Breadth First Search

https://cs.pomona.edu/classes/cs140/
Outline

Topics and Learning Objectives
• Discuss breadth first search for graphs

Exercises
• Continued from previous lecture slides
• Compute distance with Breadth-first search
Extra Resources

• Introduction to Algorithms, 3rd, Chapter 22
• Algorithms Illuminated Part 2: Chapter 8
General Algorithm

FUNCTION Connectivity(G, start_vertex)
    found = {v: FALSE FOR v IN G.vertices}
    found[start_vertex] = TRUE
    LOOP
        (vFound, vNotFound) = get_valid_edge(G.edges, found)
        IF vFound == NONE || vNotFound == NONE
            BREAK
        ELSE
            found[vNotFound] = TRUE
    RETURN found

Find an edge where one vertex has been found and the other vertex has not been found.
How do we choose the next edge?
Two common (and well studied) options

Breadth-First Search
• Explore the graph in layers
• “Cautious” exploration
• Use a FIFO data structure (can you think of an example?)

Depth-First Search
• Explore recursively
• A more “aggressive” exploration (we backtrack if necessary)
• Use a LIFO data structure (or recursion)
FUNCTION BFS(G, start_vertex)
    found = {v: FALSE FOR v IN G.vertices}
    found[start_vertex] = TRUE
    visit_queue = [start_vertex]

    WHILE visit_queue.length != 0
        vFound = visit_queue.pop()
        FOR vOther IN G.edges[vFound]
            IF found[vOther] == FALSE
                found[vOther] = TRUE
                visit_queue.add(vOther)

    RETURN found

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    RETURN found
Given a tie, visit edges are in alphabetical order

Exercise questions 2 and 3

FUNCTION BFS(G, start_vertex)
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visit_queue = [start_vertex]

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Running Time

```python
FUNCTION BFS(G, start_vertex)
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        vFound = visit_queue.pop()
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            IF found[vOther] == FALSE
                found[vOther] = TRUE
                visit_queue.add(vOther)

    RETURN found
```

What is the running time?

\[ T_{BFS}(n, m) = O(n_s + m_s) \]

How many times to we consider each edge?

where \( n_s \) and \( m_s \) are the nodes and edges findable/connected from/to the start vertex
Proof: BFS

**Claim:** BFS finds all nodes connected to the start node.

At the end of the BFS algorithm, \( v \) is marked found if there exists a path from \( s \) to \( v \)

- Note: this is just a special case of the general algorithm that we proved by contradiction

Practice for a loop invariant

Homework question
Question

The Shortest Path Problem

• How can we determine the fewest number of hops between the start vertex and all other connected vertices?
How can we determine the fewest number of hops between the start vertex and all other connected vertices?

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WHILE visit_queue.length != 0
    vFound = visit_queue.pop()
    FOR vOther IN G.edges[vFound]
        IF found[vOther] == FALSE
            found[vOther] = TRUE
            visit_queue.add(vOther)

RETURN found

Given a tie, visit edges are in alphabetical order
The Shortest Path Problem

Determine the fewest number of hops between the start vertex and all other vertices

Same algorithm as before with the following additions:

• Initialize the distances\[s\] as 0
• Initialize all other distances to infinity
• When considering an edge \((v, w)\)
  • If \(w\) is not found, then set \(\text{dist}(w)\) to \(\text{dist}(v) + 1\)
The Shortest Path Problem

After we terminate, distances[v] = “the layer that v is in”

Given a tie, visit edges are in alphabetical order
Connected Components

Let’s only consider undirected graphs for now

Let $G = (V, E)$ be an undirected graph

Goal: compute all connected components in $O(m + n)$

- A component is any group of vertices that can reach one another
- For example, if we are trying to see if a network has become disconnected

Exercise question 2:
How would you do this using our BFS procedure from before?
FUNCTION FindComponents(G)
    components = []
    found = {v: FALSE FOR v IN G.vertices}
    FOR v IN G.vertices
        IF NOT found[v]
            newly_found = BFS(G, v)
            new_component = {
                w FOR w, w_is_found IN newly_found
                    IF w_is_found
            }
            component.append(new_component)
    FOR w IN new_component:
        found[w] = TRUE
    RETURN components