn with nodes as well



2-layer network 2-layer network

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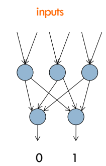
Multiple outputs



Can be used to model multiclass datasets or more interesting predictors, e.g. images

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Multiple outputs

input

output
(edge detection)

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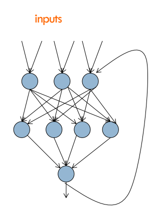
Neural networks



Recurrent network

Output is fed back to input

Can support memory!

Good for temporal/sequential data

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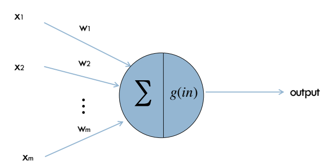
NN decision boundary



What does the decision boundary of a perceptron look like?

Line (linear set of weights)

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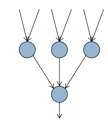
NN decision boundary



What does the decision boundary of a 2-layer network look like?
Is it linear?
What types of things can and can't it model?

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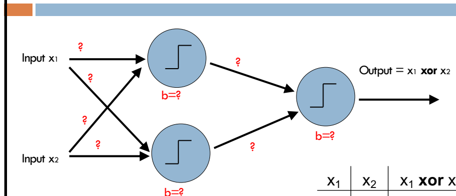
XOR



x_1	x_2	x_1 **xor** x_2
0	0	0
0	1	1
1	0	1
1	1	0

$$output = \begin{cases} 1 & \text{if } \sum_i w_i x_i + b > 0 \\ 0 & otherwise \end{cases}$$

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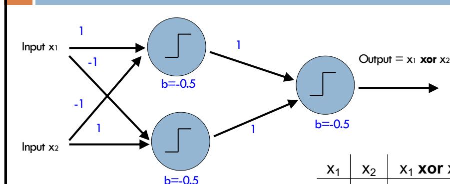
XOR



x_1	x_2	x_1 **xor** x_2
0	0	0
0	1	1
1	0	1
1	1	0

$$output = \begin{cases} 1 & \text{if } \sum_i w_i x_i + b > 0 \\ 0 & otherwise \end{cases}$$

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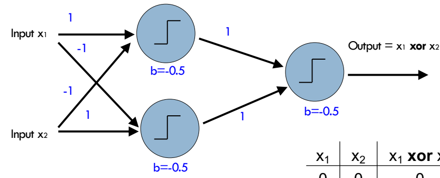
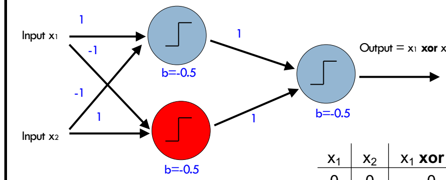
What does the decision boundary look like?



x_1	x_2	x_1 **xor** x_2
0	0	0
0	1	1
1	0	1
1	1	0

33

What does the decision boundary look like?



What does this perceptron's decision boundary look like?

x_1	x_2	x_1 **xor** x_2
0	0	0
0	1	1
1	0	1
1	1	0

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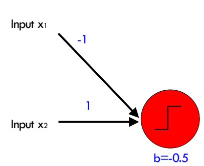
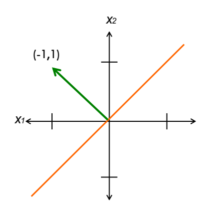
NN decision boundary





Let x_2 = 0, then:

$$-x_1 - 0.5 = 0$$
$$x_1 = -0.5$$

(without the bias)

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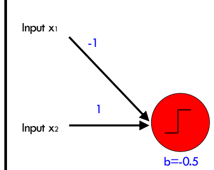
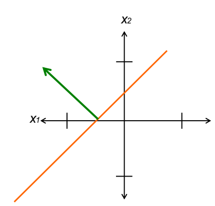
NN decision boundary





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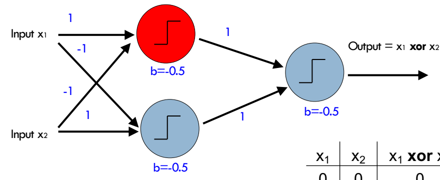
What does the decision boundary look like?



x_1	x_2	x_1 **xor** x_2
0	0	0
0	1	1
1	0	1
1	1	0

What does this perceptron's decision boundary look like?

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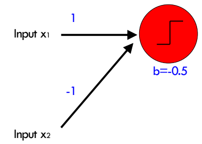
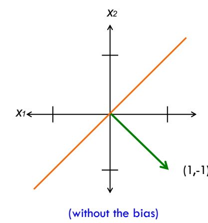
NN decision boundary



Let $x_2 = 0$, then:

$$x_1 - 0.5 = 0$$
$$x_1 = 0.5$$



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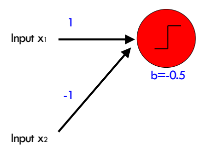
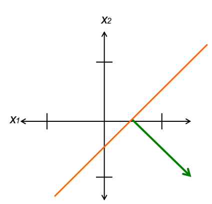
NN decision boundary





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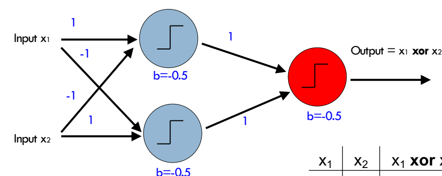
What does the decision boundary look like?



x_1	x_2	x_1 **xor** x_2
0	0	0
0	1	1
1	0	1
1	1	0

What operation does this perceptron perform on the result?

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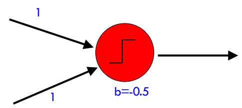
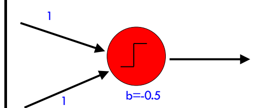
Fill in the truth table



out1	out2	
0	0	?
0	1	?
1	0	?
1	1	?

41

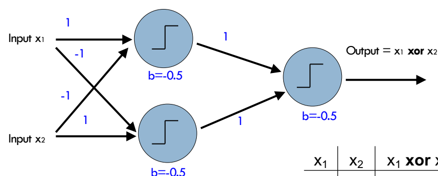
OR



out1	out2	
0	0	0
0	1	1
1	0	1
1	1	1

42

What does the decision boundary look like?



x_1	x_2	x_1 **xor** x_2
0	0	0
0	1	1
1	0	1
1	1	0

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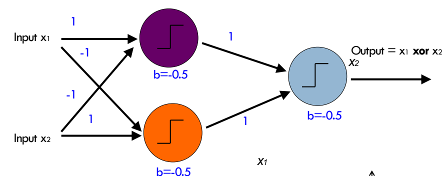

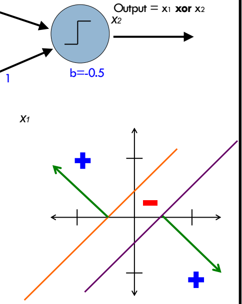

If either predicts positive, example is positive

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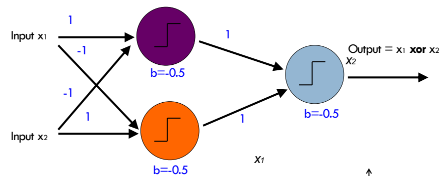
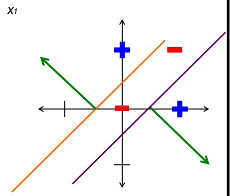
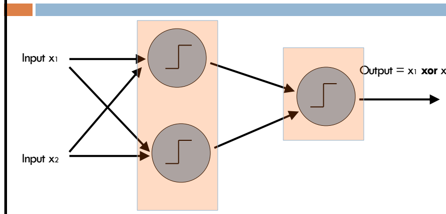




x_1	x_2	x_1 **xor** x_2
0	0	0
0	1	1
1	0	1
1	1	0

45

What does the decision boundary look like?



linear splits of the feature space

combination of these linear spaces

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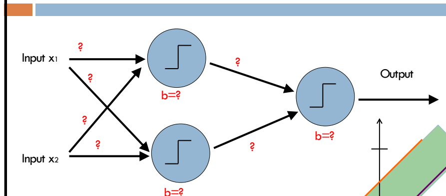
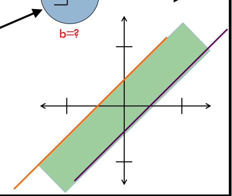
This decision boundary?



$$output = \begin{cases} 1 & \text{if } \sum_i w_i x_i + b > 0 \\ 0 & otherwise \end{cases}$$



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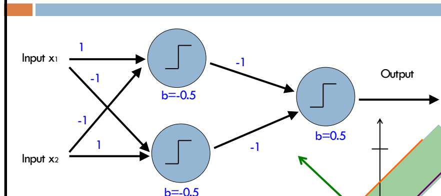
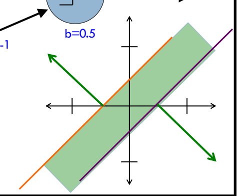
This decision boundary?



$$output = \begin{cases} 1 & \text{if } \sum_i w_i x_i + b > 0 \\ 0 & otherwise \end{cases}$$



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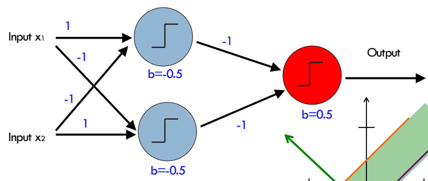
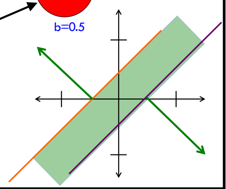
This decision boundary?





$$output = \begin{cases} 1 & \text{if } \sum_i w_i x_i + b > 0 \\ 0 & otherwise \end{cases}$$

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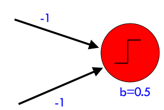
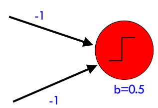


out1	out2	
0	0	?
0	1	?
1	0	?
1	1	?

50

NOR



out1	out2	
0	0	1
0	1	0
1	0	0
1	1	0

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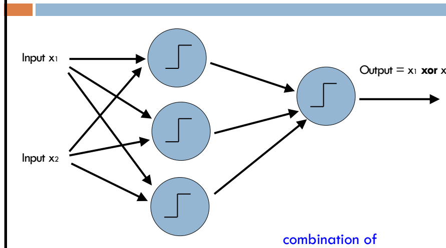
What does the decision boundary look like?



linear splits of the feature space

combination of these linear spaces

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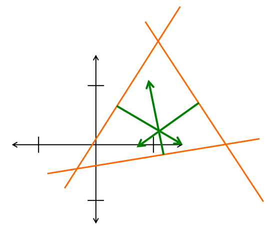
Three hidden nodes



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NN decision boundaries

Theorem 9 (Two-Layer Networks are Universal Function Approximators). *Let F be a continuous function on a bounded subset of D-dimensional space. Then there exists a two-layer neural network $\hat{F}$ with a finite number of hidden units that approximate F arbitrarily well. Namely, for all $\boldsymbol{x}$ in the domain of F, $|F(\boldsymbol{x}) - \hat{F}(\boldsymbol{x})| < \epsilon$.*

Put simply: *two-layer networks can approximate any function*

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NN decision boundaries

For DT, as the tree gets larger, the model gets more complex

The same is true for neural networks:
more hidden nodes = more complexity

Adding more layers adds even more complexity (and much more quickly)

Good rule of thumb:

$$\text{number of 2-layer hidden nodes} \leq \frac{\text{number of examples}}{\text{number of dimensions}}$$

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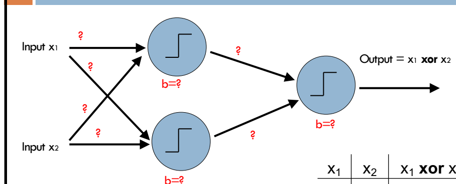
Training



x_1	x_2	x_1 **xor** x_2
0	0	0
0	1	1
1	0	1
1	1	0

How do we learn the weights?

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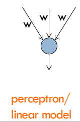
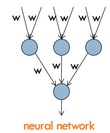
Training multilayer networks

perceptron learning: if the perceptron's **output** is different than the expected output, update the weights

gradient descent: compare **output** to label and adjust based on loss function

Any other problem with these for general NNs?





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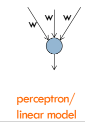
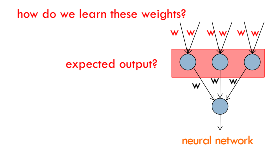
Learning in multilayer networks

Challenge: for multilayer networks, we don't know what the expected output/error is for the internal nodes!





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Backpropagation: intuition

Gradient descent method for learning weights by optimizing a loss function

1. calculate output of all nodes
2. calculate the weights for the output layer based on the error
3. "backpropagate" errors through hidden layers

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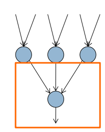
Backpropagation: intuition



We can calculate the actual error here

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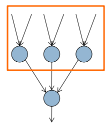
Backpropagation: intuition



Key idea: propagate the error back to this layer

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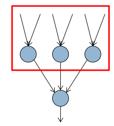
Backpropagation: intuition



"backpropagate" the error:

Assume all of these nodes were responsible for some of the error

How can we figure out how much they were responsible for?

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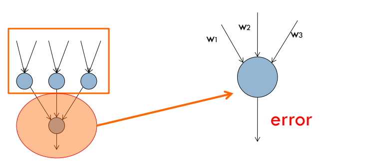
Backpropagation: intuition



error for node is ~ $\mathbf{w_i}$ * **error**

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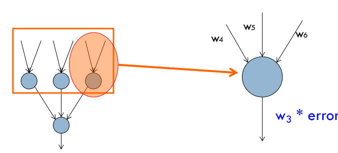
Backpropagation: intuition



Calculate as normal using this as the error

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Backpropagation: the details

Gradient descent method for learning weights by optimizing a loss function

1. calculate output of all nodes
2. calculate the updates directly for the output layer
3. "backpropagate" errors through hidden layers

What loss function?

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Backpropagation: the details

Gradient descent method for learning weights by optimizing a loss function

1. calculate output of all nodes
2. calculate the updates directly for the output layer
3. "backpropagate" errors through hidden layers

$$loss = \sum_x \frac{1}{2}(y - \hat{y})^2$$ squared error

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