Greedy Scheduling

https://cs.pomona.edu/classes/cs140/
Topics and Learning Objectives

- Introduce greedy algorithms
- Discuss the greedy scheduling algorithm
- Discuss exchange argument proofs

Exercise

- Greedy scheduling
Extra Resources

• Introduction to Algorithms, 3rd, chapter 16
Greedy Algorithms

• Iteratively make myopic (short-sighted) decisions and hope it works
• Never go back and recheck/reevaluate that you were correct

Contrasting with Divide and Conquer
• It is generally easier to create greedy algorithms (good and bad to this)
• It is typically easier to analyze greedy algorithms (no master theorem)
• It is often harder to prove/understand the correctness of greedy algorithms
• It is common for greedy algorithms to be incorrect
Greedy Algorithms

Proofs of correctness

• It can sometimes feel like more of an art than a science

1. Proof by induction on the greedy decision
2. Proof by induction on an exchange argument
   1. Either by contraction
   2. Or by exchanging with the optimal solution
3. Whatever works...
Example of a greedy algorithm

• We’ve seen one greedy algorithm before. What was it?

• What path length does Dijkstra’s output for $S \rightarrow W$?

• What is the correct shortest path length for $S \rightarrow W$?
Scheduling (ignoring concurrency)

You have a shared resource
For example, a processor
You have many jobs that need to use the resource

Each job $j$ has:
• A **Priority** $P_j$ that stands for the job’s importance
• A **Duration** $D_j$ that stands for the length of time to run the job

*In what sequence should we complete the jobs?*
Scheduling (without concurrency)

In what sequence should we complete the jobs?

• What is our criteria? What do we want to optimize?
• Let’s start by looking at job j’s completion time $C_j$
• Given three jobs: $D_1 = 1$, $D_2 = 2$, $D_3 = 3$
• What is the completion time for each if they are scheduled in order?

<table>
<thead>
<tr>
<th>Schedule</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

$P_i$, $D_i$, $C_i$
What is the completion time of Job 5?

On what does Job 5’s completion time depend?

\[ C_s = D_s + \sum D_i \]
Scheduling

Optimization objective: minimize the weighted sum of completion times

$$S_{\text{cost}} = \min \left[ \sum_{j=1}^{n} P_j C_j \right]$$

What is the weighted sum of completion times if we schedule the following jobs in order?

<table>
<thead>
<tr>
<th>Job</th>
<th>J₁</th>
<th>J₂</th>
<th>J₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration</td>
<td>D₁ = 1</td>
<td>D₂ = 2</td>
<td>D₃ = 3</td>
</tr>
<tr>
<td>Priority</td>
<td>P₁ = 3</td>
<td>P₂ = 2</td>
<td>P₃ = 1</td>
</tr>
</tbody>
</table>
Job_3

Job_1

Job_2

Priority

duration

Time
Exercise Question 1, 2, and 3
Scheduling

Calculate the weighted sum of completion times for the following jobs if they are scheduled in the order: 1, 2, 3.

<table>
<thead>
<tr>
<th>Job</th>
<th>J₁</th>
<th>J₂</th>
<th>J₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration</td>
<td>D₁ = 1</td>
<td>D₂ = 2</td>
<td>D₃ = 3</td>
</tr>
<tr>
<td>Priority</td>
<td>P₁ = 3</td>
<td>P₂ = 2</td>
<td>P₃ = 1</td>
</tr>
<tr>
<td>Completion</td>
<td>1</td>
<td>1+2</td>
<td>1+2+3=6</td>
</tr>
<tr>
<td>Weight</td>
<td>3</td>
<td>6</td>
<td>≈ 6</td>
</tr>
</tbody>
</table>

Weighted sum of completion times: ?
Greedy Scheduling

Our process for creating a greedy scheduling algorithm

1. Look at some special cases for our problem
2. Describe some possible greedy criteria
3. Compare our greedy criteria
4. Select the “best” one
5. Prove correctness if possible
Greedy Scheduling

Goal: devise a greedy algorithm to minimize the weighted sum of completion times

Why are we approaching this problem with a greedy algorithm?
• It’s a pretty easy way to start.
• Compare the approach we go through in these slides with a Divide and Conquer approach
1. What are some special cases to consider?

Consider two jobs with equal durations \( (D) \):
- These jobs have different priorities \( (P_H \text{ and } P_L) \).
- Do we schedule the lower or higher priority job first?

<table>
<thead>
<tr>
<th>Job ( \text{H} )</th>
<th>Job ( \text{L} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Higher Priority)</td>
<td></td>
</tr>
</tbody>
</table>
1. What are some special cases to consider?

Consider two jobs with equal durations (D)

• These jobs have different priorities ($P_H$ and $P_L$)

• Do we schedule the lower or higher priority job first?

$$WSCT = D_L \cdot P_L + (D_L + D_H) \cdot P_H$$
1. What are some special cases to consider?

Consider **two** jobs with **equal durations** (D)

- These jobs have different **priorities** (PH and PL)
- **Do we schedule the lower or higher priority job first?**

\[ WSCT = D_H \cdot P_H + (D_H + D_L) \cdot P_L \]
Schedule with Lower Priority First

\[ D_L \cdot P_L + (D_L + D_H) \cdot P_H \ ? \ D_H \cdot P_H + (D_H + D_L) \cdot P_L \]

\[ D_L \cdot P_L + D_L \cdot P_H + D_H \cdot P_H ? \ D_H \cdot P_H + D_H \cdot P_L + D_L \cdot P_L \]

\[ D_L \cdot P_H ? \ D_H \cdot P_L \]

\[ P_H > P_L \]

Schedule with Higher Priority First
1. What are some special cases to consider?

Consider two jobs with equal durations (D)
• These jobs have different priorities ($P_H$ and $P_L$)
• Do we schedule the lower or higher priority job first?

<table>
<thead>
<tr>
<th>Time</th>
<th>Job\textsubscript{m} (Higher Priority)</th>
<th>Job\textsubscript{k}</th>
</tr>
</thead>
</table>

Schedule higher priority jobs first so that they have the quickest completion times.
1. What are some special cases to consider?

Consider two jobs with equal priorities (P)

• These jobs have different durations ($D_E$ and $D_S$)

• Do we schedule the shorter or longer (Extended) job first?

\[ \text{Job}_E \quad \text{(Longer/Extended)} \]

\[ \text{Job}_S \]
Consider two jobs with equal priorities (P)

- These jobs have different durations ($D_E$ and $D_S$)
- **Do we schedule the shorter or longer (Extended) job first?**
1. What are some special cases to consider?

Consider two jobs with equal priorities (P)

- These jobs have different durations ($D_E$ and $D_S$)
- Do we schedule the shorter or longer (Extended) job first?
Schedule with Shorter Job First

\[ P \cdot D_S + P(D_S + D_E) > P(D_E + P(D_S + D_E)) \]

\[ P \cdot D_S + P(D_S + P_D_E) > P(D_E + P_D_S + P_D_E) \]

\[ P \cdot D_S > P \cdot D_E \]

\[ D_S < D_E \]
1. What are some special cases to consider?

Consider two jobs with equal priorities (P)

• These jobs have different durations ($D_E$ and $D_S$)

• Do we schedule the shorter or longer (Extended) job first?

Schedule shortest jobs first to minimize average completion times.
Break Video
2. Describe some possible greedy criteria

What do we do when in the more general case:

\[ P_i > P_j \text{ and } D_i > D_j \] (job \( i \) has higher priority and longer duration)

What are some simple **scoring functions** that **aggregate** our criteria?

We want a function for which jobs with a bigger score are scheduled first:

- Score increases for higher priorities
- Score increases for shorter times

1. **Greedy Criterion 1:** \( P_i - D_i \) (take the difference)

2. **Greedy Criterion 2:** \( \frac{P_i}{D_i} \) (take the ratio)
3. Compare our greedy criteria

• Jobs will be ordered from biggest to smallest value

<table>
<thead>
<tr>
<th>Job with same duration</th>
<th>Difference Metric ((P_i - D_i))</th>
<th>Ratio Metric ((P_i/D_i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Job 1: (P=2), (D=1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Job 2: (P=5), (D=1)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Which job should be scheduled first?
3. Compare our greedy criteria

- Jobs will be ordered from biggest to smallest value

<table>
<thead>
<tr>
<th>Job with same duration</th>
<th>Difference Metric ( (P_i - D_i) )</th>
<th>Ratio Metric ( (P_i/D_i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Job 1: P=2, D=1</td>
<td>( 2 - 1 = 1 )</td>
<td>( \frac{2}{1} = 2 )</td>
</tr>
<tr>
<td>Job 2: P=5, D=1</td>
<td>( 5 - 1 = 4 )</td>
<td>( \frac{5}{1} = 5 )</td>
</tr>
<tr>
<td>Highest priority</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total weighted sum</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. Compare our greedy criteria

- Jobs will be ordered from biggest to smallest value

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<tbody>
<tr>
<td>Job 1: (P=2, D=1)</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Job 2: (P=5, D=1)</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Total weighted sum</td>
<td>(5<em>1 + 2</em>2 = 9)</td>
<td>(5<em>1 + 2</em>2 = 9)</td>
</tr>
</tbody>
</table>

Highest priority

<table>
<thead>
<tr>
<th>Job with same duration</th>
<th>Difference Metric ((P_i - D_i))</th>
<th>Ratio Metric ((P_i/D_i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Job 1: (P=1, D=3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Job 2: (P=1, D=4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total weighted sum</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Which job should be scheduled first?  

Same Result
3. Compare our greedy criteria

- Jobs will be ordered from biggest to smallest value

<table>
<thead>
<tr>
<th>Job with same duration</th>
<th>Difference Metric ($P_i - D_i$)</th>
<th>Ratio Metric ($P_i/D_i$)</th>
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</thead>
<tbody>
<tr>
<td>Job 1: $P=2$, $D=1$</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Job 2: $P=5$, $D=1$</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Total weighted sum</td>
<td>$5<em>1 + 2</em>2 = 9$</td>
<td>$5<em>1 + 2</em>2 = 9$</td>
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</table>

Highest priority

<table>
<thead>
<tr>
<th>Job with same duration</th>
<th>Difference Metric ($P_i - D_i$)</th>
<th>Ratio Metric ($P_i/D_i$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Job 1: $P=1$, $D=3$</td>
<td>-2</td>
<td>1/3</td>
</tr>
<tr>
<td>Job 2: $P=1$, $D=4$</td>
<td>-3</td>
<td>1/4</td>
</tr>
<tr>
<td>Total weighted sum</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Shortest time

Which job should be scheduled first?
3. Compare our greedy criteria

- Jobs will be ordered from biggest to smallest value

<table>
<thead>
<tr>
<th>Job with same duration</th>
<th>Difference Metric (P_i - D_i)</th>
<th>Ratio Metric (P_i/D_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Job 1: (P=2, D=1)</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Job 2: (P=5, D=1)</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Total weighted sum</td>
<td>(5<em>1 + 2</em>2 = 9)</td>
<td>(5<em>1 + 2</em>2 = 9)</td>
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</table>

Highest priority

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<tr>
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<th>Ratio Metric (P_i/D_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Job 1: (P=1, D=3)</td>
<td>-2</td>
<td>1/3</td>
</tr>
<tr>
<td>Job 2: (P=1, D=4)</td>
<td>-3</td>
<td>1/4</td>
</tr>
<tr>
<td>Total weighted sum</td>
<td>(1<em>3 + 1</em>7 = 10)</td>
<td>(1<em>3 + 1</em>7 = 10)</td>
</tr>
</tbody>
</table>

Shortest time

Which job should be scheduled first?

Same Result
3. Compare our greedy criteria

• Let’s try to get them to disagree.
• Why does it matter if they don’t produce the same result?
• One scoring metric must be better than the other

• Apply the two greedy algorithms and calculate their weighted sum of completion times
3. Compare our greedy criteria

- Jobs will be ordered from biggest to smallest metric value

<table>
<thead>
<tr>
<th>Job with same duration</th>
<th>Difference Metric ((P_i - D_i))</th>
<th>Ratio Metric ((\frac{P_i}{D_i}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Job 1: P=3, D=5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Job 2: P=1, D=2</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total weighted sum</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. Compare our greedy criteria

- Jobs will be ordered from biggest to smallest metric value

<table>
<thead>
<tr>
<th>Job with same duration</th>
<th>Difference Metric ((P_i - D_i))</th>
<th>Ratio Metric ((P_i / D_i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Job 1: (P=3, D=5)</td>
<td>(3 - 5 = -2)</td>
<td>(3/5)</td>
</tr>
<tr>
<td>Job 2: (P=1, D=2)</td>
<td>(-1)</td>
<td>(1/2)</td>
</tr>
<tr>
<td>Total weighted sum</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Which job goes first?
3. Compare our greedy criteria

• Jobs will be ordered from biggest to smallest metric value

<table>
<thead>
<tr>
<th>Job with same duration</th>
<th>Difference Metric ( (P_i - D_i) )</th>
<th>Ratio Metric ( (P_i/D_i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Job 1: ( P=3, D=5 )</td>
<td>-2</td>
<td>3/5</td>
</tr>
<tr>
<td>Job 2: ( P=1, D=2 )</td>
<td>-1</td>
<td>1/2</td>
</tr>
<tr>
<td>Total weighted sum</td>
<td>( 3 \cdot 5 + 1 \cdot 1.7 = 23 )</td>
<td>( 3.5 + 1.7 = 22 )</td>
</tr>
</tbody>
</table>

Which job goes first?

What is the priority sum?
4. Select the “best” one

- Jobs will be ordered from biggest to smallest metric value

<table>
<thead>
<tr>
<th>Job with same duration</th>
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<th>Ratio Metric ((P_i/D_i))</th>
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</thead>
<tbody>
<tr>
<td>Job 1: P=3, D=5</td>
<td>-2</td>
<td>3/5</td>
</tr>
<tr>
<td>Job 2: P=1, D=2</td>
<td>-1</td>
<td>1/2</td>
</tr>
<tr>
<td>Total weighted sum</td>
<td>1<em>2 + 3</em>7 = 23</td>
<td>3<em>5 + 1</em>7 = 22</td>
</tr>
</tbody>
</table>

Which job goes first?

What is the priority sum?

Which criteria is better?
5. Prove correctness if possible

Is criteria 2 optimal?
• We don’t know yet.

Claim: Criteria 2 is optimal for minimizing the weighted sum of completion times.

• We’re going to prove this using an exchange argument!
Proof

• Assume that we have no ties (all $P_i/D_i$ are distinct numbers)
• Fix an arbitrary input with $n$ jobs
• Let’s perform a proof using an exchange argument contradiction

Let $\sigma = \text{the greedy schedule}$ and $\sigma^* = \text{the optimal schedule}$

• Let's assume that $\sigma^*$ must be better than $\sigma$ (assume greedy is not optimal)
• To perform the contradiction, we must show that $\sigma$ is better than $\sigma^*$, thus contradicting the purported optimality of $\sigma^*$
Proof

Let \( \sigma = \text{the greedy schedule} \) and \( \sigma^* = \text{the optimal schedule} \)

- Assume that: \( P_1/D_1 > P_2/D_2 > \cdots > P_n/D_n \)
- We can just rename all jobs after we calculate their scores...
- Thus, \( \sigma \) is just job 1 followed by job 2 etc. \((1, 2, \ldots, n)\)
Proof

Let $\sigma =$ the greedy schedule and $\sigma^* =$ the optimal schedule

• Assume that: $P_1/D_1 > P_2/D_2 > \ldots > P_n/D_n$
• We can just rename all jobs after we calculate their scores...
• Thus, $\sigma$ is just job 1 followed by job 2 etc. (1, 2, ..., n)
• For $\sigma^*$ there must be at least two jobs that are “out of order”
  • Specifically, jobs $i$ and $j$ where $i$ is scheduled after $j$, but $S_i > S_j$ (for example, Job$_5$ after Job$_6$)
• The greedy schedule is the only schedule where the jobs are in order
<table>
<thead>
<tr>
<th>Jobs</th>
<th>Schedule</th>
<th>Time</th>
<th>Before</th>
<th>j</th>
<th>i</th>
<th>After</th>
</tr>
</thead>
<tbody>
<tr>
<td>i and j</td>
<td>$\sigma^*$ Schedule</td>
<td>Before j</td>
<td>i</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>After</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>exchange</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>i and j</td>
<td>$\sigma$ Schedule</td>
<td>Before i</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>After</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ordered</td>
<td>based on greedy scores</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For example, $i=7$ and $j=8$
How does the exchange affect the completion time for:
1. Jobs other than i and j?
2. Job i
3. Job j

For example, i=7 and j=8
**σ** VS **σ**

(jobs i and j where i is scheduled after j, but \(P_i/D_i > P_j/D_j\))

Job i has a larger greedy score

**σ** Schedule

Before

\[\begin{array}{c}
\text{i} \\
\text{j}
\end{array}\]

After

\[\begin{array}{c}
\text{i} \\
\text{j}
\end{array}\]

Ordered based on greedy scores

**σ** Schedule

Before

\[\begin{array}{c}
\text{i} \\
\text{j}
\end{array}\]

After

\[\begin{array}{c}
\text{i} \\
\text{j}
\end{array}\]

What is the weighted sum of completion times for each schedule?

For example, i=7 and j=8
\[
\text{Cost}(\sigma^*) = \text{Cost}(\text{Before}) + P_j \times (T_b + D_j) + P_i \times (T_b + D_j + D_i) + \text{Cost}(\text{After})
\]

\[
\text{Cost}(\sigma) = \text{Cost}(\text{Before}) + P_i \times (T_b + D_i) + P_j \times (T_b + D_i + D_j) + \text{Cost}(\text{After})
\]

\[
\text{Cost}(\sigma^*) < \text{Cost}(\sigma) \quad \text{Implied by optimality of } \sigma^*
\]

\[
\text{Cost}(\text{Before}) + P_j \times (T_b + D_j) + P_i \times (T_b + D_j + D_i) + \text{Cost}(\text{After}) < \text{Cost}(\text{Before}) + P_i \times (T_b + D_i) + P_j \times (T_b + D_i + D_j) + \text{Cost}(\text{After})
\]
Cost(\(\sigma^*\)) = Cost(Before) + P_j \times (T_b + D_j) + P_i \times (T_b + D_j + D_i) + Cost(After)

Cost(\(\sigma\)) = Cost(Before) + P_i \times (T_b + D_i) + P_j \times (T_b + D_i + D_j) + Cost(After)

Cost(\(\sigma^*\)) < Cost(\(\sigma\))  \hspace{1cm} \text{Implied by optimality of } \sigma^*

\begin{align*}
\text{Cost(Before)} + P_j \times (T_b + D_j) + P_i \times (T_b + D_j + D_i) + \text{Cost(After)} \\
< \text{Cost(Before)} + P_i \times (T_b + D_i) + P_j \times (T_b + D_i + D_j) + \text{Cost(After)}
\end{align*}

\begin{align*}
P_j \times (T_b + D_j) + P_i \times (T_b + D_j + D_i) \\
< P_i \times (T_b + D_i) + P_j \times (T_b + D_i + D_j)
\end{align*}

\begin{align*}
P_j \times T_b + P_j \times D_j + P_i \times T_b + P_i \times D_j + P_i \times D_i \\
< P_i \times T_b + P_i \times D_i + P_j \times T_b + P_j \times D_i + P_j \times D_j
\end{align*}
\[
\text{Cost}(\sigma^*) = \text{Cost}(\text{Before}) + P_j \times (T_b + D_j) + P_i \times (T_b + D_j + D_i) + \text{Cost}(\text{After}) \\
\text{Cost}(\sigma) = \text{Cost}(\text{Before}) + P_i \times (T_b + D_i) + P_j \times (T_b + D_i + D_j) + \text{Cost}(\text{After})
\]

\[
\text{Cost}(\sigma^*) < \text{Cost}(\sigma) \quad \text{Implied by optimality of } \sigma^*
\]

\[
\text{Cost}(\text{Before}) + P_j \times (T_b + D_j) + P_i \times (T_b + D_j + D_i) + \text{Cost}(\text{After}) < \text{Cost}(\text{Before}) + P_i \times (T_b + D_i) + P_j \times (T_b + D_i + D_j) + \text{Cost}(\text{After})
\]

\[
P_j \times (T_b + D_j) + P_i \times (T_b + D_j + D_i) < P_i \times (T_b + D_i) + P_j \times (T_b + D_i + D_j)
\]

\[
P_j \times T_b + P_j \times D_j + P_i \times T_b + P_i \times D_j + P_i \times D_i < P_i \times T_b + P_i \times D_i + P_j \times T_b + P_j \times D_i + P_j \times D_j
\]

\[
P_i \times D_j < P_j \times D_i
\]

\[
\frac{P_i}{D_i} < \frac{P_j}{D_j}
\]

Contradiction to how they were ordered by our greedy criteria
Multiple Re-orderings

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th>i</th>
<th></th>
<th></th>
<th>j</th>
</tr>
</thead>
<tbody>
<tr>
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<td>2</td>
<td>4</td>
<td>3</td>
<td>5</td>
<td>6</td>
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<td>2</td>
<td>4</td>
<td>3</td>
<td>5</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

Our proof doesn’t account for this
## Multiple Re-orderings

<table>
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<tr>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>i</td>
<td>j</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>i</td>
<td>j</td>
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<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
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</tbody>
</table>
Example with Randomly Generated Jobs

<table>
<thead>
<tr>
<th>Job ID</th>
<th>Weight</th>
<th>Length</th>
<th>Ratio</th>
<th>Reorder</th>
<th>Greedy Time</th>
<th>Weighted</th>
<th>Unoptimized Time</th>
<th>Weighted</th>
</tr>
</thead>
<tbody>
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<td>1</td>
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<td>4</td>
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<td>14</td>
<td>14</td>
<td>4</td>
<td>4</td>
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<td>8</td>
<td>6</td>
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<td>80</td>
<td>10</td>
<td>80</td>
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<tr>
<td>3</td>
<td>6</td>
<td>1</td>
<td>6.0</td>
<td>1</td>
<td>1</td>
<td>6</td>
<td>11</td>
<td>66</td>
</tr>
<tr>
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<td>1</td>
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<tr>
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<td>1</td>
<td>9</td>
<td>0.1</td>
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<td>2</td>
<td>4</td>
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<td>28</td>
<td>196</td>
</tr>
</tbody>
</table>

**Total for Greedy**: 175
**Total for Unoptimized**: 387
Summary of Greedy Scheduling

• Given $n$ jobs, each with a priority and a duration
• Give each job a score based on their ratio of priority to duration
• Schedule jobs in decreasing order of their score
• This gives us an optimal schedule

• What do we do if we’re given more jobs while these are running?
• Any issues with this scheme?
  • Some jobs might always be postponed.