

# BEYOND BINARY CLASSIFICATION

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CS 1.58 – Spring 2022

1

## Admin

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Assignment 4






Assignment 2 graded

2

## Multiclass classification

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**examples**

	label	Same setup where we have a set of features for each example
	apple	
	orange	Rather than just two labels, now have 3 or more
	apple	
	banana	real-world examples?
	banana	
	pineapple	


3

## Real world multiclass classification


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document classification



protein classification




handwriting recognition




face recognition


most real-world applications tend to be multiclass



sentiment analysis



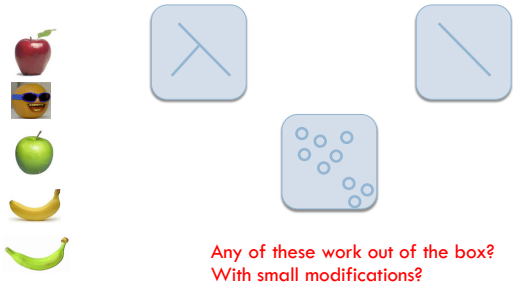
autonomous vehicles



emotion recognition

4

## Multiclass: current classifiers



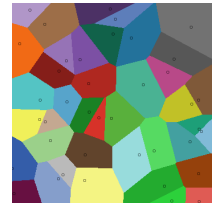
5

## k-Nearest Neighbor (k-NN)

To classify an example  $d$ :

- ▣ Find  $k$  nearest neighbors of  $d$
- ▣ Choose as the label the majority label within the  $k$  nearest neighbors

No algorithmic changes!



6

## Decision Tree learning

Base cases:

1. If all data belong to the same class, pick that label
2. If all the data have the same feature values, pick majority label
3. If we're out of features to examine, pick majority label
4. If the we don't have any data left, pick majority label of *parent*
5. If *some other stopping criteria* exists to avoid overfitting, pick majority label

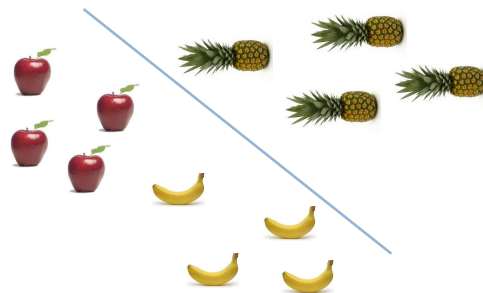
Otherwise:

- calculate the "score" for each feature if we used it to split the data
- pick the feature with the highest score, partition the data based on that data value and call recursively

No algorithmic changes!

7

## Perceptron learning

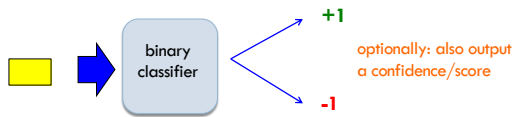


Hard to separate three classes with just one line ☹️

8

### Black box approach to multiclass

Abstraction: we have a generic binary classifier, how can we use it to solve our new problem



Can we solve our multiclass problem with this?

9

### Approach 1: One vs. all (OVA)

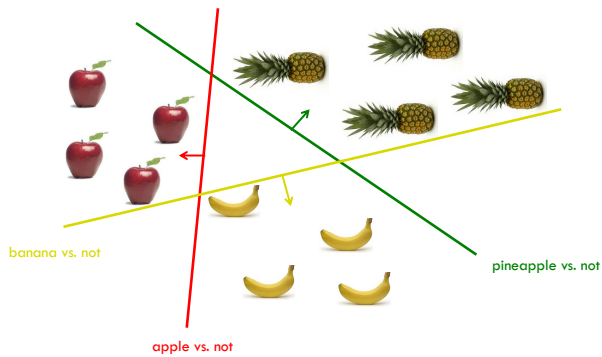
Training: for each label  $L$ , pose as a binary problem

- all examples with label  $L$  are positive
- all other examples are negative

		apple vs. not	orange vs. not	banana vs. not
	apple	+1	-1	-1
	orange	-1	+1	-1
	apple	+1	-1	-1
	banana	-1	-1	+1
	banana	-1	-1	+1

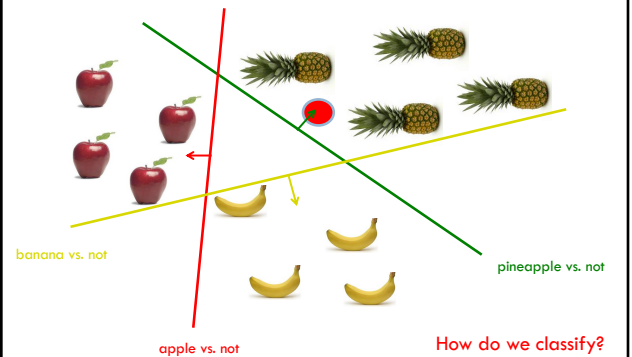
10

### OVA: linear classifiers (e.g. perceptron)

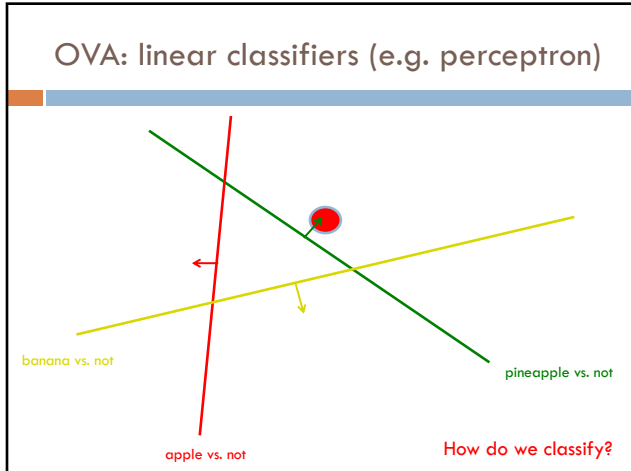


11

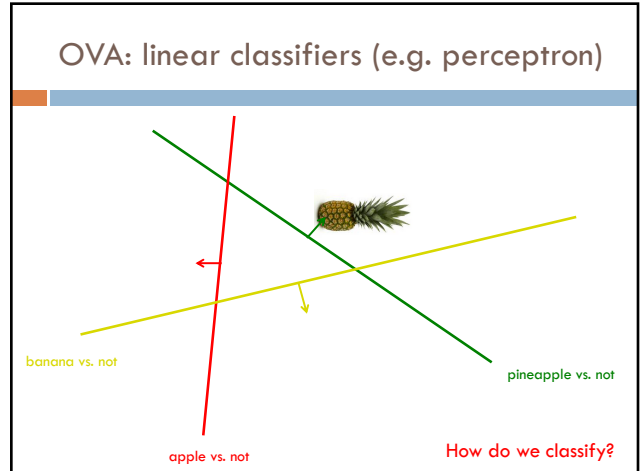
### OVA: linear classifiers (e.g. perceptron)



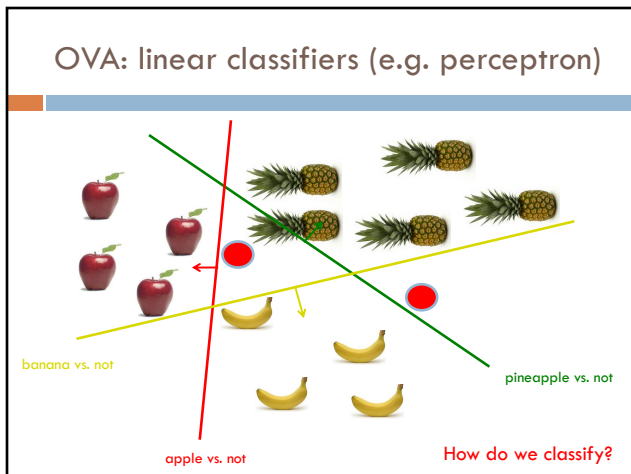
12



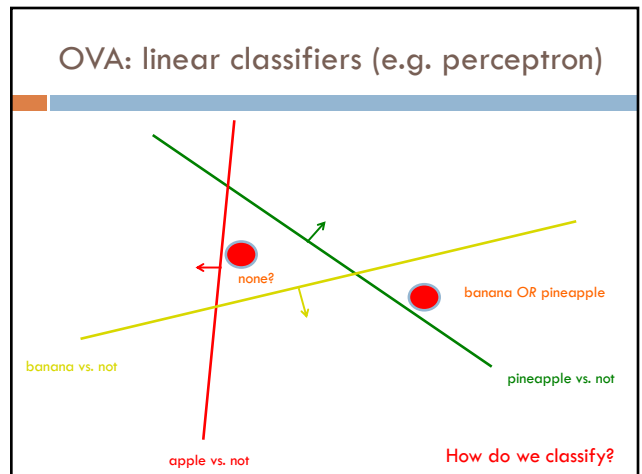
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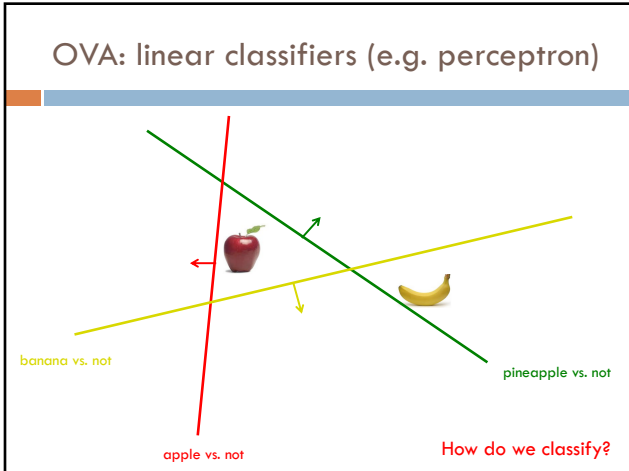
14



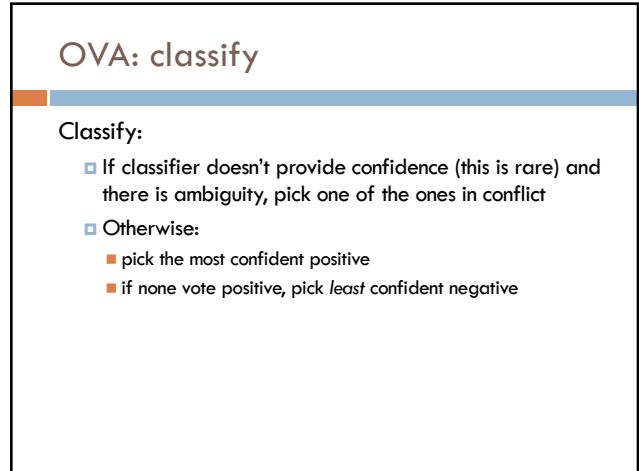
15



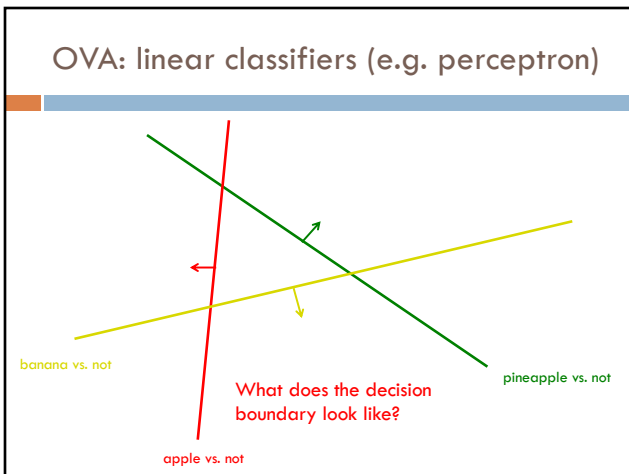
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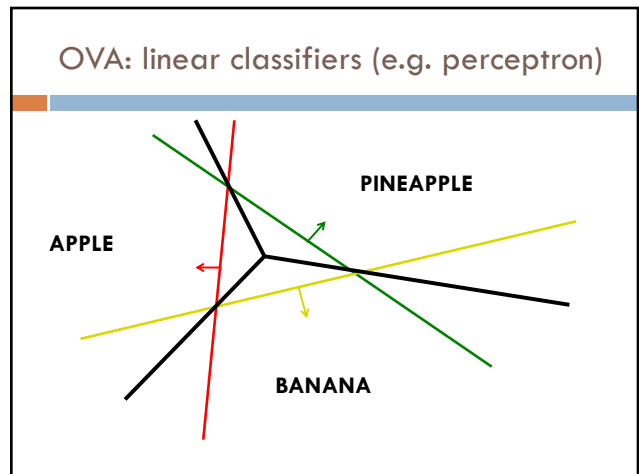
17



18



19



20

## OVA: classify, perceptron

### Classify:

- If classifier doesn't provide confidence (this is rare) and there is ambiguity, pick majority in conflict
- Otherwise:
  - pick the most **confident** positive
  - if none vote positive, pick *least* confident negative

How do we calculate this for the perceptron?

21

## OVA: classify, perceptron

### Classify:

- If classifier doesn't provide confidence (this is rare) and there is ambiguity, pick majority in conflict
- Otherwise:
  - pick the most **confident** positive
  - if none vote positive, pick *least* confident negative

$$\text{prediction} = b + \sum_{i=1}^n w_i f_i$$

Distance from the hyperplane

22

## Approach 2: All vs. all (AVA)

### Training:

For each pair of labels, train a classifier to distinguish between them

for  $i = 1$  to number of labels:

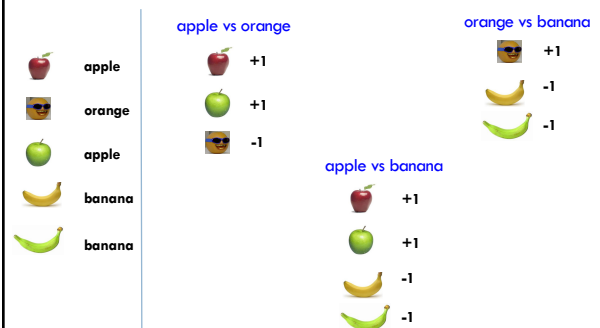
for  $k = i+1$  to number of labels:

train a classifier to distinguish between  $label_i$  and  $label_k$ :

- create a dataset with all examples *with*  $label_i$ , labeled positive and all examples with  $label_k$ , labeled negative
- train classifier on this subset of the data

23

## AVA training visualized



24

### AVA classify

apple vs orange

- 🍏 +1
- 🍏 +1 orange
- 👓 -1


apple vs banana

- 🍏 +1
- 🍏 +1 apple
- 🍌 -1
- 🍌 -1

orange vs banana

- 👓 +1
- 🍌 -1 orange
- 🍌 -1

What class?



25

### AVA classify

apple vs orange

- 🍏 +1
- 🍏 +1 orange
- 👓 -1


apple vs banana

- 🍏 +1
- 🍏 +1 apple
- 🍌 -1
- 🍌 -1

orange vs banana

- 👓 +1
- 🍌 -1 orange
- 🍌 -1

orange



In general?

26

### AVA classify

To classify example  $e$ , classify with each classifier  $f_{jk}$

We have a few options to choose the final class:

- Take a majority vote
- Take a weighted vote based on confidence
  - $y = f_{jk}(e)$
  - $\text{score}_j += y$  How does this work?
  - $\text{score}_k -= y$

Here we're assuming that  $y$  encompasses both the prediction (+1,-1) and the confidence, i.e.  $y = \text{prediction} * \text{confidence}$ .

27

### AVA classify

Take a weighted vote based on confidence

- $y = f_{jk}(e)$
- $\text{score}_j += y$
- $\text{score}_k -= y$

If  $y$  is positive, classifier thought it was of type  $j$ :

- raise the score for  $j$
- lower the score for  $k$

if  $y$  is negative, classifier thought it was of type  $k$ :

- lower the score for  $j$
- raise the score for  $k$

28

## OVA vs. AVA

Train/classify runtime?

Error? Assume each binary classifier makes an error with probability  $\epsilon$

29

## OVA vs. AVA

Train time:

AVA learns more classifiers, however, they're trained on much smaller data this tends to make it faster if the labels are equally balanced

Test time:

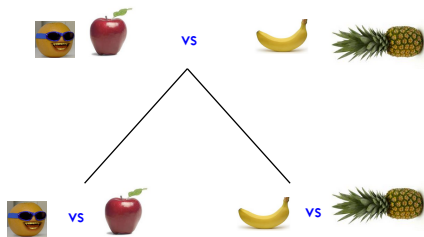
AVA has more classifiers, so often it is slower

Error (see the book for more justification):

- AVA trains on more balanced data sets
- AVA tests with more classifiers and therefore has more chances for errors
- Theoretically:
  - OVA:  $\epsilon$  (number of labels - 1)
  - AVA:  $2 \epsilon$  (number of labels - 1)

30

## Approach 3: Divide and conquer



Pros/cons vs. AVA?

31

## Multiclass summary

If using a binary classifier, the most common thing to do is OVA







Otherwise, use a classifier that allows for multiple labels:

- DT and k-NN work reasonably well
- We'll see a few more in the coming weeks that will often work better

32









## Multiclass evaluation

	label	prediction	
	apple	orange	How should we evaluate?
	orange	orange	
	apple	apple	
	banana	pineapple	
	banana	banana	
	pineapple	pineapple	






33

## Multiclass evaluation

	label	prediction	
	apple	orange	Accuracy: 4/6
	orange	orange	
	apple	apple	
	banana	pineapple	
	banana	banana	
	pineapple	pineapple	

34

## Multiclass evaluation imbalanced data

	label	prediction	
	apple	orange	Any problems? Data imbalance!
...	...	...	
	apple	apple	
	banana	pineapple	
	banana	banana	
	pineapple	pineapple	

35

## Macroaveraging vs. microaveraging

**microaveraging:** average over examples (this is the "normal" way of calculating)

**macroaveraging:** calculate evaluation score (e.g. accuracy) for each label, then average over labels

What effect does this have?  
Why include it?

36

### Macroaveraging vs. microaveraging







**microaveraging:** average over examples (this is the "normal" way of calculating)

**macroaveraging:** calculate evaluation score (e.g. accuracy) for each label, then average over labels

- Puts more weight/emphasis on rarer labels
- Allows another dimension of analysis

37

### Macroaveraging vs. microaveraging







	label	prediction	
	apple	orange	<b>microaveraging:</b> average over examples
	orange	orange	
	apple	apple	
	banana	pineapple	
	banana	banana	
	pineapple	pineapple	

**macroaveraging:** calculate evaluation score (e.g. accuracy) for each label, then average over labels

?

38

### Macroaveraging vs. microaveraging

	label	prediction	
	apple	orange	<b>microaveraging:</b> 4/6
	orange	orange	
	apple	apple	
	banana	pineapple	
	banana	banana	
	pineapple	pineapple	

**macroaveraging:**

apple = 1/2  
orange = 1/1  
banana = 1/2  
pineapple = 1/1  
total = (1/2 + 1 + 1/2 + 1)/4  
= 3/4

39

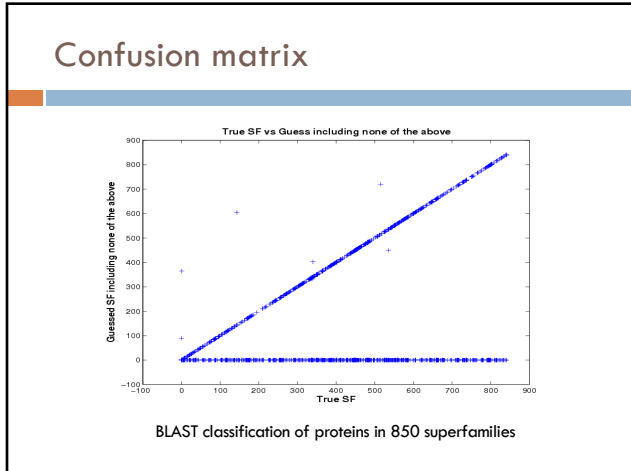
### Confusion matrix

entry  $(i, j)$  represents the number of examples with label  $i$  that were predicted to have label  $j$

another way to understand both the data and the classifier

	Classic	Country	Disco	Hiphop	Jazz	Rock
Classic	86	2	0	4	18	1
Country	1	57	5	1	12	13
Disco	0	6	55	4	0	5
Hiphop	0	15	28	90	4	18
Jazz	7	1	0	0	37	12
Rock	6	19	11	0	27	48

40



41

### Multilabel vs. multiclass classification

<ul style="list-style-type: none"> <li>• Is it edible?</li> <li>• Is it sweet?</li> <li>• Is it a fruit?</li> <li>• Is it a banana?</li> </ul>	<ul style="list-style-type: none"> <li>Is it a banana?</li> <li>Is it an apple?</li> <li>Is it an orange?</li> <li>Is it a pineapple?</li> </ul>	<ul style="list-style-type: none"> <li>Is it a banana?</li> <li>Is it yellow?</li> <li>Is it sweet?</li> <li>Is it round?</li> </ul>
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Any difference in these labels/categories?

42

### Multilabel vs. multiclass classification

<ul style="list-style-type: none"> <li>• Is it edible?</li> <li>• Is it sweet?</li> <li>• Is it a fruit?</li> <li>• Is it a banana?</li> </ul>	<ul style="list-style-type: none"> <li>Is it a banana?</li> <li>Is it an apple?</li> <li>Is it an orange?</li> <li>Is it a pineapple?</li> </ul>	<ul style="list-style-type: none"> <li>Is it a banana?</li> <li>Is it yellow?</li> <li>Is it sweet?</li> <li>Is it round?</li> </ul>
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Different structures

Nested/ Hierarchical      Exclusive/ Multiclass      General/Structured

43

### Multiclass vs. multilabel

Multiclass: each example has one label and exactly one label

Multilabel: each example has **zero or more** labels. Also called annotation

Multilabel applications?

44

## Multilabel

Image annotation

Document topics

Labeling people in a picture

Medical diagnosis

45

## Ranking problems

Suggest a simpler word for the word below:

vital

46

## Suggest a simpler word

Suggest a simpler word for the word below:

vital

word	frequency
important	13
necessary	12
essential	11
needed	8
critical	3
crucial	2
mandatory	1
required	1
vital	1

47

## Suggest a simpler word

Suggest a simpler word for the word below:

acquired

48

### Suggest a simpler word

Suggest a simpler word for the word below:

**acquired**

word	frequency
gotten	12
received	9
gained	8
obtained	5
got	3
purchased	2
bought	2
got hold of	1
acquired	1

49

### Suggest a simpler word

**vital**

important  
necessary  
essential  
needed  
critical  
crucial  
mandatory  
required  
vital

**acquired**

gotten  
received  
gained  
obtained  
got  
purchased  
bought  
got hold of  
acquired

... training data

↓ train

list of synonyms → **ranker** → list ranked by simplicity

50

### Ranking problems in general

ranking1

$f_{1,1}, f_{2,1}, \dots, f_{n,1}$
$f_{1,2}, f_{2,2}, \dots, f_{n,2}$
$f_{1,3}, f_{2,3}, \dots, f_{n,3}$
$f_{1,4}, f_{2,4}, \dots, f_{n,4}$

ranking2

$f_{1,1}, f_{2,1}, \dots, f_{n,1}$
$f_{1,2}, f_{2,2}, \dots, f_{n,2}$
$f_{1,3}, f_{2,3}, \dots, f_{n,3}$
$f_{1,4}, f_{2,4}, \dots, f_{n,4}$

ranking3

$f_{1,1}, f_{2,1}, \dots, f_{n,1}$
$f_{1,2}, f_{2,2}, \dots, f_{n,2}$
$f_{1,3}, f_{2,3}, \dots, f_{n,3}$
$f_{1,4}, f_{2,4}, \dots, f_{n,4}$

... training data: a set of rankings where each ranking consists of a set of ranked examples

↓ train

$f_{1,1}, f_{2,1}, \dots, f_{n,1}$   
 $f_{1,2}, f_{2,2}, \dots, f_{n,2}$   
 $f_{1,3}, f_{2,3}, \dots, f_{n,3}$   
 $f_{1,4}, f_{2,4}, \dots, f_{n,4}$  → **ranker** → ranking/ordering of examples

51

### Ranking problems in general

ranking1

$f_{1,1}, f_{2,1}, \dots, f_{n,1}$
$f_{1,2}, f_{2,2}, \dots, f_{n,2}$
$f_{1,3}, f_{2,3}, \dots, f_{n,3}$
$f_{1,4}, f_{2,4}, \dots, f_{n,4}$

ranking2

$f_{1,1}, f_{2,1}, \dots, f_{n,1}$
$f_{1,2}, f_{2,2}, \dots, f_{n,2}$
$f_{1,3}, f_{2,3}, \dots, f_{n,3}$
$f_{1,4}, f_{2,4}, \dots, f_{n,4}$

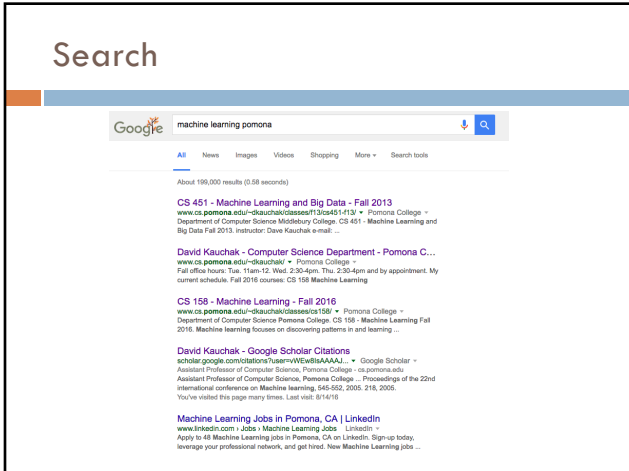
ranking3

$f_{1,1}, f_{2,1}, \dots, f_{n,1}$
$f_{1,2}, f_{2,2}, \dots, f_{n,2}$
$f_{1,3}, f_{2,3}, \dots, f_{n,3}$
$f_{1,4}, f_{2,4}, \dots, f_{n,4}$

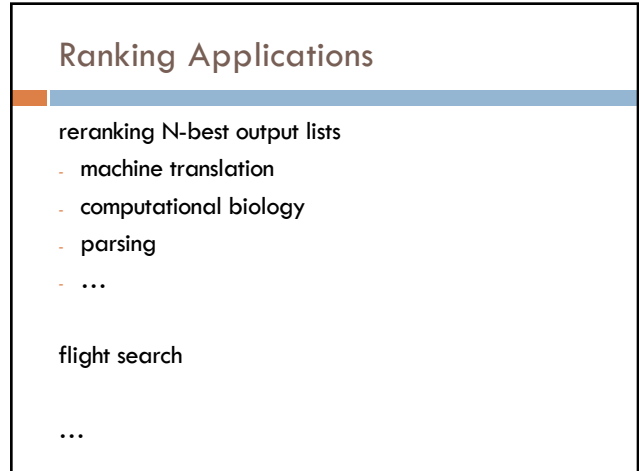
... training data: a set of rankings where each ranking consists of a set of ranked examples

Real-world ranking problems?

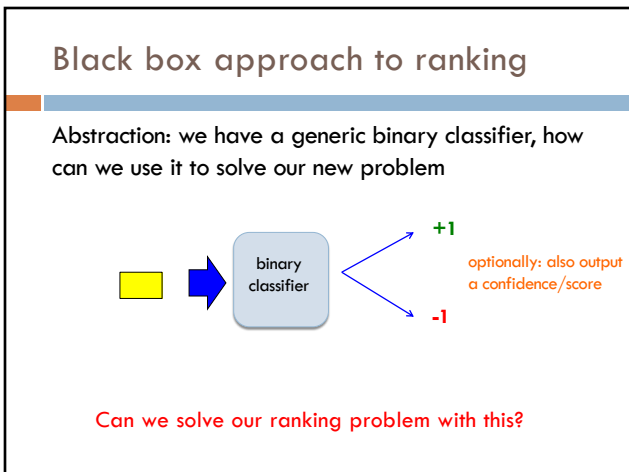
52



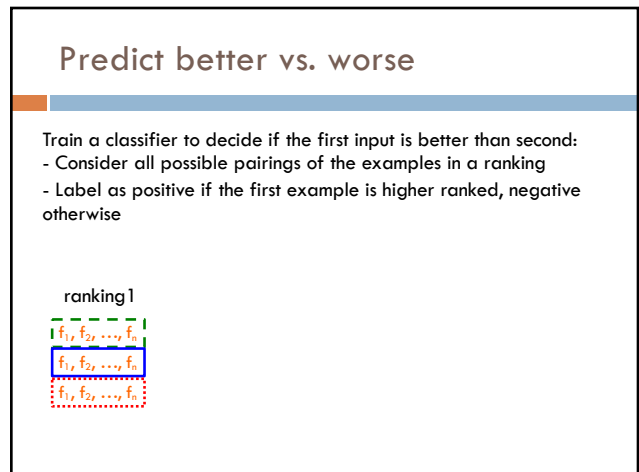
54



55



56



57

### Predict better vs. worse

Train a classifier to decide if the first input is better than second:

- Consider all possible pairings of the examples in a ranking
- Label as positive if the first example is higher ranked, negative otherwise

ranking 1

$f_{1,1}, f_{2,1}, \dots, f_{n,1}$

$f_{1,2}, f_{2,2}, \dots, f_{n,2}$

$f_{1,3}, f_{2,3}, \dots, f_{n,3}$

➔

new examples	binary label
$f_{1,1}, f_{2,1}, \dots, f_{n,1}$   $f_{1,2}, f_{2,2}, \dots, f_{n,2}$	+1
$f_{1,1}, f_{2,1}, \dots, f_{n,1}$   $f_{1,3}, f_{2,3}, \dots, f_{n,3}$	+1
$f_{1,2}, f_{2,2}, \dots, f_{n,2}$   $f_{1,3}, f_{2,3}, \dots, f_{n,3}$	-1
$f_{1,1}, f_{2,1}, \dots, f_{n,1}$   $f_{1,2}, f_{2,2}, \dots, f_{n,2}$	+1
$f_{1,1}, f_{2,1}, \dots, f_{n,1}$   $f_{1,3}, f_{2,3}, \dots, f_{n,3}$	-1
$f_{1,2}, f_{2,2}, \dots, f_{n,2}$   $f_{1,3}, f_{2,3}, \dots, f_{n,3}$	-1

58

### Predict better vs. worse

➔

binary classifier

➔ +1

➔ -1

Our binary classifier only takes one example as input

59

### Predict better vs. worse

➔

binary classifier

➔ +1

➔ -1

Our binary classifier only takes one example as input

---

$a_1, a_2, \dots, a_n$

$b_1, b_2, \dots, b_n$

➔

$f'_1, f'_2, \dots, f'_n$

How can we do this?  
We want features that compare the two examples.

60

### Combined feature vector

Many approaches! Will depend on domain and classifier

Two common approaches:

1. difference:
 
$$f'_i = a_i - b_i$$
2. greater than/less than:
 
$$f'_i = \begin{cases} 1 & \text{if } a_i > b_i \\ 0 & \text{otherwise} \end{cases}$$

61





### Testing

$f_{1j}, f_{2j}, \dots, f_{nj}$	$f_{1j}, f_{2j}, \dots, f_{nj}$	-1
$f_{1j}, f_{2j}, \dots, f_{nj}$	$f_{1j}, f_{2j}, \dots, f_{nj}$	-1
$f_{1j}, f_{2j}, \dots, f_{nj}$	$f_{1j}, f_{2j}, \dots, f_{nj}$	+1
$f_{1j}, f_{2j}, \dots, f_{nj}$	$f_{1j}, f_{2j}, \dots, f_{nj}$	+1
$f_{1j}, f_{2j}, \dots, f_{nj}$	$f_{1j}, f_{2j}, \dots, f_{nj}$	-1
$f_{1j}, f_{2j}, \dots, f_{nj}$	$f_{1j}, f_{2j}, \dots, f_{nj}$	+1

What is the ranking?  
Algorithm?

66

### Testing

for each binary example  $e_{jk}$ :

label[j] +=  $f_{jk}(e_{jk})$   
label[k] -=  $f_{jk}(e_{jk})$

rank according to label scores

$f_{1j}, f_{2j}, \dots, f_{nj}$	$f_{1j}, f_{2j}, \dots, f_{nj}$	-1
$f_{1j}, f_{2j}, \dots, f_{nj}$	$f_{1j}, f_{2j}, \dots, f_{nj}$	-1
$f_{1j}, f_{2j}, \dots, f_{nj}$	$f_{1j}, f_{2j}, \dots, f_{nj}$	+1
$f_{1j}, f_{2j}, \dots, f_{nj}$	$f_{1j}, f_{2j}, \dots, f_{nj}$	+1
$f_{1j}, f_{2j}, \dots, f_{nj}$	$f_{1j}, f_{2j}, \dots, f_{nj}$	-1
$f_{1j}, f_{2j}, \dots, f_{nj}$	$f_{1j}, f_{2j}, \dots, f_{nj}$	+1

67

### An improvement?

ranking 1	new examples	binary label
$f_{1j}, f_{2j}, \dots, f_{nj}$	$f_{1j}, f_{2j}, \dots, f_{nj}$	+1
$f_{1j}, f_{2j}, \dots, f_{nj}$	$f_{1j}, f_{2j}, \dots, f_{nj}$	+1
$f_{1j}, f_{2j}, \dots, f_{nj}$	$f_{1j}, f_{2j}, \dots, f_{nj}$	-1
$f_{1j}, f_{2j}, \dots, f_{nj}$	$f_{1j}, f_{2j}, \dots, f_{nj}$	+1
$f_{1j}, f_{2j}, \dots, f_{nj}$	$f_{1j}, f_{2j}, \dots, f_{nj}$	-1
$f_{1j}, f_{2j}, \dots, f_{nj}$	$f_{1j}, f_{2j}, \dots, f_{nj}$	-1

Are these two examples the same?

68

### Weighted binary classification

ranking 1	new examples	weighted label
$f_{1j}, f_{2j}, \dots, f_{nj}$	$f_{1j}, f_{2j}, \dots, f_{nj}$	+1
$f_{1j}, f_{2j}, \dots, f_{nj}$	$f_{1j}, f_{2j}, \dots, f_{nj}$	+2
$f_{1j}, f_{2j}, \dots, f_{nj}$	$f_{1j}, f_{2j}, \dots, f_{nj}$	-1
$f_{1j}, f_{2j}, \dots, f_{nj}$	$f_{1j}, f_{2j}, \dots, f_{nj}$	+1
$f_{1j}, f_{2j}, \dots, f_{nj}$	$f_{1j}, f_{2j}, \dots, f_{nj}$	-2
$f_{1j}, f_{2j}, \dots, f_{nj}$	$f_{1j}, f_{2j}, \dots, f_{nj}$	-1

Weight based on distance in ranking

69

### Weighted binary classification

The diagram illustrates the process of creating new examples from a single example. On the left, a single example  $f_1, f_2, \dots, f_n$  is shown with a blue box around the first two features and a red dashed box around the last two. An arrow points to a matrix of 'new examples' on the right. Each row in the matrix is a copy of the original example, but with different feature boxes and weights. The weights are: +1, +2, -1, +1, -2, -1. The weights are shown in red text.

In general can weight with any consistent distance metric  
 Can we solve this problem?

70

### Testing

If the classifier outputs a confidence, then we've learned a *distance measure* between examples

During testing we want to rank the examples based on the learned distance measure

Ideas?

71

### Testing

If the classifier outputs a confidence, then we've learned a *distance measure* between examples

During testing we want to rank the examples based on the learned distance measure

Sort the examples and use the output of the binary classifier as the similarity between examples!

72

### Ranking evaluation

	ranking	prediction
$f_1, f_2, \dots, f_n$	1	1
$f_1, f_2, \dots, f_n$	2	3
$f_1, f_2, \dots, f_n$	3	2
$f_1, f_2, \dots, f_n$	4	5
$f_1, f_2, \dots, f_n$	5	4

Ideas?

73

## Idea 1: accuracy

	ranking	prediction	
$f_1, f_2, \dots, f_n$	1	1	1/5 = 0.2
$f_1, f_2, \dots, f_n$	2	3	
$f_1, f_2, \dots, f_n$	3	2	
$f_1, f_2, \dots, f_n$	4	5	
$f_1, f_2, \dots, f_n$	5	4	

Any problems with this?

74

## Doesn't capture "near" correct

	ranking	prediction	prediction
$f_1, f_2, \dots, f_n$	1	1	1
$f_1, f_2, \dots, f_n$	2	3	5
$f_1, f_2, \dots, f_n$	3	2	4
$f_1, f_2, \dots, f_n$	4	5	3
$f_1, f_2, \dots, f_n$	5	4	2

1/5 = 0.2

75